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Estimating discretionary accruals in the cross-section of firms: A reinterpretation of the Jones model and its variants

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ABSTRACT

In this paper, we challenge a key underlying assumption of the Jones model that there is no information available about the cross-sectional distribution of the "true" profitability of firms. When we incorporate this neglected information into a Bayesian framework, the resulting model provides both a theoretical foundation and practical guidance for researchers on which variant of the Jones model is suitable in which setting. Specifically, we find that the appropriate use depends on the sampling of firms suspected of earnings management: (i) the modified Jones plus cash flow model for random sampling, (ii) the modified Jones plus profitability model for sampling on firms' reported earnings, and (iii) the modified Jones model for sampling on the Bayesian estimate of firms' true profitability.

1. Introduction

An implicit assumption of the Jones model is that there is no information available about the cross-sectional distribution of the "true" profitability of firms. This assumption is implausible, as in reality, firms' true profitability typically falls within a reasonable, predictable range. In a competitive economic environment, an extremely high or low profitability is rare, and many observations will be close to the cost of capital (see, e.g., Penman, 2011, p. 112; Palepu et al., 2022, p. 247). We posit that this prior information about the cross-sectional distribution of true profitability is useful for the decomposition of total accruals into normal and discretionary components.

To illustrate, consider the extreme example of a distribution with zero variance, i.e., true earnings scaled by total assets are constant at 10% for all firms in the sample. Then, by definition, true discretionary accruals are equal to reported earnings minus 10%, and true normal accruals are equal to 10% minus cash flows. Therefore, discretionary accruals have a cross-sectional correlation of 1 with reported earnings, and normal accruals have a cross-sectional correlation of −1 with cash flows. As a result, the sum of discretionary and normal accruals, i.e., total accruals, will also be positively related to earnings and negatively related to cash flows. Since the Jones model does not remove these cross-correlations, they will also appear in the residuals from the Jones regression.

In fact, these correlations are well-known stylized facts described in the literature (see Dechow et al., 2003 and Kothari et al., 2005 for earnings; and Dechow, 1994, Dechow et al., 1995, and Dechow et al., 1998 for cash flows). These stylized facts can impair the suitability of the residuals from the Jones model as a measure of earnings management. For example, positive residuals for IPO firms might indicate earnings management to achieve higher selling prices for the offered shares, but at the same time, positive residuals could also be due to the high profitability of IPO firms as a result of the earnings-related stylized fact. In such cases, it is not clear whether the Jones model is still appropriate for measuring earnings management. To address this issue, it is necessary to consider the source of the stylized facts by taking prior information about the cross-sectional distribution of true profitability into account.

In our model we assume a normal prior distribution of true profitability. The variance of this distribution determines the importance of the prior information. A very low variance corresponds to the extreme example above, while a very high variance renders the prior uninformative, so that no extension of the Jones model is required.

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Combining the prior information with earnings and cash flow information in a Bayesian framework, we derive the posterior distribution and the maximum likelihood estimator of true profitability, which then provides Bayesian estimators of normal and discretionary accruals.

An intriguing aspect of our model is that it can be linked to different extensions of the Jones model used in the earnings management literature. Breuer and Schütt (2023) identify 93 earnings management studies published between 2016 and 2020 in the leading accounting journals and find that 26 of these studies use the modified Jones model, 26 studies use the modified Jones model plus cash flow variables, and 42 studies add an additional control for profitability to the modified Jones model.[1] As prior literature provides no clear guidance on which setting requires the use of which variant of the Jones model to detect earnings management, researchers often use multiple models and robustness checks to mitigate concerns associated with any single measure. Although this practice reduces the risk of drawing inferences from a single (potentially biased) model variant, more guidance on which model variant is appropriate in which setting is useful. Our model implies that the most efficient way to detect earnings management depends on the sampling of firms suspected of earnings management. For random sampling, the modified Jones plus cash flow model is most efficient; for sampling with respect to reported earnings, the modified Jones plus profitability model should be used; and for sampling with respect to the Bayesian estimator of true profitability, the modified Jones model is appropriate.

In the empirical part of the paper, we test whether the recommendations derived from our model are useful in practice. It is important to note that throughout the empirical analysis, the term "true earnings" refers to a latent benchmark defined within our modeling framework. Although the concept is well defined in the model, its empirical counterpart is obtained under specific distributional and structural assumptions. Accordingly, our empirical estimates should be interpreted as model-implied measures.

We first estimate the model parameters using Compustat data for a comprehensive sample of US firms from 1987 to 2022. We then simulate the three methods of sampling of treatment firms and infuse different degrees of earnings management into the earnings of these firms. Finally, we examine the performance of different variants of the Jones model in detecting the injected earnings management. The results of our simulations and empirical analyses are consistent with our model: for random sampling, the model variants perform similarly, while for the other two sampling methods, the differences in performance are substantial, and the use of an inappropriate model variant could lead to biased results.

Our paper contributes to the literature on empirical measures of earnings quality and earnings discretion. Specifically, our findings complement previous approaches that use accounting identities to remove biases due to accrual estimation errors. The basis of these approaches is the (firm-level) accrual quality model of Dechow and Dichev (2002), which explains why accruals are negatively related to contemporaneous cash flows. The intuition of the model is that if a firm has constant predetermined earnings, the variation in accruals is entirely due to the variation in cash flows, so that accruals and cash flows are negatively correlated with a coefficient of minus one (Larson et al., 2018, p. 842). We transfer this intuition to the cross-section of firms and extend the model by making a more realistic assumption about the distribution of true profitability.

Building on Dechow and Dichev (2002), Lewellen and Resutek (2019) and Rountree et al. (2023) examine why accruals are less persistent than cash flows. One explanation originally proposed by Sloan (1996) is that measurement error in accruals makes accruals less persistent because the error component tends to reverse over time. This negative correlation gives researchers a lever to correct for the persistence bias. In this way, Lewellen and Resutek (2019) are able to test the measurement-error explanation for the low persistence of accruals against other explanations, and Rountree et al. (2023) use a similar approach to obtain an unbiased estimate of the persistence of true earnings. The main difference in our approach is that we focus on cross-sectional information to improve estimates of true earnings rather than (firm-level) time-series information from error reversals.

In a similar setup with unobservable true performance as we use, Nikolaev (2018) proposes a measure of accounting quality that corresponds to the variance ratio of two accrual components: (i) the performance measurement component, which is defined as the difference of cash flows to unobservable performance, and (ii) the accounting error component, which is defined as the difference of economic performance to earnings. While Nikolaev (2018) applies the framework at the firm level, our paper exploits cross-sectional variation of accruals. Therefore, the main elements of our paper are distinct from the study by Nikolaev (2018), namely the idea of prior information about economic performance, its relevance to the stylized facts, the Bayesian estimation of accrual components, and the resulting suggestions for identifying earnings management.

Finally, our paper contributes to the literature on Bayesian methods in accounting research and thus responds to the call for greater use of these methods by Breuer and Schütt (2023). They use Bayesian methods to obtain better estimates of normal accruals from the modified Jones model, considering uncertainty about parameter heterogeneity (at the industry-year level vs. firm level) and about the appropriate set of controls in the Jones regression. Breuer and Schütt (2023) find that Bayesian model averaging leads to superior estimates of normal accruals. Our application is different in that we address a specific bias, i.e., the stylized facts, based on specific prior information, i.e., the cross-sectional distribution of true earnings. Incorporating this prior information helps to identify the appropriate model variant. This is complementary to Breuer and Schütt (2023) because model averaging can then be based on parameter heterogeneity and different sets of controls for the chosen model variant.

The rest of the paper is structured as follows. Section 2 derives the model. Section 3 describes the estimation method and the data. Section 4 presents our empirical results. Section 5 discusses how to determine the appropriate variant of the Jones model in practical applications, and Section 6 concludes the paper.

2. Bayesian model

2.1. Variables

We use NI for reported earnings, CF for cash flow, and $TA = NI - CF$ for total accruals. The variable name NI is an acronym for net income, but our framework is also consistent with other measures of earnings.

The key element of our model is the introduction of unobservable true earnings, NI^T. These earnings are "true" in the sense that they are unambiguously defined by the applicable accounting standards.[2] If the accounting standards are not complete, we can think of NI^T as the profit or loss resulting from the most appropriate accounting judgment by a panel of neutral accounting experts.

Discretionary accruals are defined as the deviations of reported earnings from true earnings, expressed as $DA = NI - NI^T$. These deviations arise due to accounting errors and management discretion including earnings management. Similarly, normal accruals are defined

[1] The authors include studies published in *The Accounting Review*, *Journal of Accounting Research*, *Journal of Accounting and Economics*, *Review of Accounting Studies* and *Contemporary Accounting Research*.

[2] This concept corresponds to "true" or "fundamental" earnings as used by Lewellen and Resutek (2019) and Rountree et al. (2023).

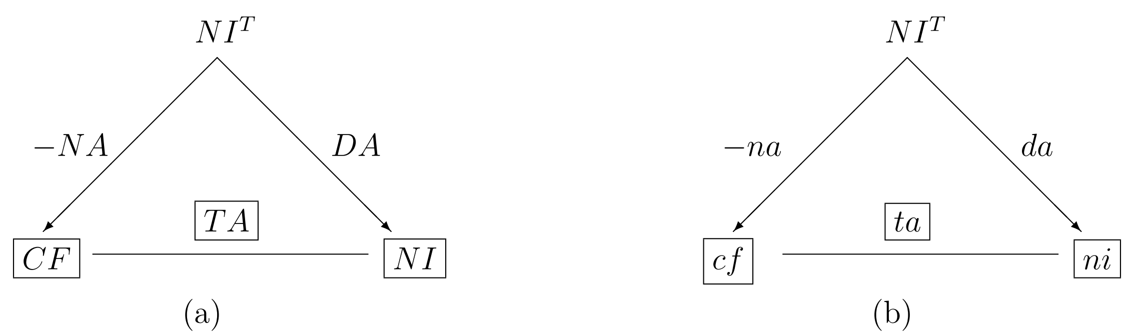

Fig. 1. Model variables and their relationships.
Total accruals (TA) are defined as reported earnings (NI) minus cash flow (CF). True earnings (NI^T) are "true" in the sense that they are unambiguously defined by the applicable accounting standards. As shown in figure (a), when true earnings (NI^T) are known, total accruals can be decomposed into discretionary accruals ($DA = NI - NI^T$) and normal accruals ($NA = NI^T - CF$) so that $TA = NA + DA$. The outside border around TA, CF, and NI indicates that the values of these variables are observable. NI^T, NA, and DA are unobservable and must be estimated. Figure (b) shows the same relationships for demeaned accruals: $na = NA - \mu_{NA}$ and $da = DA - \mu_{DA}$, where μ_{NA} and μ_{DA} are the expected values of normal and discretionary accruals. Therefore, $ni = NI^T + da = NI - \mu_{DA}$; $cf = NI^T - na = CF + \mu_{NA}$; and $ta = ni - cf = TA - \mu_{NA} - \mu_{DA}$.

as the deviations of true earnings from cash flows, given by $NA = NI^T - CF$. By definition, a firm can influence normal accruals only by altering cash flows and the corresponding operating activities. If such alterations are intended to manage earnings, they are classified as "real" earnings management.

Fig. 1(a) illustrates the relationships among the relevant variables. Panel (b) presents the same structure but for demeaned accruals, i.e., normal and discretionary accruals adjusted for their expected values, μ_{NA} and μ_{DA}, respectively. While the two structures (a) and (b) are equivalent, the demeaned version offers the advantage of clearly separating the analysis into two stages.

In the first stage, we estimate μ_{NA} and μ_{DA} and then demean the accrual variables accordingly. For this purpose, we use the modified Jones model in our empirical analysis. Under this model, μ_{NA} is a function of PPE and changes in cash revenues, and $\mu_{DA} = 0$, since the expected value of the error term of the Jones regression is zero. Alternative models from the literature could also be employed to estimate μ_{NA} and μ_{DA}. For instance, μ_{DA} may be non-zero to reflect reversals of prior discretionary accruals. As we do not contribute to the estimation of these expectations, we take the estimation model of the first step as given and then work with demeaned variables in the second step. In the first step, μ_{NA} and μ_{DA} can be firm-specific (as in the Jones model, where μ_{NA} is determined by firm-specific PPE and changes in cash revenues).

The second stage, based on the demeaned framework in Fig. 1(b), constitutes our Bayesian analysis. This step demonstrates how our approach extends beyond the traditional Jones model. We denote demeaned variables using lowercase letters: $na = NA - \mu_{NA}$ and $da = DA - \mu_{DA}$; net income becomes $ni = NI^T + da = NI - \mu_{DA}$; cash flow is $cf = NI^T - na = CF + \mu_{NA}$; and total accruals are $ta = ni - cf = TA - \mu_{NA} - \mu_{DA}$. We model the cross-sectional distributions of these variables, which aligns with the standard implementation of the Jones model as a cross-sectional regression.

2.2. General Gaussian updating problem

The structure illustrated in Fig. 1(b) can be interpreted as a signal model, where ni and cf are signals of unobservable true earnings NI^T. If the variables follow a multivariate normal distribution, the signal model is identical to the general Gaussian updating problem, and the posterior distribution of $NI^T \mid ni, cf$ is known from the literature (Murphy, 2007; Särkkä & Svensson, 2023).

Let:

$$\begin{bmatrix} NI^T \\ cf \\ ni \end{bmatrix} \sim \mathcal{N}\left(\begin{bmatrix} \mu_0 \\ \mu_0 \\ \mu_0 \end{bmatrix}, \begin{bmatrix} \sigma_0^2 & \Sigma_1^\top \\ \Sigma_1 & \Sigma_2 \end{bmatrix}\right) \quad (1)$$

where:

- μ_0 and σ_0^2 are the expected value and variance of NI^T (prior distribution),
- $\Sigma_1 = \begin{bmatrix} \mathrm{Cov}(cf, NI^T) \\ \mathrm{Cov}(ni, NI^T) \end{bmatrix}$ is the vector of covariances of the signals with NI^T,
- $\Sigma_2 = \begin{bmatrix} \sigma_{cf}^2 & \mathrm{Cov}(cf, ni) \\ \mathrm{Cov}(ni, cf) & \sigma_{ni}^2 \end{bmatrix}$ is the signal covariance matrix.

Note that the expected values of cf and ni are equal to μ_0 because we work with demeaned accruals.

Then, the posterior distribution of $NI^T \mid cf, ni$ is again normal:

$$NI^T \mid cf, ni \sim \mathcal{N}\left(\mu_0 + \Sigma_1^\top \Sigma_2^{-1}\left(\begin{bmatrix} cf \\ ni \end{bmatrix} - \begin{bmatrix} \mu_0 \\ \mu_0 \end{bmatrix}\right), \quad \sigma_0^2 - \Sigma_1^\top \Sigma_2^{-1} \Sigma_1\right). \quad (2)$$

2.3. Special case of independent signals

2.3.1. Additional assumption

In this paper, we examine a special case of the general Gaussian updating problem by introducing the additional assumption that there is no specific information indicating a relationship between (demeaned) accrual components and true earnings, or between normal and discretionary accruals. Formally, we assume:

$$Cov\left(na, NI^T\right) = Cov\left(da, NI^T\right) = Cov\left(na, da\right) = 0. \quad (3)$$

We impose these restrictions for two main reasons. First, they are implicitly embedded in the modified Jones model. Maintaining these assumptions allows us to isolate the effect of interest, namely, how the prior distribution of true earnings affects the decomposition. In the Jones model, normal accruals are determined entirely by PPE and changes in cash revenues. The residual component of total accruals is considered discretionary. As a result, there is no inherent correlation between the demeaned normal and discretionary accruals. Moreover, since NI^T is not explicitly modeled in the Jones model, there is no built-in correlation between NI^T and either na or da. Second, non-zero covariances would indicate that predictable components remain in the accrual measures after the first step. For example, if normal accruals are systematically correlated with true earnings, this predictable pattern could, in principle, be incorporated into the first-step regression. Thus, our assumption is not that such correlations cannot exist, but rather that they are addressed, if present, through the specification of the accrual model in the first step.

2.3.2. Implications

We highlight four implications of our special setting for variances and covariances of core variables.

First, assumption (3) simplifies the calculation of the variances of cash flow and reported earnings since the covariances can be ignored. The unconditional probability distributions are:

$$cf \sim \mathcal{N}\left(\mu_0, \sigma_0^2 + \sigma_{na}^2\right), \tag{4}$$

$$ni \sim \mathcal{N}\left(\mu_0, \sigma_0^2 + \sigma_{da}^2\right). \tag{5}$$

Eqs. (4) and (5) characterize the theoretical (population-level) cross-sectional distributions of cash flows and earnings. They result from a two-step procedure applied to each firm: first, the firm's true earnings are determined as a random draw from the prior distribution, and second, normal and discretionary accruals are randomly drawn, allowing cash flows and earnings to be determined as $cf = NI^T - na$ and $ni = NI^T + da$, respectively.

Second, cash flows and reported earnings in the cross-section of firms are positively related with a covariance of:

$$Cov(cf, ni) = Cov\left(NI^T - na, NI^T + da\right) = \sigma_0^2. \tag{6}$$

This positive relationship arises because both variables are derived from NI^T. The greater the uncertainty of NI^T (measured by σ_0^2), the more important the common source of uncertainty in cf and ni. Conversely, when σ_0^2 approaches zero, the covariance between cf and ni disappears because the remaining sources of uncertainty in cf and ni are the two accrual components na and da, respectively, which are independent.

Third, normal accruals are negatively related to cash flows, and discretionary accruals are positively related to earnings:

$$Cov(na, cf) = Cov\left(na, NI^T - na\right) = -\sigma_{na}^2, \tag{7}$$

$$Cov(da, ni) = Cov\left(da, NI^T + da\right) = \sigma_{da}^2, \tag{8}$$

$$Cov(na, ni) = Cov(da, cf) = 0. \tag{9}$$

The reason is that for a given NI^T, higher normal accruals are, by definition, associated with lower cash flows, and higher discretionary accruals are, by definition, associated with higher reported earnings.

Fourth, these associations are maintained when forming the sum of normal and discretionary accruals. Therefore, total accruals are also negatively related to cash flows and positively related to earnings:

$$Cov(ta, cf) = Cov(ni - cf, cf) = \sigma_0^2 - \sigma_{cf}^2 = -\sigma_{na}^2, \tag{10}$$

$$Cov(ta, ni) = Cov(ni - cf, ni) = \sigma_{ni}^2 - \sigma_0^2 = \sigma_{da}^2. \tag{11}$$

This fourth implication replicates the stylized facts mentioned in the introduction. Our framework shows that these stylized facts do not require earnings management, reversal patterns of accruals over time or causal effects of profitability (e.g., through superior growth of profitable firms leading to growing accruals). They arise naturally when true earnings in the cross-section of firms have a more or less narrow distribution. The theoretical (population-level) regression of ta on cf has a slope coefficient of $Cov(ta, cf)/\sigma_{cf}^2 = -\sigma_{na}^2/\left(\sigma_0^2 + \sigma_{na}^2\right)$, and the theoretical regression of ta on ni has a slope coefficient of $Cov(ta, ni)/\sigma_{ni}^2 = \sigma_{da}^2/\left(\sigma_0^2 + \sigma_{da}^2\right)$. Thus, for the special case of known NI^T ($\sigma_0^2 \to 0$), the slopes are equal to −1 and 1, respectively. The less precise the prior information (i.e., the higher σ_0^2), the more the slope coefficients approach zero.

2.3.3. Bayesian estimates of discretionary and normal accruals

We can summarize our signal model as follows:

$$NI^T \sim \mathcal{N}(\mu_0, \sigma_0^2),$$
$$cf = NI^T - na, \quad na \sim \mathcal{N}(0, \sigma_{na}^2),$$
$$ni = NI^T + da, \quad da \sim \mathcal{N}(0, \sigma_{da}^2),$$

where na and da are independent of each other and of NI^T.

The joint distribution is:

$$\begin{bmatrix} NI^T \\ cf \\ ni \end{bmatrix} \sim \mathcal{N}\left(\begin{bmatrix} \mu_0 \\ \mu_0 \\ \mu_0 \end{bmatrix}, \begin{bmatrix} \sigma_0^2 & \sigma_0^2 & \sigma_0^2 \\ \sigma_0^2 & \sigma_0^2 + \sigma_{na}^2 & \sigma_0^2 \\ \sigma_0^2 & \sigma_0^2 & \sigma_0^2 + \sigma_{da}^2 \end{bmatrix}\right). \tag{12}$$

This joint distribution forms the basis for computing the posterior distribution of NI^T conditional on the observed signals cf and ni. It is well known that in this special case, the general formula (2) collapses to the precision-weighted average of the three sources of information about the unobservable profit NI^T: prior information, cash flow information cf, and reported earnings ni. The posterior distribution is[3]:

$$NI^T \mid cf, ni \sim \mathcal{N}\left(\mu^*, \sigma^{*2}\right) \tag{13}$$

with

$$\mu^* = \frac{1/\sigma_0^2 \cdot \mu_0 + 1/\sigma_{na}^2 \cdot cf + 1/\sigma_{da}^2 \cdot ni}{1/\sigma_0^2 + 1/\sigma_{na}^2 + 1/\sigma_{da}^2} \tag{14}$$

and

$$\sigma^{*2} = \frac{1}{1/\sigma_0^2 + 1/\sigma_{na}^2 + 1/\sigma_{da}^2}. \tag{15}$$

The Maximum Likelihood estimator $\widehat{NI^T}$ for $NI^T \mid cf, ni$ is the posterior expected value μ^* of Eq. (14), which can be written as a weighted average of three values:

$$\widehat{NI^T} = \mu^* = w_1 \mu_0 + w_2 cf + w_3 ni, \tag{16}$$

where the weights $w_1 = \frac{1}{\sigma_0^2}\sigma^{*2}$, $w_2 = \frac{1}{\sigma_{na}^2}\sigma^{*2}$ and $w_3 = \frac{1}{\sigma_{da}^2}\sigma^{*2}$ reflect the relative precision of the three sources of information (inverse of variance). The same weights can also be related to the two stylized facts. With β_1 as the slope coefficient of the theoretical (population-level) regression of ta on cf, and β_2 as the slope coefficient of the theoretical regression of ta on ni, the weights can be expressed as[4]:

$$w_1 = \frac{-\beta_2\beta_1}{\beta_2 + \beta_2\beta_1 - \beta_1}, \tag{17}$$

$$w_2 = \frac{\beta_2 + \beta_2\beta_1}{\beta_2 + \beta_2\beta_1 - \beta_1}, \tag{18}$$

$$w_3 = \frac{\beta_2\beta_1 - \beta_1}{\beta_2 + \beta_2\beta_1 - \beta_1}. \tag{19}$$

The Bayesian estimator $\widehat{NI^T}$ of Eq. (16) leads directly to estimators for normal and discretionary accruals. Given cf and ni, the posterior distributions of da and na are normal, so that the Maximum Likelihood estimators are:

$$\widehat{da} \mid cf, ni = E\left[ni - NI^T \mid cf, ni\right] = ni - \widehat{NI^T}, \tag{20}$$

$$\widehat{na} \mid cf, ni = E\left[NI^T - cf \mid cf, ni\right] = \widehat{NI^T} - cf. \tag{21}$$

[3] Instead of deriving the posterior distribution as a special case of the general formula (2), we can also derive it directly. For the standard result with one sample, see, e.g., Leonard and Hsu (2001). A second sample can be integrated via Bayesian updating. In doing so, the posterior distribution after considering the first sample serves as the new prior distribution to be combined with the second sample information.

[4] See the expressions for the slope coefficients given in Properties 4 and 5 in Appendix A.

Appendix A sets out the properties of these estimators. The Bayesian estimators are closely related to the cash flow and profitability adjustments to the Jones model commonly applied in the earnings management literature (see the overview in Breuer & Schütt, 2023). According to Properties 4 and 5 in Appendix A, estimated discretionary accruals in our model are proportional to total accruals ta after controlling for contemporaneous cash flows (as in the modified Jones plus cash flow model), and estimated normal accruals are proportional to profitability-adjusted total accruals (as in the modified Jones plus profitability model). The intuition of this result is that our framework uses only the most basic assumptions to explain the stylized facts. Thus, our Bayesian estimators for da and na simply control for one or the other stylized fact, which is also what the variants of the Jones model do. If we tried to control for cash flow and earnings in successive regressions, we would switch back and forth between the estimators of normal and discretionary accruals (see Properties 6 and 7).

The properties in Appendix A and the covariance relationships (6), (10) and (11) derived in Section 2.3.2 do not depend on the assumption of normal distributions. They are valid for general distributions as long as the variances and covariances are defined and finite. However, outside the normal model, $\widehat{NI^T}$ according to Eq. (16) is no longer the Maximum Likelihood estimator for NI^T.

The model could be extended in different ways by considering other special cases of the general Gaussian updating problem presented in Section 2.2. For example, income smoothing could be modeled by including a smoothing parameter α such that $NI - \mu_{da} \mid NI^T \sim \mathcal{N}\left((1-\alpha)\, NI^T, \sigma^2_{da}\right)$, which results in a lower precision of the earnings information and a lower weight of earnings in the Bayesian estimator for true earnings. It is also possible that the cash flow is informative only about true earnings in one segment of the firm, because the cash budget of another segment has been fixed in advance. This knowledge would be relevant to draw the right conclusions from the cash flow signal. We do not consider such extensions in this paper. We also do not take into account how managers with certain incentives distort earnings when confronted with investors who are aware of these incentives and anticipate biased reporting (see Fischer & Verrecchia, 2000 and Dye & Sridhar, 2004).

2.4. Detecting earnings management

When a firm engages in earnings management (beyond the expected value μ_{DA}) with a profit impact of EM, its managed earnings are equal to $ni_{new} = ni + EM$.[5] With unchanged cash flow cf, the new Bayesian estimator for NI^T according to Eq. (16) is $\widehat{NI^T}_{new} = \widehat{NI^T} + w_3 EM$.[6] Thus, the new accrual estimators are:

$$\widehat{da}_{new} = ni_{new} - \widehat{NI^T}_{new} = \widehat{da} + \left(w_1 + w_2\right) EM \tag{22}$$

$$\widehat{na}_{new} = \widehat{NI^T}_{new} - cf = \widehat{na} + w_3 EM. \tag{23}$$

This means that only a share $\left(w_1 + w_2\right)$ of managed earnings appears in estimated discretionary accruals, while the remaining part is picked up by estimated normal accruals. Therefore, earnings management can, in principle, be detected in total accruals, estimated discretionary accruals, or estimated normal accruals. This raises the question of the most efficient test strategy. The following analysis compares z-statistics for known population variances. The firms suspected of earnings management are assumed to form a random sample.

Applying a z-test to total accruals, the full amount of EM is compared to the standard deviation of total accruals, so the relevant ratio is:

$$z_{ta} = \frac{EM}{\sigma_{ta}} = \frac{EM}{\sqrt{\sigma^2_{da} + \sigma^2_{na}}}. \tag{24}$$

A similar z-test based on discretionary accruals will only capture a part of EM in the numerator, but the standard deviation in the denominator is also smaller than in Eq. (24):

$$z_{da} = \frac{\left(w_1 + w_2\right) EM}{\sigma_{\widehat{da}}} = \frac{\left(w_1 + w_2\right) EM}{\sqrt{\sigma^2_{da} - \sigma^{*2}}}. \tag{25}$$

Finally, for normal accruals, we obtain:

$$z_{na} = \frac{w_3 EM}{\sigma_{\widehat{na}}} = \frac{w_3 EM}{\sqrt{\sigma^2_{na} - \sigma^{*2}}}. \tag{26}$$

A comparison of the three ratios reveals that $z_{da} > z_{ta}$ and $z_{da} > z_{na}$ so that the most powerful test is based on discretionary accruals (see Appendix C).[7] Only when $\sigma^2_0 \to \infty$ so that $w_1 \to 0$, z_{ta} and z_{na} are asymptotically equal to z_{da}.

2.5. Modified Jones model as a special case

In the modified Jones model, discretionary accruals are estimated as the error term ϵ in the cross-sectional regression of total accruals TA (not ta) on PPE and the change in cash revenues $\Delta CRev$:

$$TA = \alpha_0 + \alpha_1 PPE + \alpha_2 \Delta CRev + \epsilon, \tag{27}$$

which implies that

$$\epsilon = TA - \left(\alpha_0 + \alpha_1 PPE + \alpha_2 \Delta CRev\right). \tag{28}$$

Since ϵ are considered discretionary accruals, the implicit assumption of the Jones model about true earnings is:

$$\begin{aligned} NI^T &= NI - \epsilon && (29)\\ &= NI - TA + \left(\alpha_0 + \alpha_1 PPE + \alpha_2 \Delta CRev\right) && (30)\\ &= CF + \left(\alpha_0 + \alpha_1 PPE + \alpha_2 \Delta CRev\right) && (31)\\ &= cf. && (32) \end{aligned}$$

This means that the best estimate of true earnings are cash flows adjusted for accruals determined by PPE and $\Delta CRev$.

There are two settings in which our Bayesian model is equivalent to the modified Jones model. The first is that $\sigma^2_{na} = 0$ so that normal accruals are fully determined by PPE and $\Delta CRev$. In this case, only the cash flow signal cf is used because it is perfectly precise. As a consequence, the first stylized fact is absent (see Eq. (10)), so that $\beta_1 = 0$, $w_2 = 1$, and $w_1 = w_3 = 0$ (see Eqs. (17) to (19)). Therefore, the estimated true earnings of the Bayesian model simplify to (see Eq. (16)):

$$\widehat{NI^T} = cf, \tag{33}$$

which is consistent with the implicit assumption of the Jones model.

The Bayesian framework also collapses into the modified Jones model if no prior information about the distribution of true earnings is available. For $\sigma^2_0 \to \infty$ (uninformative prior), $w_1 = 0$, and the estimated true earnings simplify to (see Eq. (16)):

$$\widehat{NI^T} = w_2 cf + (1 - w_2) ni, \tag{34}$$

[5] In the following, we consider earnings management as a pure accounting phenomenon and thus exclude real earnings management.

[6] We assume that the firms suspected of earnings management are excluded from the sample before estimating the parameters of the Bayesian model. Therefore, the Bayesian weights are independent of earnings management and do not need to be updated here.

[7] This is consistent with the efficiency loss from a profitability adjustment shown by Keung and Shih (2014). See also Ayers et al. (2006) and Dechow et al. (2012).

which leads to the following expression for estimated discretionary accruals:

$$\widehat{da} = w_2\left(TA - \mu_{NA} - \mu_{DA}\right) = w_2\left(TA - \mu_{NA}\right) = w_2\epsilon. \quad (35)$$

This expression is up to the constant factor w_2 identical to the error term from the modified Jones model.

In general, the two models will not coincide. We use the modified Jones model only to estimate μ_{NA} as:

$$\mu_{NA} = \alpha_0 + \alpha_1 PPE + \alpha_2 \Delta CRev, \quad (36)$$

which implies that $\mu_{DA} = 0$ (zero expected value of the error term of the Jones regression). This is a natural choice to show the difference of our Bayesian model to the Jones model. (We note, however, that our Bayesian model is also compatible with any other model for estimating μ_{NA} and μ_{DA}.) We then determine ni and cf according to $ni = NI - \mu_{DA} = NI$ and $cf = CF + \mu_{NA}$ and estimate true earnings from Eq. (16) and discretionary and normal accruals from Eqs. (20) and (21). The estimation method is explained in more detail in the next section.

3. Estimation method and data

3.1. Estimation method

Both the modified Jones model and the Bayesian model are estimated for the full sample of firms. It would also be possible to exclude the firms suspected of earnings management, as including them can distort the coefficients and weaken the residual signal. The estimated parameters are then applied to the suspect firms to calculate their normal and discretionary accruals.

For the modified Jones regression, we use the following symbols: NI is the earnings measure, CF the cash flow and $TA = NI - CF$ the dependent variable. The independent variables are PPE and the change in cash revenues $\Delta CRev$, which is defined as the change in revenues (ΔREV) adjusted for the change in receivables (ΔREC). All variables are scaled by lagged total assets. Following standard practice, we estimate the regression separately for industry-year groups with a minimum of 20 observations (Breuer & Schütt, 2023, p. 743).[8] The regression equation is[9]:

$$TA_i = \alpha_{0,g} + \alpha_{1,g} PPE_i + \alpha_{2,g} \Delta CRev_i + \epsilon_i, \quad (37)$$

where subscript g indicates the industry-year group and i a firm observation within the industry-year group g, α are regression coefficients, and ϵ is the error term.

Based on (37), we define

$$\mu_{NA,i} = \alpha_{0,g} + \alpha_{1,g} PPE_i + \alpha_{2,g} \Delta CRev_i. \quad (38)$$

Since the expected value of $TA_i - \mu_{NA,i}$ is equal to zero, we have $\mu_{DA,i} = 0$. We can now compute the adjusted variables of the Bayesian model: $ta_i = TA_i - \mu_{NA,i}$; $cf_i = CF_i + \mu_{NA,i}$; and $ni_i = NI_i$.

Finally, the simplest way to estimate the Bayesian weights is to run the stylized-fact regressions (of ta on cf and ta on ni) and insert the slope coefficients in Eqs. (17) to (19). The following alternative way leads to the same result: We use the sample estimates $\hat{\sigma}^2_{cf}$ and $\hat{\sigma}^2_{ni}$ for the variances of cf and ni. According to Eq. (6), the slope coefficient of a regression of ni on cf is equal to σ_0^2/σ_{cf}^2 so that this regression provides estimate $\hat{\sigma}_0^2$. We determine the implied variation of normal and discretionary accruals as $\hat{\sigma}^2_{da} = \hat{\sigma}^2_{ni} - \hat{\sigma}^2_0$ and $\hat{\sigma}^2_{na} = \hat{\sigma}^2_{cf} - \hat{\sigma}^2_0$. The weights can then be determined from the weight definitions in Eq. (16).

Given the weight estimates, Eq. (16) is used to compute $\widehat{NI^T}$ for each firm-year. The estimated discretionary and normal accruals are then obtained from Eqs. (20) and (21).

3.2. Data and parameter estimates

To estimate our model, we use merged CRSP/Compustat data of US firms from 1987 to 2022. Our exclusion criteria are the same as in Breuer and Schütt (2023): we exclude financial firms with three-digit SIC codes between 600 and 699 and observations with lagged total assets smaller than \$10 million. We also exclude observations with financial leverage larger than 1 or total asset growth of 200% or more (to exclude M&A activity). We require the following positions to be available (Compustat data items in brackets): total assets (at); revenues ($sale$); trade receivables ($rect$); gross property, plant, and equipment ($ppegt$); long-term debt ($dltt$); current debt (dlc); cash from operating activities ($oancf$); income before extraordinary items (ib); and operating income after depreciation ($oiadp$).

We define total accruals in the Jones model (TA) as the difference between income before extraordinary items (NI) and cash flow from operating activities (CF). The exact variable definitions with reference to the Compustat data items are given in Panel A of Table 1. The definitions follow Breuer and Schütt (2023), who generally adopt the definitions of Dechow et al. (1995). Finally, we truncate the raw variables listed in Panel B of Table 1 at the first and 99th percentile. Our final sample contains 149,219 firm-year observations. Table 1 shows descriptive statistics for the raw data (Panel B) and the input and output data of the Bayesian model (Panel C).

Following the steps described in Section 3.1, we first run the modified Jones regression and then estimate the following stylized-fact regressions of total accruals ta on cash flow cf and earnings ni for our total sample (pooled regressions):

$$ta_i = \alpha_1 + \beta_1 cf_i + \epsilon_i \quad (39)$$

$$ta_i = \alpha_2 + \beta_2 ni_i + \epsilon_i, \quad (40)$$

where we expect β_1 to be significantly negative and β_2 significantly positive. This is confirmed by the following regression results, where the t-statistics in parentheses are based on standard errors clustered by industry-years[10]:

$$\hat{\alpha}_1 = -0.0011(t = -5.7); \hat{\beta}_1 = -0.0801(t = -13.1); R^2 = 0.018;$$
$$\hat{\alpha}_2 = 0.0033(t = 4.7); \hat{\beta}_2 = 0.2364(t = 24.5); R^2 = 0.185.$$

The corresponding weights for estimating true earnings are: $w_1 = 0.064$; $w_2 = 0.731$; $w_3 = 0.206$. For the standard deviations and the expected values of the model variables, we obtain the following estimates: $\hat{\sigma}_0 = 0.1479$; $\hat{\sigma}_{cf} = 0.1542$; $\hat{\sigma}_{ni} = 0.1692$; $\hat{\sigma}_{da} = 0.0823$; $\hat{\sigma}_{na} = 0.0436$; $\hat{\sigma}^* = 0.0373$; $\hat{\mu}_0 = \hat{\mu}_{cf} = \hat{\mu}_{ni} = -0.0140$.

Note that cash flows cf, reported earnings ni, and true earnings NI^T all have the same means because the cash flows CF and earnings NI are first corrected for the predictable components of normal and discretionary accruals. Therefore, the equal means do not result from a restrictive assumption, but are a direct implication of our two-step estimation procedure.

[8] We combine observations from industry-year groups that do not meet this requirement in a separate group per year. The results are very similar when using the coefficients of a pooled regression with all observations per year for this group.

[9] The estimation method described in this section can be applied regardless of the precise definition of accruals that serve as dependent variable in the Jones regression. The method is consistent with a Jones regression for the total sample or separate regressions for industry-year groups. The weaknesses of such discretionary accrual models are discussed in Dechow et al. (2010), Guay et al. (1996), Healy (1996), Holthausen et al. (1995), Kaplan (1985) and Owens et al. (2017), among others.

[10] In cross-sectional regressions per year, the slope coefficient β_1 is always negative, and it is significant at least at the 5% level in 29 of 36 years (based on standard errors clustered by industry). The slope coefficient β_2 is significantly positive at the 1% level in all 36 years.

Table 1
Descriptive statistics.

Panel A: Variable definitions

$InvAt$	Inverse of lagged total assets: $(1/at_{t-1})$
$\Delta CRev$	Change in cash revenues: $(sale_t - sale_{t-1} - (rect_t - rect_{t-1}))/at_{t-1}$
PPE	Property, plant, and equipment: $ppegt_t/at_{t-1}$
Lev	Financial leverage: $(dltt_{t-1} + dlc_{t-1})/at_{t-1}$
CF	Cash flow from operating activities (input to Jones model): $oancf_t/at_{t-1}$
NI	Net income before extraordinary items: ib_t/at_{t-1}
OI	Operating income after depreciation: $oiadp_t/at_{t-1}$
TA	Total accruals: $(ib_t - oancf_t)/at_{t-1} = NI - CF$
ta	Demeaned total accruals: $TA - \mu_{NA} - \mu_{DA}$ with $\mu_{DA} = 0$ and μ_{NA} = fitted value of the modified Jones regression
cf	Adjusted cash flow: $CF + \mu_{NA}$
ni	Reported earnings: $NI - \mu_{DA}$ with $\mu_{DA} = 0$
$\widehat{NI^T}$	Estimated true earnings: see Eq. (16)
$\widehat{na}$	Demeaned estimated normal accruals: $\widehat{NI^T} - cf$
$\widehat{da}$	Demeaned estimated discretionary accruals: $ni - \widehat{NI^T}$

Statistic	N	Mean	SD	P05	P25	Median	P75	P95
Panel B: Raw data								
$InvAt$	149,219	0.012	0.018	0.0001	0.001	0.003	0.014	0.056
$\Delta CRev$	149,219	0.076	0.218	−0.231	−0.018	0.043	0.150	0.471
PPE	149,219	0.602	0.441	0.069	0.238	0.498	0.902	1.422
Lev	149,219	0.251	0.216	0	0.051	0.227	0.385	0.661
CF	149,219	0.052	0.150	−0.246	0.011	0.074	0.132	0.241
NI	149,219	−0.014	0.169	−0.387	−0.041	0.029	0.074	0.166
OI	149,219	0.033	0.172	−0.328	−0.005	0.065	0.122	0.240
TA	149,219	−0.066	0.102	−0.246	−0.105	−0.055	−0.017	0.078
Panel C: Data Bayesian model								
ta	149,219	0	0.093	−0.161	−0.035	0.007	0.046	0.134
cf	149,219	−0.014	0.154	−0.325	−0.059	0.014	0.069	0.176
ni	149,219	−0.014	0.169	−0.387	−0.041	0.029	0.074	0.166
$\widehat{NI^T}$	149,219	−0.014	0.143	−0.310	−0.051	0.015	0.063	0.156
$\widehat{na}$	149,219	0	0.023	−0.036	−0.011	−0.0004	0.011	0.037
$\widehat{da}$	149,219	0	0.073	−0.130	−0.026	0.007	0.036	0.101

The definitions of variables in Panel A based on Compustat data are adopted from Breuer and Schütt (2023), who generally follow Dechow et al. (1995). The Compustat data items are: at is total assets; $sale$ is revenues; $rect$ is trade receivables; $ppegt$ is gross property, plant, and equipment; $dltt$ is long-term debt; dlc is current debt; $oancf$ is cash from operating activities; ib is income before extraordinary items; and $oiadp$ is operating income after depreciation. Panel B shows descriptive statistics for the variables calculated from Compustat data. Panel C shows descriptive statistics for input variables (ta, cf, ni) and output variables ($\widehat{NI^T}$, $\widehat{na}$, $\widehat{da}$) of the Bayesian model. The final sample of 149,219 observations consists of 16,873 firms from 1987 to 2022. Our exclusion criteria are the same as in Breuer and Schütt (2023): we exclude financial firms with three-digit SIC codes between 600 and 699 as well as observations with lagged total assets smaller than \$10 million, financial leverage Lev larger than 1, or total asset growth of 200% or more. We only keep observations with non-missing values for all variables listed in Panel B. Finally, we truncate the variables listed in Panel B at the first and 99th percentile. We estimate the Jones regression for industry-year groups according to Eq. (37).

Standard Compustat data include restated financials, which is problematic in studies of earnings management because restatements may partially remove aggressive accounting choices. Historically, many benchmark studies relied on Compustat data downloaded using the DATAFMT = STD filter, which at the time provided unrestated data (Dechow et al., 2012). However, Compustat's data structure has changed: the DATAFMT = STD filter now delivers restated values, while unrestated observations are, in principle, provided under the DATAFMT = PRE_AMENDS flag. However, hardly any data is available under this flag, so the earlier practice described in Dechow et al. (2012) is no longer feasible in Compustat.

Dechow et al. (1995) collect original, unrestated numbers for a small SEC enforcement sample of 56 firm-years and correct the restated numbers before estimating the modified Jones model. We conduct a robustness analysis in the spirit of Dechow et al. (1995). Specifically, we use the AAER database as an extended SEC enforcement sample and exclude all firm-years associated with an AAER event (N = 941) from the estimation sample. We then re-estimate the model parameters and replicate all analyses. We find that the resulting parameter estimates under our Bayesian framework are virtually identical to those obtained from the full sample, and all results remain essentially the same.

4. Empirical results

The simulations in Section 4.1 are based on empirical parameter estimates assuming normal distributions, while the analysis in Section 4.2 is based on actual empirical distributions. In the following, we use the term "Model ta" when (demeaned) total accruals (ta) serve as the measure of earnings management, and analogously refer to "Model da" and "Model na" when using (demeaned) discretionary accruals (da) and normal accruals (na), respectively.

4.1. Simulations

In this section, we simulate the normal distributions of the Bayesian model based on the parameter estimates reported in Section 3.2. The treatment firms are selected randomly (Section 4.1.1), based on reported earnings (Section 4.1.2), or based on estimated true earnings (Section 4.1.3).

4.1.1. Random sampling

Fig. 2 shows a simulation of 2000 firm-years (grey scatterplot) together with a random sample of 100 of these observations (treatment group; blue points) without earnings management ($EM = 0$).

From the left to the right column, the x-axis of the diagrams in Fig. 2 corresponds to cash flows, earnings, and estimated true earnings. From the top to the bottom row, the y-axis corresponds to the accrual measures ta, $\widehat{na}$, and $\widehat{da}$. Each diagram also shows the regression line for the grey scatterplot of all observations. In the diagrams on the diagonal from bottom-left to top-right, the x and y variables are, by design,

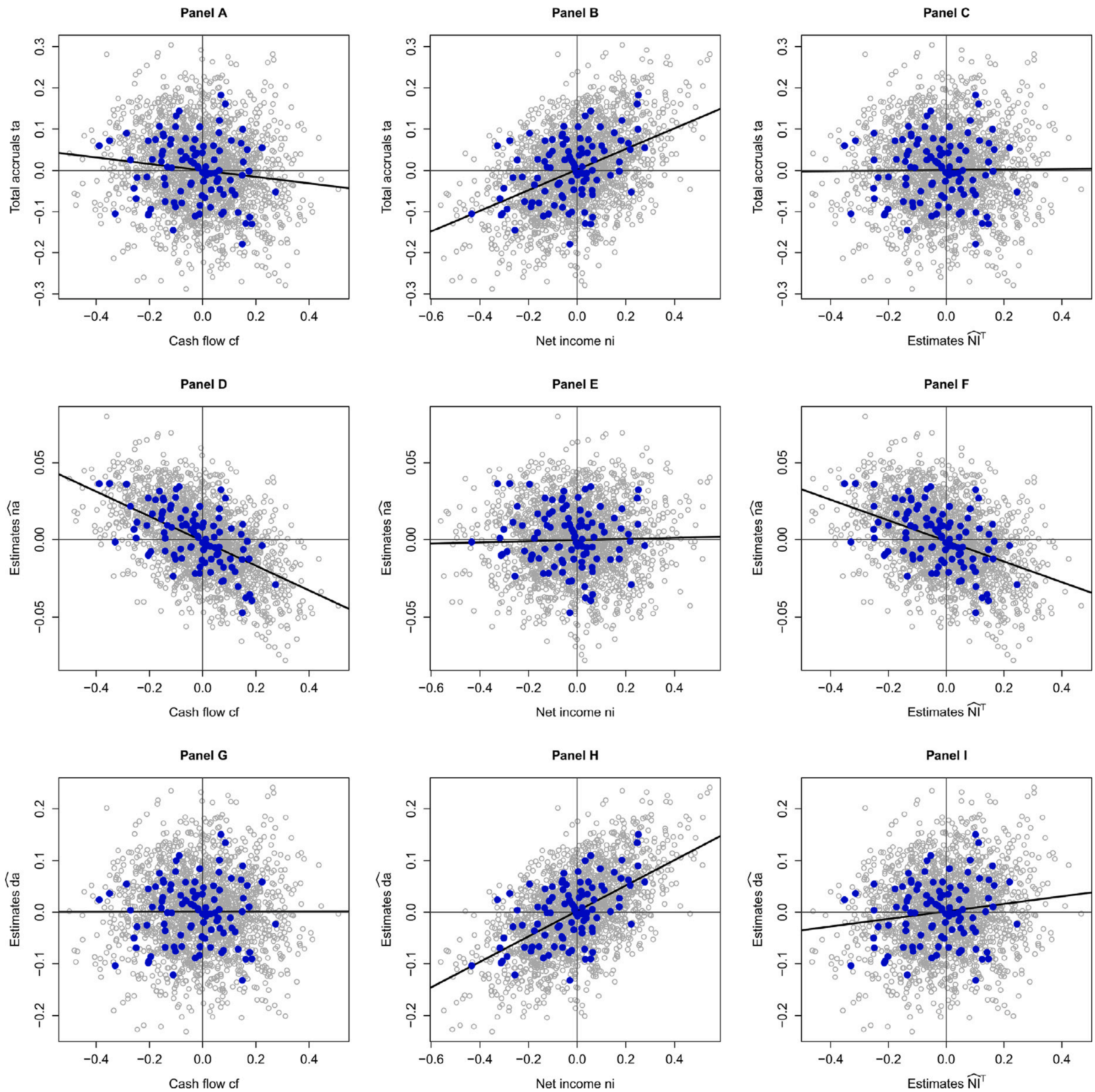

Fig. 2. Illustration of random sampling.
The grey scatterplot shows 2000 simulated observations (firm-years). The simulations are based on the empirical parameter estimates obtained in Section 3.2. The blue points mark a random sample of 100 firm-years (treatment sample) drawn from the 2000 total observations. No earnings management is infused ($EM = 0$). The black line shows the linear regression based on the total observations. Variable ta is total accruals, $\widehat{na}$ is the Bayesian estimate of normal accruals, $\widehat{da}$ is the Bayesian estimate of discretionary accruals, ni is earnings, cf is cash flow, and $\widehat{NI^T}$ is the Bayesian estimate of true earnings. All variables are scaled by lagged total assets.

independent of each other ($\widehat{da}$ and cf in Panel G; $\widehat{na}$ and ni in Panel E; ta and $\widehat{NI^T}$ in Panel C).

Panels A and B show that total accruals are negatively related to cf and positively related to ni. These are the two stylized facts at the core of this paper. Panel D shows the negative relation of $\widehat{na}$ to cf (with the same slope as in Panel A), Panel H the positive relation of $\widehat{da}$ to ni (with the same slope as in Panel B). $\widehat{na}$ is not only negatively related to cf (Panel D) but also to $\widehat{NI^T}$ (Panel F), and $\widehat{da}$ is not only positively related to ni (Panel H) but also to $\widehat{NI^T}$ (Panel I).

In the case of random sampling, the expected values of all three accrual measures (ta, $\widehat{na}$, $\widehat{da}$) in the treatment group are zero. Therefore, all three measures can be used to detect earnings management, but as shown in Section 2.4, using $\widehat{da}$ is most efficient.

Following prior research, we now run simulations to test how often earnings management of a given size infused in certain firm-years is detected (see, e.g., Dechow et al., 2012, 1995; Keung & Shih, 2014; Kothari et al., 2005). We first create 10,000 simulated observations (firm-years).[11] From these 10,000 observations, we randomly draw 100 firm-years (treatment group) that are infused with earnings management of a fixed size (from −3% to + 3%). We use a two-sided t-test to test whether the mean discretionary accruals (Model da, corresponding to the modified Jones plus cash flow model), the mean normal accruals (Model na, corresponding to the modified Jones plus profitability model), and the mean total accruals (Model ta, corresponding to the modified Jones model) in the treatment group are different from zero (5% significance level). We repeat the random drawing of the treatment sample 1000 times.

[11] This number of simulated firm-years is large enough so that excluding the treatment firms would have a negligible effect on the estimated model parameters.

Table 2
Detecting earnings management in simulated data.

EM	−3%	−2%	−1%	0%	1%	2%	3%
Panel A: Random sampling							
Model da	90.6 / 0	60.6 / 0	21.5 / 0	NA/ 4.6	14.4 / 0.2	54.1 / 0	87.6 / 0
Model na	79 / 0	46.5 / 0	18.4 / 0.1	NA/ 4.6	12.3 / 0.4	37.5 / 0	71.5 / 0
Model ta	89.9 / 0	59.7 / 0	21.7 / 0	NA/ 4.2	14.2 / 0.2	52.5 / 0	87.4 / 0
Panel B: Sampling related to reported earnings							
Model da	0.1 / 24.3	0 / 67.2	0 / 94.1	NA/ 99.8	100 / 0	100 / 0	100 / 0
Model na	77.3 / 0	42.6 / 0	16.4 / 0.2	NA/ 4.9	14.4 / 0.4	41.4 / 0.1	71.5 / 0
Model ta	1.2 / 5.1	0 / 28.6	0 / 69	NA/ 94.5	99.5 / 0	100 / 0	100 / 0
Panel C: Sampling related to the Bayesian estimate of true earnings							
Model da	57.7 / 0	19.1 / 0	2.4 / 1.4	NA/ 15.1	55.8 / 0	85.5 / 0	98.7 / 0
Model na	100 / 0	100 / 0	99.7 / 0	NA/ 97.4	0 / 77.4	0 / 45.2	0 / 13.8
Model ta	88.8 / 0	55.9 / 0	19.9 / 0	NA/ 4	15.8 / 0.2	50.9 / 0	86 / 0

We first create 10,000 simulated observations (firm-years) based on the empirical parameter estimates obtained in Section 3.2. For each column in the table, we then repeat the following three steps 1000 times: (1) randomly draw 100 firm-years (without replacement); (2) infuse the respective level of earnings management (EM from −3% to + 3%) into these 100 firm-years; and (3) run a two-sided t-test for the three models to test the null hypothesis of no earnings management. In Panel A, the random drawing in step (1) is from all 10,000 observations. In Panel B, the random drawing is from the observations with above median reported earnings (ni). In Panel C, the random drawing is from the observations with above median Bayesian estimate of true earnings ($\widehat{NI^T}$). The first number in each cell indicates the percentage of rejecting the null hypothesis at the 5% level (out of the 1000 repetitions) with the right sign of the t-statistic. The second number indicates the percentage of significant cases with the wrong sign of the t-statistic. Model da uses discretionary accruals, Model na normal accruals, and Model ta total accruals as defined in the Bayesian model.

Panel A of Table 2 reports the percentage of samples with a significant t-statistic that has the right sign (1st number) and the percentage of samples with a significant t-statistic that has the wrong sign (2nd number). As expected, in the case of no earnings management (EM = 0%), the null hypothesis is rejected with a similar likelihood, i.e., between 4% and 5% out of the 1000 repetitions. Positive and negative EM levels are detected with similar likelihoods. Model da performs better than Model na, which is in line with our theoretical considerations. However, Model ta performs just as well as Model da. The reason for this result is that our set of parameters places a strong emphasis on cash flow information ($w_2 = 0.731$), so that total accruals ($ni - cf$) provide similar results as estimated discretionary accruals ($ni - \widehat{NI^T}$) in this setting of random sampling.

4.1.2. Sampling related to reported earnings

The inferences change if the firms suspected of earnings management are no longer a random sample. In particular, the composition of the treatment group might be related to the firms' profitability. To simulate this situation, we assume that the treated firm-years stem from the observations with above median earnings ni. Fig. 3 and Panel B of Table 2 show the results in the same format as before.

Panel H in Fig. 3 shows that the test based on discretionary accruals is heavily biased because the estimated discretionary accruals are positively related to earnings. Therefore, the selection of firm-years with high ni mechanically results in high da estimates. Even in the case of $EM = -3\%$, 24.3% of the t-statistics of Model da are significantly *positive* (see Panel B of Table 2). Model ta is biased in the same direction. This is exactly the setting where a profitability-adjustment is necessary, which is in essence Model na. Panel E of Fig. 3 illustrates that $\widehat{na}$ is indeed unbiased in this setting. The results of Model na reported in Panel B are of similar quality as the results for random sampling in Panel A.

4.1.3. Sampling related to the Bayesian estimate of true earnings

We now assume that firms suspected of earnings management belong to the observations with above median $\widehat{NI^T}$. As our estimate of true profitability is positively related to ni, one might consider a profitability adjustment as in the last Section 4.1.2. However, Panel C of Table 2 shows extremely poor results for the corresponding Model na. The model is biased downwards, so that in 77.4% of the cases with earnings management of $EM = +1\%$, the t-statistic is significantly *negative*. Model da performs substantially better, but with a slight bias in the opposite direction. Notably, in line with the theoretical expectation, Model ta performs well. In fact, Model ta's performance is similar to the performance in the case of random sampling in Panel A.

Fig. 4 illustrates why this is the case. Panel F reveals the negative association of $\widehat{na}$ to $\widehat{NI^T}$. Thus, selecting treatment firm-years with high $\widehat{NI^T}$ will result in negative mean $\widehat{na}$ values. The positive association of $\widehat{da}$ to $\widehat{NI^T}$ (as shown in Panel I) is less pronounced, so that Model da is less strongly biased. Model ta, however, is unbiased because ta is not related to $\widehat{NI^T}$, as shown in Panel C (flat regression line).

4.2. Empirical analysis

We now examine how well artificially infused earnings management is recognized in empirical data. A key difference from the simulations in the last section is that we use actual empirical data instead of normally distributed variables generated based on our empirical parameter estimates. If the tests are robust with respect to the normal distribution assumption, the results should be similar.

The infused earnings management (EM) again ranges from −3% to +3%. The number N of treated firm-years is 10, 25, 50 or 100. As before, we consider three types of sampling: random sampling among all firm-years (Table 3); sampling from firms with earnings ni above the median (Table 4); and sampling from firms with Bayesian estimates of true earnings $\widehat{NI^T}$ above the median (Table 5). We examine 1000 samples drawn from the total sample of 149,219 observations (as reported in Table 1) and show the percentage of significant t-statistics with the right/wrong sign in each cell of Tables 3, 4, and 5.

The results generally confirm our findings from the previous simulations. Random sampling (Table 3) is the least problematic setting: All three models are unbiased, but Models da and ta are more efficient than Model na. When sampling with respect to ni (Table 4), Model na is the only appropriate model, as Models da and ta are biased towards more positive earnings management. The only noteworthy difference to the simulations occurs when sampling is related to $\widehat{NI^T}$ (Table 5). Here, the estimates of Model ta are slightly downward biased so that Model da produces better results. However, the main simulation result is confirmed: Model na has a strong negative bias in this setting. Its performance shows that an inappropriate model can lead not only to high rates of false positives, but also to inflated likelihoods of significant results in the expected direction. For example, for $N = 100$ (Panel D in Table 5), Model na *always* confirms the hypothesis of

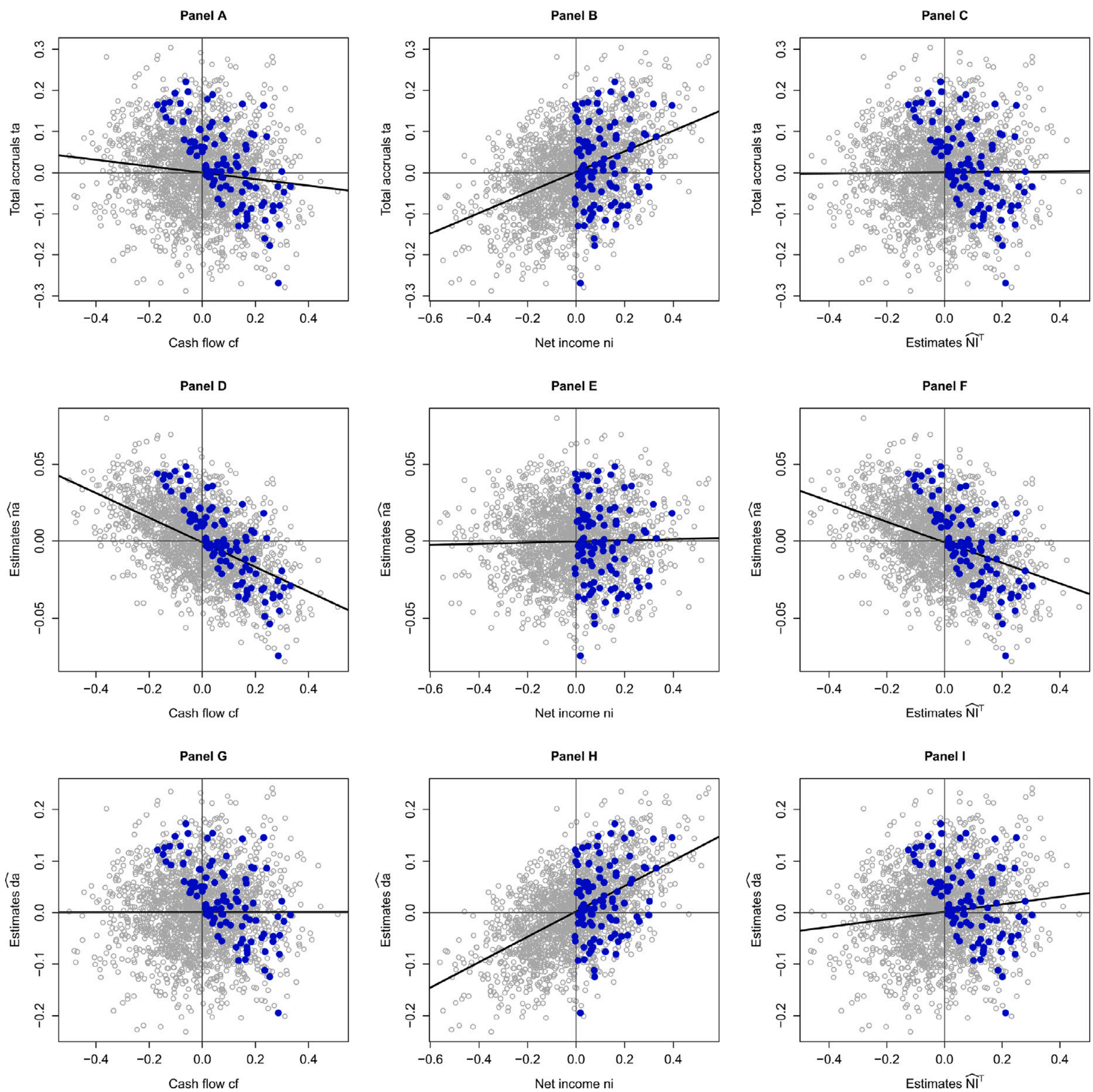

Fig. 3. Illustration of sampling related to reported earnings.
The grey scatterplot shows 2000 simulated observations (firm-years). The simulations are based on the empirical parameter estimates obtained in Section 3.2. The blue points mark a sample of 100 firm-years (treatment sample) randomly drawn from observations with above median reported earnings (ni). No earnings management is infused ($EM = 0$). The black line shows the linear regression based on the total observations. Variable ta is total accruals, $\widehat{na}$ is the Bayesian estimate of normal accruals, $\widehat{da}$ is the Bayesian estimate of discretionary accruals, ni is earnings, cf is cash flow, and $\widehat{NI^T}$ is the Bayesian estimate of true earnings. All variables are scaled by lagged total assets.

negative EM when the hypothesis is actually true (values 100/0 for $EM < 0$). Therefore, the model suggests significant results consistent with the postulated earnings management, but only because the model is biased in the "right" direction.

5. Practical implementation: Case study of AAER data

To assess the external validity of the Bayesian measures in detecting earnings management and to illustrate their practical implementation, we analyze the updated AAER dataset originally compiled by Dechow et al. (2011). The dataset contains Accounting and Auditing Enforcement Releases (AAERs) issued by the SEC between May 1982 and December 2021. The annual file comprises 1809 AAERs covering 741 firms. Of these, 941 AAERs for 376 firms overlap with our final sample. After excluding 37 AAERs that involve earnings understatements, the final sample consists of 904 AAER firm-year observations.

We first estimate the modified Jones model and the parameters of the Bayesian model as described in Section 3.2. Using these estimates, we compute mean-adjusted total accruals (ta) as well as discretionary and normal accruals ($\widehat{da}$ and $\widehat{na}$) for the AAER firm-years. As shown in the theoretical model (Section 2.4), all three measures are affected by earnings management. Because the AAERs in our sample involve alleged earnings overstatements, we expect these measures to be positive on average. Consistent with this prediction, the mean values are 0.0134 ($t = 3.92$) for ta, 0.0111 ($t = 4.21$) for $\widehat{da}$, and 0.0022 ($t = 2.75$) for $\widehat{na}$.

While all three measures indicate statistically significant earnings overstatements, the magnitude of the corresponding t-statistics varies substantially, ranging from 2.75 to 4.21. This dispersion suggests that identifying the most reliable measure remains informative. Our specification test exploits the theoretical properties of the measures. By construction, in the full sample, (i) a regression of ta on $\widehat{NI^T}$, (ii) a regression of $\widehat{da}$ on cf, and (iii) a regression of $\widehat{na}$ on ni yield slope

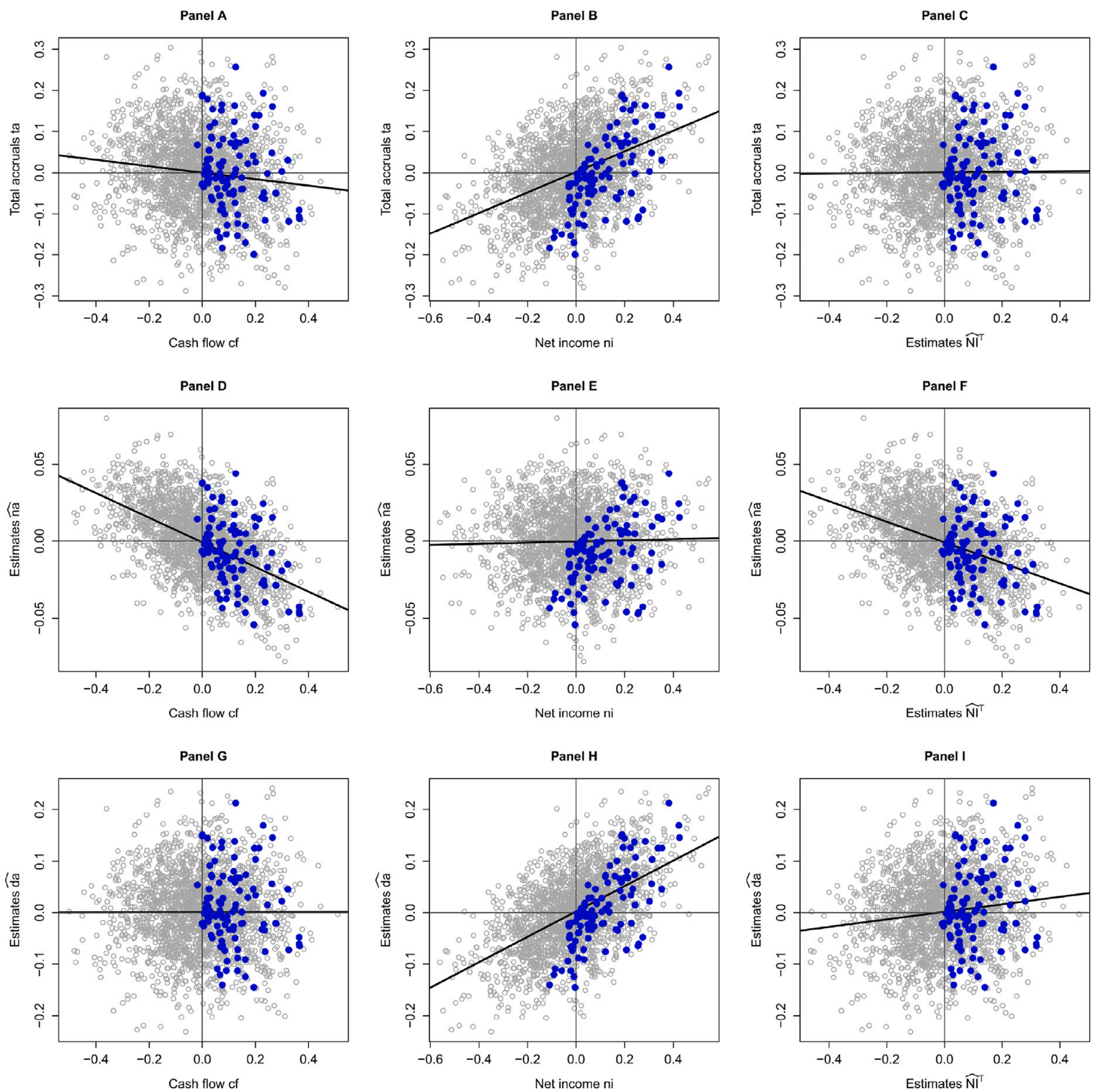

Fig. 4. Illustration of sampling related to the Bayesian estimate of true earnings.
The grey scatterplot shows 2000 simulated observations (firm-years). The simulations are based on the empirical parameter estimates obtained in Section 3.2. The blue points mark a sample of 100 firm-years (treatment sample) randomly drawn from observations with above median Bayesian estimate of true earnings ($\widehat{NI^T}$). No earnings management is infused ($EM = 0$). The black line shows the linear regression based on the total observations. Variable ta is total accruals, $\widehat{na}$ is the Bayesian estimate of normal accruals, $\widehat{da}$ is the Bayesian estimate of discretionary accruals, ni is earnings, cf is cash flow, and $\widehat{NI^T}$ is the Bayesian estimate of true earnings. All variables are scaled by lagged total assets.

coefficients of zero. We refer to these regressions as the characteristic regressions of the respective measures. Our model analysis shows that, when these regressions are estimated in a non-random test sample suspected of earnings management, systematic relationships emerge if the underlying stylized facts are present. Accordingly, the specification criterion is to select the measure whose characteristic regression does not exhibit a statistically significant slope coefficient.

For the AAER sample, the results of the characteristic regressions are as follows:

Regression of ta on $\widehat{NI^T}$: slope – 0.0921 ($t = -1.74$), $R^2 = 0.0075$.

Regression of $\widehat{da}$ on cf : slope – 0.1383 ($t = -4.06$), $R^2 = 0.0388$.

Regression of $\widehat{na}$ on ni : slope 0.0494 ($t = 5.60$), $R^2 = 0.0699$.

Among the three regressions, only the regression involving ta yields both an insignificant slope coefficient and a negligible R^2. The specification test therefore favors the ta measure. Based on this measure, we conclude with strong statistical confidence ($t = 3.92$) that earnings are, on average, overstated in the AAER sample.

6. Conclusion

In summary, we can describe our study from three different angles. First, we propose a new explanation for the empirical observation that the residuals from the modified Jones model tend to be positively related to earnings and negatively related to operating cash flows in the cross-section of firms. We argue that these stylized facts arise naturally when unobservable true profitability has a more or less narrow cross-sectional distribution, which is likely to be the case in a competitive economic environment. Second, we extend the modified Jones model

Table 3
Detecting earnings management in empirical data: random sampling.

EM	−3%	−2%	−1%	0%	1%	2%	3%
Panel A: N = 10							
Model da	14 / 0.1	5.5 / 0.3	1.5 / 1.7	NA/ 4.4	9.2 / 0.1	17.5 / 0	27.8 / 0.2
Model na	15.5 / 0.1	7.9 / 0.2	4 / 1	NA/ 5.2	4.2 / 0.8	9 / 0.5	13.7 / 0.2
Model ta	15.1 / 0.1	6.5 / 0.2	2 / 1.5	NA/ 4.1	7.7 / 0.1	15.8 / 0	26.4 / 0.2
Panel B: N = 25							
Model da	33.4 / 0	15.6 / 0.1	4.7 / 0.8	NA/ 6.7	13.1 / 0.3	25.6 / 0.2	42.5 / 0
Model na	27.5 / 0	16.6 / 0	7.2 / 0.6	NA/ 6.1	5.9 / 0.7	16.2 / 0.2	27.9 / 0
Model ta	34.1 / 0	17.1 / 0.1	5.4 / 0.9	NA/ 6.7	11.4 / 0.3	24.7 / 0.2	40.5 / 0
Panel C: N = 50							
Model da	62.1 / 0	29 / 0	8.8 / 0.3	NA/ 4.5	14 / 0.2	36 / 0.1	61.3 / 0
Model na	46.3 / 0	24.3 / 0.1	8.9 / 0.4	NA/ 5.8	9 / 0.6	23.6 / 0.1	49.9 / 0
Model ta	60.4 / 0	29.8 / 0	9.1 / 0.3	NA/ 4.6	12.7 / 0.2	35.4 / 0.1	61 / 0
Panel D: N = 100							
Model da	92.5 / 0	61.1 / 0	17.8 / 0.2	NA/ 5.7	20.7 / 0.2	59.8 / 0	88.4 / 0
Model na	77 / 0	46.1 / 0	14 / 0.3	NA/ 5	14.3 / 0.2	46 / 0	78.5 / 0
Model ta	92.1 / 0	60.3 / 0	17.8 / 0.3	NA/ 5.7	19.4 / 0.2	60 / 0	87.8 / 0

The table reports results on how well artificially infused earnings management is recognized in empirical data. For each combination of infused earnings management (from −3% to +3%) and number N of treated firm-years (10, 25, 50 or 100), we repeat the following three steps 1000 times: (1) randomly draw N firm-years (without replacement) from all 149,219 observations of our sample; (2) infuse the respective level of earnings management (EM) into these N firm-years; and (3) run a two-sided t-test for the three models to test the null hypothesis of no earnings management. The first number in each cell indicates the percentage of rejecting the null hypothesis at the 5% level (out of the 1000 repetitions) with the right sign of the t-statistic. The second number indicates the percentage of significant cases with the wrong sign of the t-statistic. Model da uses discretionary accruals, Model na normal accruals, and Model ta total accruals as defined in the Bayesian model.

Table 4
Detecting earnings management in empirical data: sampling related to reported earnings.

EM	−3%	−2%	−1%	0%	1%	2%	3%
Panel A: N = 10							
Model da	4.3 / 1	0.9 / 3.5	0.1 / 9.6	NA/ 24.8	46.5 / 0	66.5 / 0	76.8 / 0
Model na	24.7 / 0	15.1 / 0	8.3 / 0.4	NA/ 4.5	3.2 / 1.1	8 / 0.4	12.3 / 0.1
Model ta	8 / 0.1	2.1 / 2	0.6 / 4.5	NA/ 14.4	27.8 / 0.1	47.5 / 0	60.6 / 0
Panel B: N = 25							
Model da	4.4 / 1.1	0.5 / 8.4	0.1 / 28.2	NA/ 59.1	87.2 / 0	96.3 / 0	99.3 / 0
Model na	44.2 / 0	27.5 / 0	11.2 / 0.1	NA/ 6.3	5 / 1.3	16.9 / 0.2	30.8 / 0
Model ta	10.6 / 0.2	2.5 / 1.9	0.3 / 12.4	NA/ 36.3	65 / 0	86.2 / 0	96.6 / 0
Panel C: N = 50							
Model da	4.4 / 1.2	0 / 16.4	0 / 55.7	NA/ 88.5	98.9 / 0	100 / 0	100 / 0
Model na	67.9 / 0	43.8 / 0	16.9 / 0	NA/ 5.8	7.5 / 0.8	24.7 / 0	57.4 / 0
Model ta	13.4 / 0.3	1.6 / 4.1	0 / 23.3	NA/ 62.6	89.4 / 0	98.6 / 0	100 / 0
Panel D: N = 100							
Model da	3.8 / 1.7	0.1 / 28	0 / 85.2	NA/ 99.4	100 / 0	100 / 0	100 / 0
Model na	92.5 / 0	68 / 0	27.9 / 0.1	NA/ 6.7	12.9 / 0.3	50.8 / 0	86.7 / 0
Model ta	19.4 / 0.1	1.4 / 5.1	0 / 44.8	NA/ 90.8	99.7 / 0	100 / 0	100 / 0

The table reports results on how well artificially infused earnings management is recognized in empirical data. For each combination of infused earnings management (from −3% to +3%) and number N of treated firm-years (10, 25, 50 or 100), we repeat the following three steps 1000 times: (1) randomly draw N firm-years (without replacement) from the observations with above median reported earnings (ni) among the 149,219 observations of our sample; (2) infuse the respective level of earnings management (EM) into these N firm-years; and (3) run a two-sided t-test for the three models to test the null hypothesis of no earnings management. The first number in each cell indicates the percentage of rejecting the null hypothesis at the 5% level (out of the 1000 repetitions) with the right sign of the t-statistic. The second number indicates the percentage of significant cases with the wrong sign of the t-statistic. Model da uses discretionary accruals, Model na normal accruals, and Model ta total accruals as defined in the Bayesian model.

by replacing the implicit assumption of an uninformative prior distribution of true profitability with a more realistic informative prior distribution. Third, we provide a theoretical basis for distinguishing between three regression approaches commonly used in the earnings management literature, namely the modified Jones model, the modified Jones plus cash flow model, and the modified Jones plus profitability model. Which approach is appropriate, depends on the sampling of firms suspected of earnings management.

For random sampling, the modified Jones plus cash flow model is most efficient. For sampling with respect to reported earnings, a profitability adjustment is necessary, which is equivalent to using the modified Jones plus profitability model. For sampling with respect to the Bayesian estimate of true earnings, the modified Jones model is appropriate without adjustment.

Our simulations and tests based on empirical data for a comprehensive sample of US firms from 1988 to 2022 show that the choice of model is relevant when the treatment sample is not random. We propose a simple regression-based diagnostic to identify the relevant setting. However, while this diagnostic can provide researchers with a useful tool for deciding which variant of the Jones model to apply, we note that in empirical applications it may sometimes be difficult to distinguish sharply between the different settings.

To apply the Bayesian model, the expected values of normal accruals and discretionary accruals (μ_{NA} and μ_{DA}) must first be estimated. In our empirical study, we used the modified Jones model for this purpose. This means that the parameter μ_{NA} for a given firm is equal to the fitted value of the modified Jones regression, while μ_{DA} is equal to zero. The modified Jones model was a natural choice given its role

Table 5
Detecting earnings management in empirical data: sampling related to the Bayesian estimate of true earnings.

EM	−3%	−2%	−1%	0%	1%	2%	3%
Panel A: N = 10							
Model da	31 / 0	15 / 0	5.4 / 0.5	NA/ 4.6	9.1 / 0.2	23.9 / 0	38.5 / 0
Model na	77.8 / 0	65.9 / 0	48.1 / 0	NA/ 29.1	0.1 / 15.5	0.4 / 6.2	2.2 / 2.6
Model ta	44.9 / 0	23.7 / 0	11.4 / 0.1	NA/ 4.9	3.7 / 0.9	12.6 / 0.2	23.5 / 0
Panel B: N = 25							
Model da	64.7 / 0	34.3 / 0	9.3 / 0.4	NA/ 5	15 / 0.1	39.4 / 0	66.3 / 0
Model na	99.3 / 0	96.3 / 0	88.1 / 0	NA/ 70.2	0 / 43.8	0 / 18.8	0.8 / 5.2
Model ta	80.6 / 0	55.5 / 0	25 / 0	NA/ 5.8	5.4 / 1	20.4 / 0	42.9 / 0
Panel C: N = 50							
Model da	91.7 / 0	58.6 / 0	16.8 / 0.2	NA/ 5.9	25.3 / 0	60.7 / 0	86.4 / 0
Model na	100 / 0	100 / 0	99.3 / 0	NA/ 93.1	0 / 75.9	0 / 37.9	0.5 / 10.9
Model ta	98.3 / 0	84 / 0	47.2 / 0	NA/ 13.3	6 / 0.5	28.3 / 0	63.9 / 0
Panel D: N = 100							
Model da	99.5 / 0	87.5 / 0	30.7 / 0.2	NA/ 4.5	37.3 / 0	86.4 / 0	99 / 0
Model na	100 / 0	100 / 0	100 / 0	NA/ 99.9	0 / 96.5	0 / 69.4	0.3 / 18
Model ta	100 / 0	98.7 / 0	75.8 / 0	NA/ 19.9	7.4 / 0.6	45.5 / 0	89.4 / 0

The table reports results on how well artificially infused earnings management is recognized in empirical data. For each combination of infused earnings management (from −3% to +3%) and number N of treated firm-years (10, 25, 50 or 100), we repeat the following three steps 1000 times: (1) randomly draw N firm-years (without replacement) from the observations with above median Bayesian estimates of true earnings ($\widehat{NI^T}$) among the 149,219 observations of our sample; (2) infuse the respective level of earnings management (EM) into these N firm-years; and (3) run a two-sided t-test for the three models to test the null hypothesis of no earnings management. The first number in each cell indicates the percentage of rejecting the null hypothesis at the 5% level (out of the 1000 repetitions) with the right sign of the t-statistic. The second number indicates the percentage of significant cases with the wrong sign of the t-statistic. Model da uses discretionary accruals, Model na normal accruals, and Model ta total accruals as defined in the Bayesian model.

in the literature, but the Bayesian model is also compatible with any other method of estimating μ_{NA} and μ_{DA}. For example, if time series information suggests a reversal of accruals, this information can be used to improve the estimates of the parameters μ_{NA} and μ_{DA} before applying the Bayesian model to incorporate cross-sectional information. Our model is therefore complementary to firm-level accrual models.

An open question is how the Bayesian model would perform across the range of empirical settings examined in prior studies, and in which cases it can provide additional insights. Addressing this question requires systematic replications of existing work, which is beyond the scope of this paper.

The proposed Bayesian model has several limitations. In particular, it only considers accounting earnings management and does not cover real earnings management. It also does not model specific incentives for earnings management such as income smoothing. Both of these limitations, which also apply to the Jones model, could be addressed in future research. A restrictive assumption is that the model is based on normal distributions. However, a normal cross-sectional distribution of true earnings seems a natural choice in an attempt to overcome the unrealistic implicit assumption of the Jones model that no information on the distribution of true earnings is available.

CRediT authorship contribution statement

Peter Fiechter: Writing – review & editing, Validation, Resources, Data curation. **Martin Wallmeier:** Writing – review & editing, Writing – original draft, Software, Methodology, Formal analysis, Data curation, Conceptualization.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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Appendix A. Properties of Bayesian estimators

The following properties apply to the Bayesian estimators (20) and (21) in Section 2.3.3. The variances and covariances of Properties 1. to 3. are straight-forward to derive. We prove Property 5. in Appendix B. Properties 4. and 6. to 9. can be derived analogously.

1. The unconditional variances are: $Var\left(\widehat{NI^T}\right) = \sigma_0^2 - \sigma^{*2}$; $Var\left(\widehat{da}\right) = \sigma_{da}^2 - \sigma^{*2}$; and $Var\left(\widehat{na}\right) = \sigma_{na}^2 - \sigma^{*2}$. The deduction of σ^{*2} reflects a shrinkage towards the prior expected value, which is a well-known characteristic of Bayesian estimators. Therefore, the variances of the estimators are smaller than the variances of the unobservable actual variables.
2. The estimators $\widehat{da}$ and $\widehat{na}$ have a positive covariance of $Cov\left(\widehat{da}, \widehat{na}\right) = \sigma^{*2}$ even though da and na are independent. The posterior uncertainty σ^{*2} about the size of NI^T reflects the remaining uncertainty about the decomposition of total accruals into normal and discretionary accruals. As a consequence, higher total accruals tend to be attributed in part to both components, which is the source of their covariance.
3. The covariances of discretionary and normal accruals with ni and cf are preserved in the Bayesian estimators: $Cov\left(\widehat{da}, ni\right) = Cov(da, ni) = \sigma_{da}^2$; $Cov\left(\widehat{da}, cf\right) = Cov(da, cf) = 0$; $Cov\left(\widehat{na}, cf\right) = Cov(na, cf) = -\sigma_{na}^2$; and $Cov\left(\widehat{na}, ni\right) = Cov(na, ni) = 0$.
4. The estimator $\widehat{da}$ is proportional to the error term of a theoretical (population-level) linear regression of total accruals ta on cash flow cf. The regression equation is $ta = \alpha + \beta \cdot cf + \epsilon$, with constant α, slope $\beta = -\sigma_{na}^2/\sigma_{cf}^2 = -w_1/(w_1 + w_2)$, and error term ϵ. Then, $\widehat{da} = (w_1 + w_2) \cdot \epsilon$.
5. The estimator $\widehat{na}$ is proportional to the error term of a theoretical linear regression of total accruals ta on earnings ni. The regression equation is $ta = \alpha + \beta \cdot ni + \epsilon$, with constant α,

slope $\beta = \sigma_{da}^2/\sigma_{ni}^2 = w_1/(w_1 + w_3)$, and error term ϵ. Then, $\widehat{na} = (w_1 + w_3) \cdot \epsilon$.

6. The estimator $\widehat{da}$ is proportional to the error term of a theoretical linear regression of $\widehat{na}$ on cf. The regression equation is $\widehat{na} = \alpha + \beta \cdot cf + \epsilon$, with constant α, slope $\beta = -\sigma_{na}^2/\sigma_{cf}^2 = -w_1/(w_1+w_2)$, and error term ϵ. Then, $\widehat{da} = (w_1 + w_2)/w_3 \cdot \epsilon$. Together with Property 4 this implies that the error term of regressing $\widehat{na}$ on cf is equal to w_3 times the error term of regressing ta on cf.
7. The estimator $\widehat{na}$ is proportional to the error term of a theoretical linear regression of $\widehat{da}$ on ni. The regression equation is $\widehat{da} = \alpha + \beta \cdot ni + \epsilon$, with constant α, slope $\beta = \sigma_{da}^2/\sigma_{ni}^2 = w_1/(w_1 + w_3)$, and error term ϵ. Then, $\widehat{na} = (w_1 + w_3)/w_2 \cdot \epsilon$. Together with Property 5 this implies that the error term of regressing $\widehat{da}$ on ni is equal to w_2 times the error term of regressing ta on ni.
8. Total accruals ta are proportional to the error term of a theoretical linear regression of $\widehat{da}$ on $\widehat{NI^T}$. The regression equation is $\widehat{da} = \alpha + \beta \cdot \widehat{NI^T} + \epsilon$, with constant α, slope $\beta = \sigma^{*2}/(\sigma_0^2 - \sigma^{*2}) = w_1/(1 - w_1)$, and error term ϵ. Then, $ta = (1 - w_1)/w_2 \cdot \epsilon$.
9. Total accruals ta are proportional to the error term of a theoretical linear regression of $\widehat{na}$ on $\widehat{NI^T}$. The regression equation is $\widehat{na} = \alpha + \beta \cdot \widehat{NI^T} + \epsilon$, with constant α, slope $\beta = -\sigma^{*2}/(\sigma_0^2 - \sigma^{*2}) = -w_1/(1 - w_1)$, and error term ϵ. Then, $ta = (1 - w_1)/w_3 \cdot \epsilon$.

Appendix B. Proof of property 5. in Appendix A

To show that

$$\widehat{na} = (w_1 + w_3)\,\epsilon.$$

we write the left-hand side as

$$\widehat{na} = \widehat{NI^T} - cf = w_1\mu_0 + w_2 cf + w_3 ni - cf$$

and insert on the right-hand side

$$\epsilon = ta - \alpha - \frac{w_1}{w_1 + w_3} ni = ni - cf - \alpha - \frac{w_1}{w_1 + w_3} ni,$$

with

$$\alpha = \mu_{ta} - \frac{w_1}{w_1 + w_3}\mu_{ni} = -\frac{w_1}{w_1 + w_3}\mu_0.$$

Collecting terms related to ni, cf, and μ_0 leads to the stated result.

Appendix C. Most efficient test with random sampling

In the following, we prove the property mentioned in Section 2.4 that the most powerful test for detecting earnings management in a setting with random sampling is based on discretionary accruals. The following applies: $w_1, w_2, w_3 > 0$; $w_1 + w_2 + w_3 = 1$; $\sigma^{*2} = w_1\sigma_0^2 = w_2\sigma_{na}^2 = w_3\sigma_{da}^2$; $\sigma_{da}^2 - \sigma^{*2} > 0$; $\sigma_{na}^2 - \sigma^{*2} > 0$. We first compare ratio z_{da} as defined in Eq. (25) with ratio z_{ta} as defined in Eq. (24):

$$\begin{aligned}
& z_{da} > z_{ta} \\
\Leftrightarrow\ & (w_1 + w_2)\sqrt{\sigma_{da}^2 + \sigma_{na}^2} > \sqrt{\sigma_{da}^2 - \sigma^{*2}} \\
\Leftrightarrow\ & (w_1 + w_2)^2 \left(\sigma_{da}^2 + \sigma_{na}^2\right) > \sigma_{da}^2 - \sigma^{*2} \\
\Leftrightarrow\ & \sigma^{*2} > \sigma_{da}^2 - (w_1 + w_2)^2 \left(\sigma_{da}^2 + \sigma_{na}^2\right) \\
\Leftrightarrow\ & \sigma^{*2} > \sigma_{da}^2 - (1 - w_3)^2 \sigma_{da}^2 - (w_1 + w_2)^2 \sigma_{na}^2
\end{aligned}$$

Multiplying out and using $\sigma^{*2} = w_1\sigma_0^2 = w_2\sigma_{na}^2 = w_3\sigma_{da}^2$, we obtain:

$$\begin{aligned}
& \sigma^{*2} > \sigma^{*2}\left(2 - w_3 - 2w_1 - w_2 - w_1\frac{\sigma_{na}^2}{\sigma_0^2}\right) \\
\Leftrightarrow\ & 1 > 2 - (w_1 + w_2 + w_3) - w_1\left(1 + \frac{w_1}{w_2}\right) \\
\Leftrightarrow\ & 0 > -w_1\left(1 + \frac{w_1}{w_2}\right)
\end{aligned}$$

This inequality is true since $w_1, w_2, w_3 > 0$.

Comparing z_{da} with z_{na} as defined in Eq. (26), we obtain:

$$\begin{aligned}
& z_{da} > z_{na} \\
\Leftrightarrow\ & (w_1 + w_2)^2 \left(\sigma_{na}^2 - \sigma^{*2}\right) > w_3^2 \left(\sigma_{da}^2 - \sigma^{*2}\right) \\
\Leftrightarrow\ & (1 - w_3)^2 \left(\sigma_{na}^2 - \sigma^{*2}\right) - w_3^2 \left(\sigma_{da}^2 - \sigma^{*2}\right) > 0 \\
\Leftrightarrow\ & (1 - w_3)^2 \sigma_{na}^2 - (1 - w_3)^2 \sigma^{*2} - w_3\sigma^{*2} + w_3^2\sigma^{*2} > 0 \\
\Leftrightarrow\ & (1 - w_3)^2 \sigma_{na}^2 - (1 - w_3)^2 \sigma^{*2} - w_3(1 - w_3)\sigma^{*2} > 0 \\
\Leftrightarrow\ & (1 - w_3)\sigma_{na}^2 - (1 - w_3)\sigma^{*2} - w_3\sigma^{*2} > 0 \\
\Leftrightarrow\ & (1 - w_3)\sigma_{na}^2 - \sigma^{*2} > 0 \\
\Leftrightarrow\ & (1 - w_3)\sigma_{na}^2 - w_2\sigma_{na}^2 > 0 \\
\Leftrightarrow\ & 1 - w_3 > w_2 \\
\Leftrightarrow\ & 1 - w_3 > 1 - w_3 - w_1 \\
\Leftrightarrow\ & w_1 > 0.
\end{aligned}$$

Data availability

The link to our data and code is available at: https://researchbox.org/4446.

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