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How to promote R&D-based growth?
Public education expenditure on scientists
and engineers versus R&D subsidies

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Abstract

This paper compares the positive and normative implications of two alternative measures to promote R&D-based growth: R&D subsidies to firms and publicly provided education targeted to the development of science and engineering (S&E) skills. The model accounts for the specificity of S&E skills, where individuals with heterogeneous ability choose their type of education. Although intertemporal knowledge spillovers are the only R&D externality, the analysis suggests that R&D subsidies may be detrimental to both productivity growth and welfare. Moreover, they raise earnings inequality. In contrast to R&D subsidies, publicly provided education targeted to S&E skills are found to be unambiguously growth-promoting and neutral with respect to the earnings distribution.
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1. Introduction

R&D may have positive as well as negative externalities. This leaves the question whether there is under- or overinvestment in R&D (compared to the social optimum) theoretically ambiguous.[1] By trying to shed light into this ambiguity, both empirical evidence and calibration exercises in R&D-based models of economic growth strongly suggest that the social return to R&D significantly exceeds the private return to R&D (e.g., Jones and Williams, 1998, 2000; Park, 2004; Alvarez-Pelaez and Groth, 2005). For instance, Jones and Williams (1998) argue that a ''conservative estimate indicates that optimal investment in research is more than two to four times actual investment'' (p. 1134).

Such evidence has alarmed policy makers. For instance, the Barcelona European Council 2002 has brought consensus among EU members ''to increase the average research investment level from 1.9% of GDP today to 3% of GDP by 2010, of which 2/3 should be funded by the private sector'' (COM, 2003, p. 3). In particular, the European Commission seems to be ready to provide firms with more financial incentives to invest in R&D, arguing that ''[p]ublic support is justified by the recognised failure of the market to induce business investment in research at an optimal level'' (COM, 2003, p. 19). However, as pointed out by Romer (2000) in his informal discussion about US government policies to encourage R&D spending, ''[f]ew participants in [the political debate surrounding demand-subsidy policies] seem to have considered the broad range of alternative programs that could be considered'' (pp. 5–6). So the question is not whether public policy should promote R&D but *how* to do it.

This paper attempts to contribute to this debate by comparing positive and normative effects of two alternative measures to foster R&D-based growth: (demand-side) R&D subsidies and (supply-side) publicly provided education targeted to the development of science and engineering (S&E) skills. It develops an overlapping generations model where productivity growth is driven by in-house R&D of monopolistically competitive firms. It rests on three novel elements. First, R&D occupations require *specific skills*, which may differ substantially from skills applicable in non-R&D tasks. For instance, scientists or engineers need different skills than machine operators or lawyers. Second, and related to this, individuals choose which skill type to acquire: S&E skills for employment in R&D jobs, skills applicable for non-R&D tasks, or to remain unskilled. Third, individuals are heterogeneous in their ability to perform R&D jobs when choosing to acquire S&E skills. Focussing on this heterogeneity captures that R&D activities require extraordinary talent.[2]

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[1] At least four externalities have been identified by the literature (see e.g. Jones, 2005). First, innovators do not take into account that their R&D output may enhance capabilities of future innovators, which has been called ''standing on shoulders'' or ''intertemporal knowledge spillover'' effect, introduced by Romer (1990). Second, the equilibrium mark up which innovators can charge for a new design may not coincide with the consumer surplus created by a new good, i.e., innovating firms can appropriate only part of the surplus (Jones and Williams, 2000). These two distortions promote underinvestment in R&D activities. Third, when new goods replace older goods, gains from past innovating effort is lost. This ''business-stealing'' effect, introduced by Grossman and Helpman (1991) and Aghion and Howitt (1992), serves as a negative externality of R&D investments. Finally, overinvestment in R&D is also promoted by patent races, in which different firms work on similar R&D projects in the hope to be the first to be assigned a patent for their innovation (''duplication externality'').

[2] Growth theory has successfully integrated models in which R&D and human capital accumulation are engines of growth by emphasizing the complementarity between these two factors for the process of development (e.g., Redding, 1996; Arnold, 1998; Funke and Strulik, 2000; Strulik, 2004). However, to the best of my knowledge, the literature has yet not accounted for an endogenous formation of specific S&E skills and heterogeneity in ability.

Intertemporal knowledge spillovers are the only externality from R&D in the model. According to conventional wisdom, this calls for positive R&D subsidies to firms in order to induce a reallocation of labor towards R&D activity. This policy recommendation critically depends on the assumption, however, that the (high-skilled) labor force is capable to perform both R&D and production activities without having to adjust to a change in occupation. One obvious drawback of this assumption is that labor supply of scientists and engineers is rather inelastic in the short-run, i.e., R&D subsidies are absorbed by rising wage rates for R&D labor services (e.g. Romer, 2000).

The present analysis is consistent with this evidence on short-run effects. But more interestingly, it suggests that even by accounting for skill supply responses of R&D subsidies, earnings of scientists and engineers rise unambiguously. This result is driven by the heterogeneity of individuals in ability. It is consistent with empirical evidence by Goolsbee (1998), who finds that a 10% increase in government spending on R&D affects both income and hourly wages of scientists and engineers by 3% even in the longer run. Moreover, we find that despite a positive externality from R&D activity, R&D subsidies may be *detrimental* to both productivity growth and welfare.

Fortunately, the analysis suggests a sensible and straightforward alternative to promote R&D-based growth: to target public R&D spending directly to the supply of skills. First, promotion of S&E talent does not affect the distribution of earnings in the proposed framework, in contrast to the impact of higher R&D subsidies to firms on earnings inequality. Moreover, and also in contrast to R&D subsidies, higher education spending on S&E skills unambiguously raises productivity growth. The socially optimal *structure* of education expenditure (allocation of public education funds to the development of S&E and non-S&E skills) depends on the interaction between the relative effectiveness of the education system across skills and the effectiveness of private-sector R&D spending relative to the output elasticity of non-S&E skills.

The critical role of education in scientific and technical fields in the model is consistent with recent empirical literature which emphasizes the importance of specific R&D skills for innovations and performance of firms. For instance, technical skills cluster in firms which invest in R&D. More importantly, there is a positive interaction between technical skills and innovation output which determines the profitability of firms, controlling for R&D intensity (see Leiponen, 2005, and the references therein). This strongly suggests that successful innovations depend on S&E skills in a firm rather than R&D spending per se. Colombo and Grilli (2005) examine the role of specific human capital in scientific and technical fields vis-à-vis other education fields of entrepreneurs for growth of new firms in high-tech sectors. They find that the average years of founders' education has no significant effect whereas education in S&E fields has large positive effects.

The paper is organized as follows. Section 2 presents the structure of the model. Section 3 derives both the short-run and long-run effects of public policy measures. Section 4 studies the socially optimal policy design with respect to both R&D subsidies and public education expenditure. Section 5 discusses the results and concludes. All proofs are relegated to Appendix.

2. The model

Consider the following overlapping-generations economy, where each generation is populated by L individuals.

2.1. Individuals

Individuals live for two periods. In the first period of life, they live with their parents and decide whether to specialize in S&E skills (i.e., to work as scientist or engineer), to acquire skills applicable in more routinized production processes, or to remain unskilled. There is a unit time endowment in the first period of life, devoted to the acquisition of skills and leisure. Acquiring S&E skills is necessary to perform R&D tasks.[3] It requires $z^R \in (0,1)$ units of time, whereas acquiring production skills requires $z^S \in (0,1)$ units of time. In the second period of life (adulthood), individuals supply their skills inelastically to a perfect labor market. Individuals differ in the ability to perform R&D tasks after having acquired S&E skills, denoted by a (as will be specified in Section 2.3). In order to focus the analysis on an ability type which is relevant for knowledge spillovers and growth, this is the only source of individual heterogeneity in the model.[4]

Intertemporal preferences of an individual i born in $t-1$ (i.e., a member i of generation $t-1$) are defined over leisure time in the first period of life, $d_{t-1}(i)$, and consumption during adulthood. The utility function is specified as

$$U_{t-1}(i) = \ln d_{t-1}(i) + \ln \left(\int_0^{n_t} (\tilde{x}_t(i,j))^{\frac{\eta-1}{\eta}} \mathrm{d}j \right)^{\frac{\eta}{\eta-1}}, \tag{1}$$

$\eta > 1$, where $\tilde{x}_t(i,j)$ denotes the quantity of a good $j \in [0,n_t]$ consumed by member i of generation $t-1$ in period t. The measure n_t is referred to as the "number of products" in t.

2.2. Goods production and productivity

Each producer manufactures one variety of the differentiated goods in monopolistic competition. Firms have the following constant-returns-to-scale production technology:

$$x_t(j) = A_t(j)F(l_t^S(j), l_t^U(j)) \equiv A_t(j)l_t^U(j)f(\chi_t(j)), \quad \chi_t(j) \equiv l_t^S(j)/l_t^U(j), \tag{2}$$

where $l_t^S(j)$ and $l_t^U(j)$ denote efficiency units of skilled and unskilled *production-related* labor employed in firm j at date t, respectively, whereas $x_t(j)$ and $A_t(j)$ are output and total factor productivity of firm j in t. $f(\cdot)$ is an increasing and strictly concave function.

In each period t, firm j can affect productivity $A_t(j)$ by employing scientists and engineers.[5] In line with growth theory based on in-house R&D (e.g., Young, 1998) and the IO literature on innovation activities (e.g., Sutton, 1998), R&D outlays are

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[3] This captures the notion that, say, a lawyer or bookkeeper is not capable to develop computer processors or software. The set up allows for the possibility that S&E skills are applicable in skill-intensive non-R&D jobs and that skilled workers can be employed (in the same occupations) as unskilled workers. As will become apparent, however, such situations do not arise in equilibrium.

[4] This is not to deny that, for instance, there are productivity differences among students graduating in law, but these skills do not seem to foster growth. For instance, Murphy et al. (1991) present empirical cross-country evidence that the fraction of students in engineering fields (around 10% on average in their sample) is positively related to growth, whereas the fraction of law students (around 9% on average) even adversely affects growth.

[5] An alternative formulation is that firms have to incur R&D expenditure one period in advance of production (financed by borrowing), like in the (discrete-time) infinite-horizon growth model of Young (1998). However, this assumption seems to be less plausible in an OLG model. Rather, for simplicity (since irrelevant for the main arguments of this paper), the analysis abstracts from savings and asset markets.

(endogenous) sunk costs for firms. Productivity $A_t(j)$ of firm j in any period $t \geqslant 0$ evolves according to

$$A_t(j) = \overline{S}_{t-1} h(l_t^R(j)), \tag{3}$$

where $l_t^R(j)$ denotes the efficiency units of R&D labor investments of firm j in t and function $h(\cdot)$ is increasing. The term $\overline{S}_{t-1}$ captures public knowledge at time t from previous investments of firms in R&D. It depends on the average productivity of firms in $t-1$, $\overline{A}_{t-1} \equiv \int_0^{n_{t-1}} A_{t-1}(j)\,\mathrm{d}j$. That is, innovations create proprietary knowledge for one period only. Moreover, possibly $\overline{S}_{t-1}$ depends on the number of firms, n_{t-1}. ($\overline{S}_{-1} > 0$ is given.) Formally, let

$$\overline{S}_{t-1} = \overline{A}_{t-1}(n_{t-1})^{1-\varepsilon}, \tag{4}$$

$0 < \varepsilon \leqslant 1$. The number component captures that innovations of firms are not "equivalent" in their contribution to public knowledge which can be accessed by firms for future innovations. Two remarks are in order. First, if each firm chooses the same R&D labor investment, i.e., if $l_t^R(j) = l_t^R$ for all j (which will be the case in equilibrium), then the growth rate of average productivity, $\vartheta_t = \overline{A}_t / \overline{A}_{t-1} - 1$, is given by

$$\vartheta_t = (n_{t-1})^{1-\varepsilon} h(l_t^R) - 1, \tag{5}$$

$t \geqslant 1$. Hence, in steady state (where n and l^R are time-invariant), there is balanced growth. Second, as will become apparent, a *scale effect* with respect to the economy's growth rate (i.e., ϑ_t rises with population size L) occurs if $\varepsilon < 1$ but not if $\varepsilon = 1$. This is because a higher population size leaves equilibrium R&D labor investment per firm (l^R) unaffected, whereas it raises the number of firms, n.

There is free entry of firms into the economy, with a large number of potential entrants. At all times, firms have to incur standard fixed cost $\bar{l} > 0$ in terms of unskilled labor.[6] Since $\bar{l}$ has to be incurred each period and innovations remain private knowledge by firms for only one period, the length of the planning horizon of firms is exactly one period (Young, 1998).

2.3. Educational production and government spending

To focus on public policy issues, education is publicly financed.[7] Denote public expenditure levels for S&E skills and production skills of generation $t-1$ by G_{t-1}^R and G_{t-1}^S, and the population share of either type of worker in period t (one period after receiving education) by s_t^R and s_t^S, respectively; thus, $s_t^U = 1 - s_t^R - s_t^S$ is the population share of unskilled workers in t. The respective spending levels per student are given by

$$g_{t-1}^R = \frac{G_{t-1}^R}{s_t^S L}, \quad g_{t-1}^S = \frac{G_{t-1}^S}{s_t^S L}. \tag{6}$$

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[6] The main results are unaffected if production would also require a fixed staff of skilled, non-R&D workers. However, the additional analytical complexity would be substantial.

[7] One standard justification for public finance of education is the incapability of individuals to borrow for educational purposes. It is straightforward to allow for certain forms of private human capital investments as well, without affecting the main results of this study.

Suppose that efficiency units of workers who specialize in a type of skill depend on education expenditure per student (g^R, g^S). Thus, the rivalry of educational spending creates a negative externality from educational choice: given total expenditure levels, an individual who decides to acquire education does not take into account the negative effect on efficiency units of others within the skill group.

Formally, denote the set of individuals (of generation $t-1$) who supply S&E skills and production skills in period t by $\mathscr{R}_t$ and $\mathscr{S}_t$, respectively. (Thus, $s_t^R L = \int_{\mathscr{R}_t} di$ and $s_t^S L = \int_{\mathscr{S}_t} di$.) An individual $i \in \mathscr{R}_t$ with ability $a(i)$ acquires

$$e_t^R(a(i)) = a(i)\tilde{e}^R(g_{t-1}^R) \tag{7}$$

efficiency units of R&D labor. For simplicity, suppose ability a is uniformly distributed on the unit interval, $a \sim uniform[0,1]$.[8] Individuals $i \in \mathscr{S}_t$ acquire

$$e_t^S = \tilde{e}^S(g_{t-1}^S) \tag{8}$$

efficiency units of production skills. If remaining unskilled, an individual owns one unit of unskilled labor ($e_t^U = 1$).

Let w_t^R and w_t^S denote the wage rate *per efficiency unit* of S&E skills and production skills at date t, respectively. Unskilled labor is chosen as numeraire ($w_t^U = 1$). Using (7) and (8), the nominal income level of a member i of generation $t-1$ (with ability $a(i)$), conditional on her educational choice, is thus given by

$$I_t(i) = \begin{cases} w_t^R a(i)\tilde{e}^R(g_{t-1}^R) \equiv I_t^R(a(i)), & \text{if } i \in \mathscr{R}_t, \\ w_t^S \tilde{e}^S(g_{t-1}^S) \equiv I_t^S, & \text{if } i \in \mathscr{S}_t, \\ 1, & \text{otherwise.} \end{cases} \tag{9}$$

Let L_t^R, L_t^S and L_t^U denote aggregate supply of efficiency units of R&D labor, skilled production labor, and unskilled labor in t, respectively, where initial values L_0^R, L_0^S and L_0^U are given.

At each t, besides financing education, the government may subsidize R&D spending of firms at rate $\mu_t \in [0,1]$. For the analysis of dynamic policy effects, suppose that μ_t is announced at least one period in advance. (This implies that members of generation $t-1$ take μ_t into account when choosing among educational fields.) Both education expenditure and R&D subsidies are financed by a proportional income tax on workers, where $\tau_t \in [0,1)$ denotes the tax rate at date t. The government budget is balanced each period.

3. Equilibrium analysis

Suppose that functions f and h in (2) and (3), respectively, have isoelastic forms:

$$f(\chi) = \chi^\alpha, \quad h(l^R) = (l^R)^\gamma, \tag{10}$$

where $\alpha \in (0,1)$ is the output elasticity of production skills and elasticity $\gamma > 0$ measures the effectiveness of R&D. As argued below, these specifications do not systematically bias policy implications, but considerably improve analytical tractability.

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[8] As will become apparent, heterogeneity in ability drives dynamic policy effects on the income distribution.

Individual demand functions, $\tilde{x}_t^D(i,j) = (1 - \tau_t)I_t(i)p_t(j)^{-\eta} / \int_0^{n_t} p_t(j)^{1-\eta} \mathrm{d}j$ in period t, implied by (1), give rise to the following demand functions faced by firm j:

$$x_t^D(j) = \frac{E_t p_t(j)^{-\eta}}{P_t^{1-\eta}}, \quad P_t \equiv \left(\int_0^{n_t} p_t(j)^{1-\eta} \mathrm{d}j \right)^{\frac{1}{1-\eta}}, \tag{11}$$

where $E_t \equiv (1 - \tau_t)(w_t^R L_t^R + w_t^S L_t^S + L_t^U)$ is aggregate (nominal) expenditure for consumption goods (which equals aggregate disposable income of generation $t - 1$ during adulthood) and $p_t(j)$ denotes the price of good j in t. Using (11), output prices are set according to the well-known formula

$$p_t(j) = \frac{\eta}{\eta - 1} c_t(j), \tag{12}$$

$t \geqslant 0$ (Dixit and Stiglitz, 1977), where

$$c_t(j) = \frac{\alpha^{-\alpha}(1 - \alpha)^{-(1-\alpha)}(w_t^S)^\alpha}{A_t(j)} \tag{13}$$

is marginal production cost (recall $w_t^U = 1$). Profits of firm j in t, $\pi_t(j)$, are given by

$$\pi_t(j) = (p_t(j) - c_t(j))x_t^D(j) - (1 - \mu_t)w_t^R l_t^R(j) - \bar{l}. \tag{14}$$

Thus, using (3), (11), (12), (13) and (14), firm j solves

$$\max_{l_t^R(j)} \left\{ \left(\frac{P_t \overline{S}_{t-1} h(l_t^R(j))}{(w_t^S)^\alpha} \right)^{\eta-1} \Gamma E_t - (1 - \mu_t)w_t^R l_t^R(j) - \bar{l} \right\}, \tag{15}$$

where $\Gamma \equiv \alpha^{\alpha(\eta-1)}(1 - \alpha)^{(1-\alpha)(\eta-1)}(\eta - 1)^{\eta-1}/\eta^\eta > 0$, $t \geqslant 0$. (Firms take μ_t, w_t^S, w_t^R, E_t and P_t as given.) The first-order condition for the optimal choice of R&D labor implies that the marginal benefit of an increase in R&D labor in t must equal its marginal cost, where the latter is decreasing in the R&D subsidy rate μ_t. Moreover, $l_t^R(j) = l_t^R$, and thus, $A_t(j) = \overline{A}_t$, $c_t(j) = c_t$, $p_t(j) = p_t$, $x_t^D(j) = x_t^D$, $l_t^S(j) = l_t^S$ and $l_t^U(j) = l_t^U$ for all j, i.e., there is symmetry in equilibrium. Using (10) and (15), it is easy to show that for the second-order condition of a profit maximum to hold, $1 > \gamma(\eta - 1)$ is required, which is assumed throughout the paper.

The following equilibrium conditions must hold for any $t \geqslant 0$:

(E1) $x_t^D = \overline{A}_t F(l_t^S, l_t^U)$ (goods market equilibrium),
(E2) $n_t(l_t^U + \bar{l}) = L_t^U$, $n_t l_t^R = L_t^R$, and $n_t l_t^S = L_t^S$ (labor market clearing),
(E3) $\pi_t(j) = 0$ for all $j \in [0, n_t]$, i.e., $(p_t - c_t)x_t^D = (1 - \mu_t)w_t^R l_t^R + \bar{l}$ (free entry).

Zero-profit condition (E3), which says that gross profits equal sunk costs of each firm due to free entry, will imply a unique equilibrium number of firms, n. This is because the "love of variety" property in utility function (1) implies that consumers' willingness to pay for each good decreases as the number of goods increases (Dixit and Stiglitz, 1977). Hence, under symmetry, gross profits of each firm are decreasing in n.

3.1. Equilibrium for given educational choice

We are interested in policy implications of education spending on either type of skill vis-à-vis R&D subsidies to firms. For this purpose, it is helpful to first derive the equilibrium

income levels for given educational choices, which determine education incentives for individuals. (All results are proven in Appendix.)

Lemma 1. *For given $\mathscr{R}_t$ and $\mathscr{S}_t$, in equilibrium we have*

$$I_t^{\mathrm{R}}(a(i)) = \frac{a(i)\gamma(\eta - 1)L(1 - s_t^{\mathrm{R}} - s_t^{\mathrm{S}})}{\Xi(1 - \mu_t)\int_{\mathscr{R}_t} a(i)\,\mathrm{d}i}, \tag{16}$$

$$I_t^{\mathrm{S}} = \frac{\alpha(\eta - 1)(1 - s_t^{\mathrm{R}} - s_t^{\mathrm{S}})}{\Xi s_t^{\mathrm{S}}}, \tag{17}$$

$t \geqslant 1$, *where* $\Xi \equiv 1 + (1 - \alpha - \gamma)(\eta - 1) > 0$.

We first turn to analyze comparative-static effects with respect to policy variables for given educational choices. One may refer to this as static policy effects. These capture the impact of policy changes which are unanticipated by individuals.

Proposition 1 (Static policy effects). *For given educational choice, an increase in μ_t affects the income distribution in favor of scientists and engineers without affecting productivity, $\overline{A}_t$, $t \geqslant 0$. An increase in G_{t-1}^{R} raises $\overline{A}_t$ without affecting the income distribution. An increase in G_{t-1}^{S} does neither affect $\overline{A}_t$ nor the income distribution, $t \geqslant 1$.*

Proposition 1 implies that an increase in R&D subsidies to firms, which is unanticipated by individuals, merely serves as a windfall gain for individuals who happen to possess S&E skills. Thus, inequality across educational groups is raised, without reducing R&D costs of firms. Consequently, inventive activity remains unchanged. These results are an implication of the assumption that S&E skills need time to develop, i.e., are in inelastic supply in the short-run, as has been argued (informally) in the previous literature (Goolsbee, 1998; Romer, 2000). In contrast, higher educational spending targeted to S&E (but not to other) skills raises efficiency units of R&D labor per firm next period and thus increases productivity, for given educational choices. Finally, as immediately implied by Lemma 1 and discussed below in some detail, education policy does not affect the income distribution.

3.2. Equilibrium with endogenous educational choice

As will become apparent, interestingly, dynamic effects of public policy, taking into account adjustments in education choices, are quite similar to static policy effects.

3.2.1. Equilibrium income levels

Substituting individual demand functions, $\tilde{x}_t^{\mathrm{D}}(i, j)$, into utility function (1), using (9), and observing time requirements z^{R} and z^{S} for the acquisition of skills, indirect life-time utility of individual i from generation $t - 1$, $V_{t-1}(i)$, reads

$$V_{t-1}(i) = \begin{cases} \ln(1 - z^{\mathrm{R}}) + \ln\left(\frac{(1 - \tau_t)I_t^{\mathrm{R}}(a(i))}{P_t}\right), & \text{if } i \in \mathscr{R}_t, \\ \ln(1 - z^{\mathrm{S}}) + \ln\left(\frac{(1 - \tau_t)I_t^{\mathrm{S}}}{P_t}\right), & \text{if } i \in \mathscr{S}_t, \\ \ln\left(\frac{1 - \tau_t}{P_t}\right), & \text{otherwise,} \end{cases} \tag{18}$$

$t \geqslant 1$. Since individuals differ only in the ability to perform R&D tasks after acquiring S&E skills, in equilibrium, each production worker must be indifferent whether to acquire

production skills or to remain unskilled. Thus, $(1 - z^S)I_t^S = 1$, according to (18). More-over, (18) implies that individuals choose to become scientist or engineer if $(1 - z^R)I_t^R(a) \geqslant 1$. Since $I_t^R(a)$ is increasing in a (Lemma 1), there exists a unique thresh-old ability level at each date t, denoted $\tilde{a}_t$, which is given by $(1 - z^R)I_t^R(\tilde{a}_t) = 1$.[9] Conse-quently, for any $t \geqslant 1$, the set of workers who acquire S&E skills is given by $\mathscr{R}_t = \{i | a(i) \geqslant \tilde{a}_t\}$. Recalling that ability a is uniformly distributed on the unit interval, this implies $s_t^R = (\int_{\mathscr{R}_t} \mathrm{d}i)/L = \int_{\tilde{a}_t}^1 \mathrm{d}a = 1 - \tilde{a}_t$, $t \geqslant 1$. The following proposition summa-rizes these results and states, in addition, how educational shares s_t^R, s_t^S and s_t^U, and equi-librium income levels of scientists and engineers, $I_t^R(a)$, depend on policy parameters, μ_t, G_{t-1}^R and G_{t-1}^S.

Proposition 2 (Educational choice and equilibrium income). *For any* $t \geqslant 1$,

(a) $I_t^S = (1 - z^S)^{-1} > 1$;
(b) *there exists a unique threshold ability level*

$$\tilde{a}_t = \left[\frac{(1 - \mu_t)\Theta}{2(1 - z^R)\gamma(\eta - 1) + (1 - \mu_t)\Theta} \right]^{\frac{1}{2}} \equiv a^*(\mu_t), \tag{19}$$

$\Theta \equiv 1 + (1 - \gamma - z^S \alpha)(\eta - 1) > 0$, *such that all members of generation* $t - 1$ *with* $a(i) \geqslant \tilde{a}_t$ *become R&D workers*;
(c) $\tilde{a}_t = a^*(\mu_t)$ *is decreasing in* μ_t, *and thus*, $s_t^R = 1 - \tilde{a}_t$ *is increasing in* μ_t; *moreover, both* $\tilde{a}_t$ *and* s_t^R *are independent of* G_{t-1}^R, G_{t-1}^S;
(d) *both* s_t^S *and* s_t^U *are decreasing in* μ_t *and independent of* G_{t-1}^R, G_{t-1}^S;
(e) *finally, we have*

$$I_t^R(a) = \frac{a}{(1 - z^R)\tilde{a}_t}. \tag{20}$$

Thus, for all $a \in (\tilde{a}_t, 1]$, $I_t^R(a)$ *is increasing in* μ_t *and independent of* G_{t-1}^k, $k = $ R, S.

Comparative-static results in Proposition 2 can be understood as follows. Since changes in public education spending, G_{t-1}^R or G_{t-1}^S, have no impact on income levels for given edu-cational choices of generation $t - 1$, according to Lemma 1, they do not affect educational choices. In contrast, an increase in the R&D subsidy rate, μ_t, by raising demand for R&D labor, has a positive impact on the fraction of scientists and engineers in the population, s_t^R. Thus, an increase in μ has a negative impact on both s_t^S and s_t^U. The additional supply of scientists or engineers induced by an increase in μ_t stems from workers with mediocre abilities. Thus, implementing a R&D subsidy raises labor income for all individuals who would become researchers even if $\mu_t = 0$. In other words, an increase in μ_t raises the *return to ability*, i.e., income $I_t^R(a)$ is increasing in μ_t whenever $a > \tilde{a}_t$. Noteworthy, this occurs *despite* an increase in the number of workers who acquire S&E skills, $s_t^R L$.[10]

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[9] Educational choices do not depend on the income tax rate, τ_t. Thus, there are no distortions of educational decisions through income taxation which would arise from, say, a progressive tax system. Consequently, no problem of indeterminacy of equilibrium arises under a balanced budget rule of the government, unlike in the neoclassical growth model (Schmitt-Grohé and Uribe, 1997).

[10] This effect is similar to one derived in Galor and Moav (2000) in a different context in which an increase in technological progress raises the return to innate (and heterogeneous) cognitive ability.

3.2.2. Income distribution

To illustrate the latter point in more detail, let us consider the impact of an increase in μ_t on two inequality measures: first, on earnings inequality within the group of R&D workers, and second, on inequality between R&D labor and production workers. As income of R&D workers is proportional to ability and ability is uniformly distributed, it is appropriate to define a measure of *within-group* inequality in period t as ratio of the top to bottom earners within this group,[11] $\sigma_t^R \equiv I_t^R(1)/I_t^R(\tilde{a}_t)$. Thus, $\sigma_t^R = 1/\tilde{a}_t$, according to (20). Moreover, *between-group* inequality, denoted $\sigma_t^{R/P}$, is defined as ratio of average income levels between R&D workers, $\bar{I}_t^R \equiv (1/s_t^R) \int_{\tilde{a}_t}^{1} I_t^R(a) \, da$, and production workers, $\bar{I}_t^P \equiv (s_t^S I_t^S + s_t^U)/(s_t^S + s_t^U)$, i.e., $\sigma_t^{R/P} \equiv \bar{I}_t^R/\bar{I}_t^P$. The following result arises.

Proposition 3 (Dynamic distributional policy effects). *Both σ_t^R and $\sigma_t^{R/P}$ are increasing in μ_t, and independent of G_{t-1}^R, G_{t-1}^S.*

Thus, even if R&D subsidies are fully taken into account by individuals in their educational choice, R&D subsidies are positively related to income inequality according to both measures, within-group and between-group inequality. Noteworthy, this result does not hinge on a weak short-run supply elasticity of S&E skills (compare with Proposition 1). Rather it is an implication of the heterogeneity in ability. This is a novel aspect in the literature on R&D subsidies. In contrast, public provision of education of either kind does not affect earnings inequality, according to Proposition 3. Hence, whether public policy addresses demand or supply of S&E skills has very different distributional effects. Whereas the analysis suggests that public provision of education may be neutral to inequality, R&D subsidies to firms are not.

3.2.3. Productivity growth

As will become apparent, public education expenditure on scientists and engineers also fosters productivity growth unambiguously, such that there is no trade-off between equity and efficiency with respect to this policy measure. In contrast, higher R&D subsidies may depress growth. To see this, and for deriving the socially optimal policy design (Section 4), the following result is helpful.

Lemma 2. *For any $t \geq 1$, in equilibrium with endogenous educational choice:*

$$n_t = \frac{La^*(\mu_t)[1 - \gamma(\eta - 1)]}{\Theta \bar{l}} \equiv n^*(\mu_t), \tag{21}$$

$$l_t^R = \frac{\bar{l}\Theta\xi^R \tilde{e}^R\left(\frac{G_{t-1}^R/L}{1-a^*(\mu_t)}\right)[1 - a^*(\mu_t)^2]}{2[1 - \gamma(\eta - 1)]a^*(\mu_t)} \equiv l^{R*}(\mu_t, G_{t-1}^R), \tag{22}$$

$$w_t^S = \frac{1}{(1 - z^S)\tilde{e}^S\left(\frac{\Theta G_{t-1}^S/L}{(1-z^S)\alpha(\eta-1)a^*(\mu_t)}\right)} \equiv w^{S*}(\mu_t, G_{t-1}^S). \tag{23}$$

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[11] A similar measure has been applied by Galor and Moav (2000), who also assume a uniform ability distribution. Ability has a different interpretation in their model, however.

[Proposition 2](#) and [Lemma 2](#) imply that the economy is in its steady state from period 1 onwards and there is balanced growth of productivity if public policy does not change over time,[12] i.e., if

$$\mu_t = \bar{\mu}, \quad G_t^R = \overline{G}^R, \quad \text{and } G_t^S = \overline{G}^S \quad \text{for all } t \geqslant 1. \tag{24}$$

Also note that, as claimed in Section [2.2](#), because the equilibrium number of firms is rising in population size L, in the case where $\varepsilon < 1$ there is a scale effect in the growth rate for $t \geqslant 1$, $\vartheta_{t+1} = (n_t)^{1-\varepsilon} h(l_{t+1}^R) - 1$. In this case, the number of firms matter for knowledge spillovers, in addition to average productivity. In contrast, holding per capita spending on S&E skills, G^R/L, constant and assuming $\varepsilon = 1$ removes the scale effects regarding growth. Hence, the case where $\varepsilon = 1$ seems to be more consistent with empirical evidence than the case $\varepsilon < 1$ (see e.g. [Jones, 2005](#)).

The next result shows the dynamic effects of public policy for productivity growth. It immediately follows from [Lemma 2](#).

Proposition 4 (Dynamic effects of policy on productivity growth). *If $\mu_t = \bar{\mu}$ for $t \geqslant 1$, the impact of an increase in $\bar{\mu}$ on ϑ_{t+1} is generally ambiguous. Moreover, for all $t \geqslant 1$, an increase in G_t^R positively affects ϑ_{t+1}, whereas ϑ_{t+1} is independent of G_t^S.*

An increase in the R&D subsidy rate, $\mu_t = \bar{\mu}$, raises the incentive for individuals with mediocre abilities to acquire S&E skills, i.e., $\tilde{a}_t$ declines. This shift in the employment structure away from production activities lowers profits of firms due to a reduction in output, all other things equal. Thus, an increase in R&D subsidies adversely affects the equilibrium number of firms, n^*, according to [(21)](#). Moreover, and somewhat surprisingly at the first glance, [(22)](#) implies that efficiency units of S&E skills per firm, l^{R*}, may decrease with μ, despite the fact that a larger fraction of individuals chooses education in a S&E field. This possibility arises because public education is a rival good. Consequently, given total education spending, G^R, there is a "congestion effect" : effective R&D labor of an individual with some ability a, $e^R(a) = a\bar{e}^R(g^R)$, is decreasing if s^R increases. Stated differently, if an individual chooses to acquire S&E skills, triggered by an increase in μ, it exerts a negative externality on effective R&D labor of others. In sum, because an increase in μ lowers the number of firms, n^*, and may decrease R&D labor investment per firm, l^{R*}, R&D subsidies may be harmful for productivity growth.

In contrast to R&D subsidies, an increase in public education expenditure of either kind, G^R or G^S, leaves population shares, s^R, s^S and s^U, unchanged ([Proposition 2](#)). Consequently, an increase in G^R does not affect firm number n^*, but raises effective R&D labor per firm, l^{R*}; as a result, productivity growth is unambiguously promoted. An increase in G^S, which raises effective labor supply of skilled production workers, does neither affect n^* nor l^{R*}, and thereby leaves the growth rate unchanged. This is consistent with evidence that educational spending on scientists and engineers but not on other skilled workers is positively related to productivity growth (e.g., [Murphy et al., 1991](#)). Public education targeted to production skills affects welfare, however, as will become apparent in Section [4](#).

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[12] The absence of transitional dynamics greatly simplifies the normative analysis in the following section. It arises because we abstract from capital accumulation, firms have to re-establish (via fixed cost $\bar{l}$) each period, and there is no linkage with respect to education levels across generations. These features of the model are not critical for the basic mechanisms highlighted in the analysis.

In sum, the analysis suggests that R&D subsidies may be a rather ineffective way to stimulate R&D activity and productivity growth. Rather, one may conclude that the primary policy goal should not necessarily be to raise the fraction of scientists and engineers in the population,[13] but to promote the skill development of the best talents, i.e., to emphasize excellence in the education system.

3.2.4. Discussion of robustness

It is useful to look more deeply into the basic mechanisms which drive policy effects and to discuss robustness of the main positive results: the invariance of the income distribution with respect to education spending on S&E skills, in contrast to R&D subsidies (Propositions 1–3), and the growth effects of alternative R&D policies (Proposition 4).

The critical property of the model which gives rise to the effects of R&D policy on the income distribution is that in equilibrium for given educational choice, income levels of scientists and engineers, I^R, and therefore education incentives do not depend on government spending targeted to S&E skills, G^R, whereas an increase in the R&D subsidy rate, μ, raises I^R (Lemma 1). Regarding education spending, an increase in G^R has two opposing effects on income levels of scientists and engineers, which exactly cancel under the employed specifications. (Analogously for a change in G^S.) First, observing (6), efficiency units of each R&D worker are enhanced, according to (7). This raises income levels of workers with S&E skills, according to (9), when holding the wage rate per efficiency unit, w^R, constant. Second, however, if G^R increases, wage rate w^R declines, due to an increase in the effective aggregate supply of S&E skills, L^R.

How robust is this neutrality result, $\partial I^R/\partial G^R = 0$, in Propositions 1 and 2? Allowing for general functional forms with respect to production technology and innovation technology, $f(\chi)$ and $h(l^R)$, respectively, reveals the following (the formal analysis is not shown here for the sake of brevity but is available upon request): First, one obtains $\partial I^R/\partial G^R = 0$ whenever $\tilde{\gamma}(l^R) \equiv h'(l^R)l^R/h(l^R) = \text{const.}$, irrespective of the function $f(\chi)$. (Similarly, $\partial I^S/\partial G^S = 0$ if $f(\chi) = \chi^{\alpha}$, irrespective of function $h(l^R)$.) Moreover, one can show that in the empirically relevant case in which the relative wage elasticity of substitution between skilled and unskilled labor in production is not below unity, the sign of $\partial I^R/\partial G^R$ is ambiguous irrespective whether $\tilde{\gamma}' > 0$ or $\tilde{\gamma}' < 0$. Finally, one can show that irrespective of functional forms of $f(\chi)$ and $h(l^R)$, an increase in the R&D subsidy rate, μ, *unambiguously* raises income levels of scientists and engineers. One may therefore conclude that the first three Propositions do not systematically alter when departing from the isoelastic forms of $f(\chi)$ and $h(l^R)$ assumed in (10). Generally, it may well be the case that equilibrium income levels of scientists and engineers, and therefore earnings inequality, even shrinks when G^R increases.

Also Proposition 4 is robust to functional forms. First, as R&D subsidies generally raise income levels of R&D workers, the congestion effect on the quality of S&E education is always present. Second, this educational quality, and therefore the effective amount of S&E skills employed in firms, will typically increase with public education expenditure G^R, irrespective of the net effects of an increase in G^R on education incentives.

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[13] To avoid misunderstandings, this presumes that there are no obstacles to attract the best talents to S&E fields. For instance, a much discussed policy debate are gender-specific attitudes to S&E fields in particular and problems to attract ethnic minorities to tertiary education in general (see e.g. European Commission, 2003).

4. Normative analysis

This section examines implications of the positive analysis on both the desirability of R&D subsidies and the optimal structure of public education expenditure from a normative point of view. To obtain easily interpretable and closed-form solutions, the education technology in (7) and (8) is specified as

$$\tilde{e}^{R}(g_{t-1}^{R}) = (g_{t-1}^{R})^{\beta^{R}} \quad \text{and} \quad \tilde{e}^{S}(g_{t-1}^{S}) = (g_{t-1}^{S})^{\beta^{S}}, \tag{25}$$

where elasticities $\beta^{R} > 0$ and $\beta^{S} > 0$ measure the effectiveness of education expenditure targeted to S&E skills and production skills, respectively.

Suppose that the social planner maximizes the discounted sum of welfare of each generation, employing an utilitarian welfare function. For simplicity, the initial generation is neglected and policy variables μ_0, G_0^{R}, G_0^{S} are treated as pre-determined in the subsequent analysis.[14] The social welfare function is then given by

$$W = \sum_{t=1}^{\infty} \rho^{t} \int_{i \in [0,L]} V_{t-1}(i) \, \mathrm{d}i, \tag{26}$$

where $\rho \in (0,1)$ is the time preference rate of the social planer. (Recall that $V_{t-1}(i)$ is indirect utility of member i of generation $t-1$.)

Tax rate τ_t is determined by the balanced budget constraint of the government. It is increasing in contemporary policy parameters, μ_t, G_t^{R}, G_t^{S} (see Appendix). Due to the lack of transitional dynamics in the model, the social planning problem entails that (24) holds, i.e., policy variables are time-invariant for $t \geqslant 1$. The socially optimal R&D subsidy, given constraint $\bar{\mu} \geqslant 0$ (a non-negative R&D subsidy rate is imposed in order to allow for a well-defined corner solution), can be characterized as follows.

Proposition 5 (Optimal R&D subsidy). *For all $t \geqslant 1$, under plausible parameter configurations, the socially optimal R&D subsidy rate may be given by $\mu_t = \bar{\mu} = 0$. Provision of R&D subsidies is "more likely" to be detrimental to social welfare, the higher β^{R} or the lower ε.*

According to Proposition 5, although a positive intertemporal spillover effect is the only externality from R&D, it may well be the case that providing R&D subsidies reduces utilitarian welfare. This is because there are two other externalities from R&D subsidies. One is the congestion effect due to the rivalry of public education, which gives rise to a negative impact of an increase in $\bar{\mu}$ on educational quality of an R&D worker. This effect is stronger, the higher the effectiveness of education expenditure targeted to S&E skills, β^{R}. Moreover, an increase in $\mu_t = \bar{\mu}$ lowers the number of firms and products, $n_t = n^{*}(\bar{\mu})$, according to Lemma 2, which has two adverse effects on social welfare. First, utility of all individuals declines due to the love-of-variety property of preferences. Second, if $\varepsilon < 1$, steady state

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[14] For the initial generation, the allocation of skills across adult individuals is already given. The social planning problem with respect to initial policy variables is analyzed in the working paper version of this article. Its analysis leads to similar insights than the analysis of optimal policy in $t \geqslant 1$.

productivity growth (driven by knowledge spillovers) is negatively affected by a decrease in n.[15]

Proposition 6 (Optimal structure of public education). *The socially optimal structure of public education expenditure can be characterized as*

$$\frac{G_t^R}{G_t^S} = \frac{\overline{G}^R}{\overline{G}^S} = \frac{(2-\rho)\gamma\beta^R}{(1-\rho)\alpha\beta^S}. \tag{27}$$

Moreover, for a given R&D-subsidy rate, $\overline{G}^R$ is increasing in β^R, and decreasing in β^S; the opposite holds for $\overline{G}^S$.

According to Proposition 6, the socially optimal structure of education spending, $\overline{G}^R/\overline{G}^S$, positively depends on both the relative effectiveness of the education technology, β^R/β^S, and the effectiveness of R&D relative to the output elasticity of production skills, γ/α. Moreover, the educational production technology and technologies of firms interact: the higher the relative effectiveness of R&D, γ/α, the higher the impact of an increase in the relative effectiveness of the education technology, β^R/β^S, on the optimal relative education spending on S&E skills, $\overline{G}^R/\overline{G}^S$, and vice versa. Finally, expenditure levels for each type of skills positively depend on the effectiveness of developing this type of skills in the education technology and are adversely related to the effectiveness of developing the other skill type. Notably, although education expenditure targeted to non-S&E skills does not affect growth, it affects welfare. This is because an increase in $\overline{G}^S$ reduces goods prices through lowering wage rate w^S (see (23) in Lemma 2), thereby reducing the marginal cost of production.

5. Concluding remarks

The question how to promote R&D activity in order to enhance productivity growth is at the center of public policy debates. Almost a third of R&D expenditure in the OECD is financed by the government sector, e.g., through grants, project funding or tax incentives (OECD, 1999).

Contrary to conventional wisdom, the present analysis suggests that productivity growth and welfare may not increase in response to higher R&D subsidies. This holds true although intertemporal knowledge spillovers are the only externality from R&D spending of firms in the model. The main reason for a potentially adverse growth and welfare impact of higher R&D subsidies is a congestion effect under a public education system. If more individuals choose to acquire S&E skills, in response to enhanced demand of R&D labor by firms which is triggered by higher R&D subsidies, educational quality declines for given public education expenditure. This insight arises from taking into account both that R&D activity primarily requires human resources with specialized skills and that public education is a rival good.

Moreover, it has been shown that higher R&D subsidies raise the dispersion of labor income across skill groups (R&D and production workers) as well as within the group

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[15] More generally, under a non-utilitarian welfare function, also distributional effects matter for the socially optimal policy design. For instance, the introduction of a R&D subsidy would be more likely to harm social welfare, the lower social preferences for high-ability types, according to Proposition 3.

of scientists and engineers. This is because R&D subsidies raise the return to ability of scientists and engineers. For instance, in view of the roughly 11 million people in the US who graduated in a S&E field (National Science Board, 2002), macroeconomic distribution effects may be non-negligible.

The analysis suggests that a more desirable measure to promote R&D is to increase public expenditure targeted to the education of scientists and engineers. It has been shown that this alternative does not systematically affect the income distribution, but unambiguously raises productivity growth.

Interestingly, the widely-recognized "Sapir-Report" of a group of top economists on growth-promoting policies for Europe (on the initiative of the President of the European Commission) recommends a "*substantial increase in government and EU spending for [...] postgraduate education, but at the same time putting the main emphasis on excellence when allocating the new additional funds*" (Sapir et al., 2004, p. 134; italics original). This paper has given a theoretical foundation to the policy prescription to support the best talents by providing a high-standard S&E education. The analysis not only suggests that such a policy fosters growth but also that the often heard distributional concern to an education system which aims at promoting excellence of students and researchers is mistaken. Moreover, the analysis has accounted for the welfare-enhancing effects of public education targeted to non-S&E fields as well. The optimal structure of public education spending towards different skills depends on the relative effectiveness of the education sector across fields and its interaction with the technological characteristics of firms' R&D and production activity.

By and large, the analysis suggests to reconsider the policy-mix of public expenditure to promote growth. Although a special subsidy on R&D equipment (rather than a general subsidy) may be still desirable, it is fair to say that demand-side R&D policy has mainly to be evaluated on the basis whether or not it stimulates employment of S&E skills, since R&D capital expenditure typically accounts for only 10–13% of business R&D (e.g., Hall and van Reenen, 2000). Directly promoting employment of S&E skills by education targeted to scientists and engineers seems to be preferable in many respects.

Acknowledgements

Appendix

Proof of Lemma 1. First, using both (10) and the fact that, under symmetry, $(p_t - c_t)x_t^{\mathrm{D}} = [\overline{S}_{t-1}h(l_t^{\mathrm{R}})P_t/(w_t^{\mathrm{S}})^{\alpha}]^{\eta-1}\Gamma E_t$, the first-order condition for profit maximization problem (15) implies

$$(p_t - c_t)x_t^{\mathrm{D}}(\eta - 1)\gamma = (1 - \mu_t)w_t^{\mathrm{R}}l_t^{\mathrm{R}} \tag{A.1}$$

for any $t \geqslant 0$ in symmetric equilibrium. Combining (A.1) with the free entry condition (E3), we obtain

$$w_t^{\mathrm{R}}l_t^{\mathrm{R}} = \frac{\bar{l}\gamma(\eta - 1)}{(1 - \mu_t)[1 - \gamma(\eta - 1)]}. \tag{A.2}$$

Note that $p_t - c_t = c_t/(\eta - 1)$ in symmetric equilibrium, according to (12). Thus, substituting both goods market clearing condition (E1) and (A.2) into (A.1), as well as observing $F(l^S, l^U) = (l^S)^{\alpha}(l^U)^{1-\alpha}$, leads to

$$c_t \overline{A}_t (l_t^S)^{\alpha}(l_t^U)^{1-\alpha} = \frac{\overline{l}(\eta - 1)}{1 - \gamma(\eta - 1)}. \tag{A.3}$$

Moreover, the wage rate per efficiency unit of skilled labor in production (relative to unskilled labor) fulfills

$$w_t^S = \frac{\alpha}{1 - \alpha} \frac{l_t^U}{l_t^S} \left[= \frac{\partial F/l^S}{\partial F/l^U} \right] \tag{A.4}$$

(recall $w_t^U = 1$). Substituting (13) into (A.3) and observing (A.4) yields

$$l_t^U = \frac{(1 - \alpha)\overline{l}(\eta - 1)}{1 - \gamma(\eta - 1)} \tag{A.5}$$

for any $t \geqslant 0$. Substituting (A.5) into the labor market clearing condition (E2) for unskilled labor, we find that, for any $t \geqslant 0$, the number of firms is given by

$$n_t = \frac{L_t^U[1 - \gamma(\eta - 1)]}{\overline{l} \Xi}, \tag{A.6}$$

where $\Xi = 1 + (1 - \alpha - \gamma)(\eta - 1)$ has been used. ($\Xi > 0$ is implied by $\alpha < 1$ together with assumption $1 > \gamma(\eta - 1)$.) Consequently, combining (A.6) with condition (E2) for skilled labor of type $k = R, S$, respectively, one obtains

$$l_t^k = \frac{L_t^k \overline{l} \Xi}{L_t^U[1 - \gamma(\eta - 1)]}. \tag{A.7}$$

For $k = R$, substituting (A.7) into (A.2) implies that, for $t \geqslant 0$, the wage rate per efficiency unit of R&D labor is given by

$$w_t^R = \frac{L_t^U \gamma(\eta - 1)}{(1 - \mu_t)L_t^R \Xi}. \tag{A.8}$$

Similarly, substituting both (A.7) for $k = S$ and (A.5) into (A.4) yields, for $t \geqslant 0$,

$$w_t^S = \frac{L_t^U \alpha(\eta - 1)}{L_t^S \Xi}. \tag{A.9}$$

Next, note that for all $t \geqslant 1$, total efficiency units of R&D labor are given by

$$L_t^R = \int_{\mathscr{R}_t} e_t^R(a(i)) \, \mathrm{d}i = \tilde{e}^R(g_{t-1}^R) \int_{\mathscr{R}_t} a(i) \, \mathrm{d}i, \tag{A.10}$$

where (7) has been used for the latter equation. Similarly, using (8), one finds

$$L_t^S = s_t^S L \tilde{e}^S(g_{t-1}^S) \tag{A.11}$$

for the total efficiency units of skilled production labor. Total supply of unskilled labor is given by

$$L_t^U = L s_t^U = L\left(1 - s_t^R - s_t^S\right). \tag{A.12}$$

Using (A.10)–(A.12), (A.8) and (A.9) can be written as

$$w_t^{\mathrm{R}} = \frac{L\left(1 - s_t^{\mathrm{R}} - s_t^{\mathrm{S}}\right)\gamma(\eta - 1)}{(1 - \mu_t)\Xi\xi^{\mathrm{R}}(g_{t-1}^{\mathrm{R}})^{\beta^{\mathrm{R}}} \int_{\mathcal{R}_t} a(i)\,\mathrm{d}i}, \tag{A.13}$$

$$w_t^{\mathrm{S}} = \frac{\left(1 - s_t^{\mathrm{R}} - s_t^{\mathrm{S}}\right)\alpha(\eta - 1)}{s_t^{\mathrm{S}}\tilde{e}^{\mathrm{S}}(g_{t-1}^{\mathrm{S}})\Xi}. \tag{A.14}$$

Finally, substituting (A.13) and (A.14) into (9) confirms (16) and (17), respectively. □

Proof of Proposition 1. First, according to Lemma 1, for $t \geqslant 1$, an increase in μ_t raises $I_t^{\mathrm{R}}(a)$ but does not affect I_t^{S}. Moreover, an increase in μ_0 raises w_0^{R} but does not affect w_0^{S}, according to (A.8) and (A.9), respectively. This confirms the result regarding the relationship between R&D subsidies and the income distribution. For the impact of an increase in μ_t on $\overline{A}_t = \overline{S}_{t-1}h(l_t^{\mathrm{R}})$, note that $l_t^{\mathrm{R}} = L_t^{\mathrm{R}}/n_t$ from (E2). Also recall that S_{-1}, L_0^{R}, L_0^{S} and L_0^{U} are exogenously given. Thus, for any $t \geqslant 0$, n_t is unaffected for given educational choices, according to (A.6). According to (A.10), the same is true regarding L_t^{R}, $t \geqslant 1$. This concludes the proof of the first part of Proposition 1. Now consider public education policy. According to Lemma 1, an increase in education expenditure does neither affect $I_t^{\mathrm{R}}(a)$ nor I_t^{S}. It remains to examine the impact of an increase in G_{t-1}^{R} and G_{t-1}^{S} on $\overline{A}_t$, which positively depends on $l_t^{\mathrm{R}} = L_t^{\mathrm{R}}/n_t$. As n_t is not affected, $\overline{A}_t$ can only change if L_t^{R} changes. Recalling $g_{t-1}^{\mathrm{R}} = G_{t-1}^{\mathrm{R}}/(s_t^{\mathrm{R}}L)$ from (6) and observing (A.10) proves the results. □

Proof of Proposition 2. Part (a) follows from equilibrium condition $(1 - z^{\mathrm{S}})I_t^{\mathrm{S}} = 1$. To prove part (b), first, combine (17) and $I_t^{\mathrm{S}} = (1 - z^{\mathrm{S}})^{-1}$, and then use $\tilde{a}_t = 1 - s_t^{\mathrm{R}}$ to obtain

$$s_t^{\mathrm{S}} = \frac{(1 - z^{\mathrm{S}})\alpha(\eta - 1)\tilde{a}_t}{\Theta}, \tag{A.15}$$

where $\Theta = 1 + (1 - \gamma - z^{\mathrm{S}}\alpha)(\eta - 1)$ has been used. ($\Theta > 0$ is implied by $\alpha < 1$, $z^{\mathrm{S}} < 1$ and $\gamma(\eta - 1) < 1$. Also recall $\Xi = 1 + (1 - \alpha - \gamma)(\eta - 1)$, when using (17).) Combining (A.15) and $\tilde{a}_t = 1 - s_t^{\mathrm{R}}$ yields

$$s_t^{\mathrm{U}} = 1 - s_t^{\mathrm{R}} - s_t^{\mathrm{S}} = \frac{\Xi\tilde{a}_t}{\Theta}. \tag{A.16}$$

Moreover, recalling that $a \sim uniform[0,1]$, we have

$$\int_{\mathcal{R}_t} a(i)\,\mathrm{d}i = L\int_{\tilde{a}_t}^1 a\,\mathrm{d}a = \frac{L(1 - (\tilde{a}_t)^2)}{2} \tag{A.17}$$

for all $t \geqslant 1$. Substituting both (A.16) and (A.17) into (16), we obtain

$$I_t^{\mathrm{R}}(a) = \frac{a\gamma(\eta - 1)}{(1 - \mu_t)\Theta}\frac{2\tilde{a}_t}{1 - (\tilde{a}_t)^2}. \tag{A.18}$$

Thus, combining (A.18) with equilibrium condition $(1 - z^{\mathrm{R}})I_t^{\mathrm{R}}(\tilde{a}_t) = 1$ and rearranging terms confirms part (b). Part (c) follows from (19) and the fact that $s_t^{\mathrm{R}} = 1 - \tilde{a}_t$. Moreover, according to (A.15) and (A.16), respectively, both s_t^{S} and s_t^{U} are decreasing in μ_t and are independent of both G_{t-1}^{R} and G_{t-1}^{S}. This confirms part (d). Finally, substituting (19) into

(A.18) confirms (20). Thus, using the fact that $\tilde{a}_t$ is decreasing in μ_t proves part (e). This concludes the proof. $\square$

Proof of Proposition 3. The result that $\sigma^R = 1/\tilde{a}_t$ increases in μ_t immediately follows from part (c) of Proposition 2. For the impact of an increase of μ_t on $\sigma_t^{R/P} = \overline{I}_t^R/\overline{I}_t^P$, first, note that one can write

$$\overline{I}_t^P = \frac{(s_t^S/s_t^U)I_t^S + 1}{s_t^S/s_t^U + 1}. \tag{A.19}$$

Since $I_t^S = (1 - z^S)^{-1}$ (see part (a) of Proposition 2) and $s_t^S/s_t^U = (1 - z^S)\alpha(\eta - 1)/\Xi$, according to (A.15) and (A.16), $\overline{I}_t^P$ is independent of policy parameters μ_t, G_{t-1}^R and G_{t-1}^S. Substituting both (20) and $s_t^R = 1 - \tilde{a}_t$ into $\overline{I}_t^R = (1/s_t^R) \int_{\tilde{a}_t}^1 I_t^R(a)\,\mathrm{d}a$ leads to

$$\overline{I}_t^R = \frac{1}{2(1 - z^R)}\left(\frac{1}{\tilde{a}_t} + 1\right). \tag{A.20}$$

Hence, $\overline{I}_t^R$, and thus $\sigma_t^{R/P} = \overline{I}_t^R/\overline{I}_t^P$ are increasing in μ_t, according to part (c) of Proposition 2. This concludes the proof. $\square$

Proof of Lemma 2. Note that, for any $t \geq 1$, $L_t^U = L\Xi\tilde{a}_t/\Theta$ in equilibrium, according to (A.12) and (A.16). Substituting this into (A.6) and using $\tilde{a}_t = a^*(\mu_t)$ from (19) confirms (21). To prove (22), first, substitute (A.17) into (A.10) to find $L_t^R = \tilde{e}^R(g_{t-1}^R)L(1 - (\tilde{a}_t)^2)/2$. Now, use $l_t^R = L_t^R/n_t$ from (E2) with n_t as given by (21), and recall $g_{t-1}^R = G_{t-1}^R/(s_t^R L)$, $t \geq 1$, $s_t^R = 1 - \tilde{a}_t$ and $\tilde{a}_t = a^*(\mu_t)$. To prove (23), substitute $g_{t-1}^S = G_{t-1}^S/(s_t^S L)$ into (A.14), and use (A.15) and (A.16). This concludes the proof. $\square$

Proof of Proposition 5. At each date $t \geq 0$, the government budget constraint reads

$$\tau_t Y_t = G_t^R + G_t^S + \mu_t w_t^R L_t^R, \tag{A.21}$$

where $Y_t \equiv w_t^R L_t^R + w_t^S L_t^R + L_t^U$ is aggregate (nominal) income. Using (A.8) and (A.9), it is easy to show that Y_t is given by

$$Y_t = \frac{L_t^U}{\Xi}\left(\eta + \frac{\gamma(\eta - 1)\mu_t}{1 - \mu_t}\right). \tag{A.22}$$

(Recall $\Xi = 1 + (1 - \alpha - \gamma)(\eta - 1)$.) Substituting (A.8) and (A.22) into (A.21), using $L_t^U = Ls_t^U$ and s_t^U as given by (A.16), and rearranging terms, leads to tax rate

$$\tau_t = \frac{\frac{(G_t^R + G_t^S)\Theta}{La^*(\mu_t)} + \frac{\gamma(\eta - 1)\mu_t}{1 - \mu_t}}{\eta + \frac{\gamma(\eta - 1)\mu_t}{1 - \mu_t}} \equiv \tilde{\tau}(G_t^R, G_t^S, \mu_t), \tag{A.23}$$

$t \geq 1$. Using derivative $\mathrm{d}a^*/\mathrm{d}\mu < 0$ from part (c) of Proposition 2, we have $\partial\tau_t/\partial\mu_t > 0$ if $\tau_t < 1$. Moreover, $\partial\tau_t/\partial G_t^k > 0$, $k = R, S$.

Next, social welfare is derived as function of policy variables. Substituting both $I_t^S = (1 - z^S)^{-1}$ and (20) from Proposition 2 into (18) yields

$$V_{t-1}(i) = \begin{cases} \ln(1 - \tau_t) - \ln P_t + \ln(a(i)/\tilde{a}_t), & \text{if } i \in \mathcal{R}_t, \\ \ln(1 - \tau_t) - \ln P_t, & \text{otherwise.} \end{cases} \tag{A.24}$$

Making use of the fact that there is symmetry in equilibrium, the definition of the price index in (11) implies $P_t = (n_t)^{\frac{1}{1-\eta}} p_t$. Thus, using (12) and (13),

$$-\ln P_t = \frac{\ln n_t}{\eta - 1} - \alpha \ln w_t^{\mathrm{S}} + \ln \overline{A}_t - \kappa, \tag{A.25}$$

where $\kappa \equiv \ln[\alpha^{-\alpha}(1-\alpha)^{-(1-\alpha)}\eta/(\eta-1)]$. Substituting (A.24) into (26), and using (A.25), social welfare can be written as

$$W = \sum_{t=1}^{\infty} \rho^t \left[\ln(1 - \tau_t) + \frac{\ln n_t}{\eta - 1} - \alpha \ln w_t^{\mathrm{S}} + \ln \overline{A}_t - \kappa + \Omega(\tilde{a}_t) \right], \tag{A.26}$$

where

$$\Omega(\tilde{a}_t) \equiv \int_{\tilde{a}_t}^{1} \ln(a/\tilde{a}_t)\, \mathrm{d}a. \tag{A.27}$$

According to (3), (4) and $h(l^{\mathrm{R}}) = (l^{\mathrm{R}})^{\gamma}$, $\overline{S}_0 = \overline{A}_0(n_0)^{1-\varepsilon} = \overline{S}_{-1}(n_0)^{1-\varepsilon}(l_0^{\mathrm{R}})^{\gamma}$, where n_0 and l_0^{R} are given by (A.6) and (A.7), respectively. Moreover, in symmetric equilibrium, $\overline{A}_t = \overline{S}_0(n^*)^{(t-1)(1-\varepsilon)}(l^{\mathrm{R}*})^{t\gamma}$ for $t \geqslant 1$. Substituting this expression for $\overline{A}_t$ together with (19), (21), (22) and (23) into (A.26), observing (24), and making use of the facts $\sum_{t=1}^{\infty} \rho^t = \rho/(1-\rho)$ and $\sum_{t=1}^{\infty} \rho^t t = \rho/(1-\rho)^2$, eventually yields $W = W^*(\overline{G}^{\mathrm{R}}, \overline{G}^{\mathrm{S}}, \bar{\mu})$, where

$$
\begin{aligned}
W^*(\overline{G}^{\mathrm{R}}, \overline{G}^{\mathrm{S}}, \bar{\mu}) \equiv &\ \mathrm{const.} + \frac{\rho}{1-\rho} \\
&\times \Bigg\{ \ln(1 - \tilde{\tau}(\overline{G}^{\mathrm{R}}, \overline{G}^{\mathrm{S}}, \bar{\mu})) + \rho \left[\frac{(2-\rho)\gamma\beta^{\mathrm{R}} \ln \overline{G}^{\mathrm{R}}}{1-\rho} + \alpha\beta^{\mathrm{S}} \ln \overline{G}^{\mathrm{S}} \right] \\
&+ \left(\frac{1}{\eta - 1} + \frac{(1-\varepsilon)\rho}{1-\rho} - \alpha\beta^{\mathrm{S}} \right) \ln a^*(\bar{\mu}) + \Omega(a^*(\bar{\mu})) \\
&+ \frac{\gamma}{1-\rho} \left[\ln\left(1 + \frac{1}{a^*(\bar{\mu})}\right) + (1-\beta^{\mathrm{R}}) \ln\left(1 - a^*(\bar{\mu})\right) \right] \Bigg\}.
\end{aligned} \tag{A.28}
$$

Next, verify from (A.27) that $\Omega'(\tilde{a}_t) = 1 - 1/\tilde{a}_t < 0$. Thus, according to (A.30),

$$\frac{\partial W^*(\overline{G}^{\mathrm{R}}, \overline{G}^{\mathrm{S}}, \bar{\mu})}{\partial \bar{\mu}} = \frac{\rho}{1-\rho} \left[-\frac{\partial \tilde{\tau}/\partial \bar{\mu}}{1-\tilde{\tau}} + \Lambda(a^*) \frac{\mathrm{d}a^*(\bar{\mu})/\mathrm{d}\bar{\mu}}{a^*} \right], \tag{A.29}$$

where

$$\Lambda(a^*) \equiv \frac{1}{\eta - 1} + \frac{(1-\varepsilon)\rho}{1-\rho} - \alpha\beta^{\mathrm{S}} - (1 - a^*) - \frac{\gamma}{1-\rho} \left[\frac{1}{a^* + 1} + \frac{(1-\beta^{\mathrm{R}})a^*}{1 - a^*} \right]. \tag{A.30}$$

According to (A.29), making use of the facts $\partial \tilde{\tau}/\partial \bar{\mu} > 0$ and $\mathrm{d}a^*/\mathrm{d}\bar{\mu} < 0$, we have $\partial W^*/\partial \bar{\mu} < 0$ for all $\bar{\mu} \in [0,1)$ if, for instance, $\Lambda(a^*(\bar{\mu})) \geqslant 0$ for all $\bar{\mu} \in [0,1)$. In this case, $\bar{\mu} = 0$ is socially optimal. To confirm that $\bar{\mu} = 0$ is possible in social optimum for plausible parameter values, suppose $\beta^{\mathrm{R}} = \beta^{\mathrm{S}} = \varepsilon = 1$, $\alpha = \gamma = 0.25$, $z^{\mathrm{R}} = z^{\mathrm{S}} = 0.5$, $\rho = 0.9$ and $\eta = 1.2$. Thus, $a^*(\bar{\mu}) = \sqrt{(1-\bar{\mu})/[1/22 + 1 - \bar{\mu}]}$, according to (19), i.e., for $\bar{\mu} = 0$, the population share of R&D workers roughly is 2.2%. $\bar{\mu} = 0$ is socially optimal in this numerical example if $\Lambda(a^*) = 4.5 - (1 - a^*) - 2.5/(a^* + 1) \geqslant 0$, which clearly holds. $\square$

Proof of Proposition 6. Note that W^* is strictly concave as a function of both $\overline{G}^{\mathrm{R}}$ and $\overline{G}^{\mathrm{S}}$, according to (A.29) and (A.30); moreover, $\partial^2 W^*/\partial \overline{G}^{\mathrm{R}} \partial \overline{G}^{\mathrm{S}} = 0$. According to (A.23), (A.29) and (A.30), it is straightforward to show that first-order conditions $\partial W^*/\overline{G}^{\mathrm{R}} = \partial W^*/\overline{G}^{\mathrm{S}} = 0$ imply

$$\left(\eta - \frac{\Theta(\overline{G}^{\mathrm{R}} + \overline{G}^{\mathrm{S}})}{La^*(\mu_t)} \right) \frac{\rho(2-\rho)\gamma\beta^{\mathrm{R}}}{(1-\rho)\overline{G}^{\mathrm{R}}} = \frac{\Theta}{La^*(\mu_t)}, \tag{A.31}$$

$$\left(\eta - \frac{\Theta(\overline{G}^{\mathrm{R}} + \overline{G}^{\mathrm{S}})}{La^*(\mu_t)} \right) \frac{\rho\alpha\beta^{\mathrm{S}}}{\overline{G}^{\mathrm{S}}} = \frac{\Theta}{La^*(\mu_t)}, \tag{A.32}$$

Combining (A.31) and (A.32) confirms (27). Holding $\mu_t = \bar{\mu}$ constant and applying the implicit function theorem to equation system (A.31) and (A.32), confirms the comparative-static results. This concludes the proof.   □

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