Firm size and diversification: Multiproduct firms in asymmetric oligopoly

Volker Grossmann *

University of Fribourg, Department of Economics, Bd. de Pérolles 90, CH-1700 Fribourg, Switzerland
CESifo, Poschingerstr. 5, D-81679 Munich, Germany
Institute for the Study of Labor (IZA), P.O. Box 7240, D-53072 Bonn, Germany

Abstract

This paper presents an oligopoly model of multiproduct firms in which firms are endowed with possibly different marginal cost and product quality, and choose product ranges before product market competition. Consistent with empirical evidence on a positive relationship between firm size and product diversification, the analysis suggests that firms with higher quality–cost margins typically have both larger size and larger product ranges. The main results are proved for Cournot competition and linear demand with differentiated products. They also hold for duopoly under Bertrand competition in the nested multinomial logit model, and, under some restrictions, for Bertrand competition with linear demand.
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1. Introduction

By using various indicators, empirical evidence for the manufacturing sector strongly suggests that there is a positive relationship between the size and diversification of firms.[1] Moreover, there

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 [1] This pattern seems to be consistent over time at least from the 1950s onwards. Well-known studies supporting this conclusion are Gort (1962), Gollop and Monahan (1991), Lichtenberg (1992) and Markides (1995) for the US as well as Amey (1964) and Utton (1977) for the UK. More recent studies include Aw and Batra (1998) for Taiwan, Davies et al. (2001) for a sample of European firms and Gourlay and Seaton (2004) for the UK.

is evidence on an important role of firm-specific characteristics for both diversification patterns and firm size.[2] For instance, Davies et al. (2001) conclude that "many empirical studies confirm positive statistical associations between diversification, firm size, R&D and advertising" (p. 1317) and argue that "diversification is driven […] by a desire to exploit a specific asset" (p. 1334).

This paper presents an oligopoly model of asymmetric multiproduct firms in order to examine the apparent link between firm size, diversification and specific characteristics of firms. The set up may be described as follows. Potential firms decide whether or not to enter at some fixed cost. They are endowed with some possibly different (and immutable) marginal cost and product quality. These characteristics are of public good nature from the perspective of a firm, i.e., apply to any good within a firm's product line. After entering the economy, firms choose the number of products offered to the market (stage 1), and then enter product market competition (stage 2).

The main contribution of the paper is twofold. First, it derives basic properties of profit functions of multiproduct firms for the widely used linear-demand model with differentiated goods under *Cournot* competition, for a given configuration of product ranges (stage 2 equilibrium). Second, using these properties, the analysis shows that, typically, firms with more favorable quality–cost margins have both larger size (measured by total sales) and larger product ranges, consistent with the empirical evidence outlined above.

Moreover, the analysis seeks to identify determinants of average industry diversification, in addition to those of the size–diversification relationship. For instance, Gorecki (1975, p. 134) suggests that "specific assets of a technological nature formed the basis of much [industry] diversification" in the UK, whereas Baldwin et al. (2000) find no evidence of a role of technological characteristics for average diversification in Canada. The present analysis supports the findings of Gorecki (1975) by showing that, for a given number of symmetric firms, an increase in quality–cost margins raises product ranges. In contrast, higher substitutability of products reduces diversification of product lines.

The mechanisms which give rise to a positive size–diversification relationship suggest more generality beyond the Cournot model. For this reason, and in order to capture the notion that products offered by a firm are closer substitutes for each other than for products sold by other firms (unlike in the linear-demand model), Bertrand competition in the *nested multinomial logit* model is examined (e.g. Anderson and de Palma, 1992; Anderson et al., 1992). Restricting the analysis to duopoly for tractability reasons, it is also shown that in the nested multinomial logit model a larger firm has a more diversified product line.[3]

There is a considerable literature on the determinants of corporate diversification.[4] Besides the emphasis of empirical researchers on the role of technological characteristics, at least three further sources of diversification are frequently mentioned in the literature. First, there is the "agency view", according to which "a manager might direct a firm's diversification in a way that increases the firm's demand for his or her particular skills" (Montgomery, 1994, p. 166). Second, it has been suggested that diversification contributes to risk management of firms.

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[2] Roberts and Supina (2000) report a negative correlation between firm size and marginal costs among U.S. manufacturing firms. Moreover, using micro-level data from the 'Longitudinal Research Database' (developed by the U. S. Bureau of the Census), Baily et al. (1992) find that the size of U.S. manufacturing firms is positively related to their total factor productivity (see their Tables 8 and 9).

[3] Bertrand competition under linear demand is also analyzed. General results are difficult to obtain however, as reaction functions at stage 1 may not be well-behaved everywhere. The underlying reason is that, compared to the Cournot case, diversification incentives may be considerably weaker.

[4] Montgomery (1994) provides an excellent literature review of this topic.

Third, diversification may be a mean to extend the boundaries of a firm in the presence of internal coordination problems. Whereas the first two of these views do not seem to imply a particular size–diversification relationship, the latter is potentially interesting in this respect as internal coordination problems naturally arise in large firms. To the best of my knowledge, however, the theoretical literature has not yet focussed on the relationship between product diversification and firm size.[5]

The remainder of the paper is organized as follows. Section 2 presents the linear-demand model with Cournot competition and asymmetric multiproduct firms. Section 3 analyzes the equilibrium for this model in the light of the empirical regularities outlined above. Section 4 examines the size–diversification relationship in alternative multiproduct models. The last section concludes.

2. The Cournot model

Consider a market for differentiated goods and let $\mathcal{K}$ be the set of all varieties in the market. The varieties are produced by a set $\mathcal{I} = \{1,\dots,I\}$ of firms (indexed by i), which is determined under free entry. Let $\mathcal{N}_i$ be the set of goods produced by firm i, in (endogenous) number N_i. The inverse demand function for variety $k \in \mathcal{K}$ has the familiar linear form

$$p_k = a_k - \beta x_k - \gamma \sum_{l \neq k} x_1, \tag{1}$$

$\beta > \gamma > 0$, where p_k and x_k denote the price and quantity of product k, respectively. Suppose that there is a large (but finite) number of potential entrants. In order to enter, firms have to incur sunk cost $F > 0$. Initially, all firms draw a quality parameter A_i and a (constant) marginal production cost c_i from some joint distribution function $g(A, c)$ which has support $(0, \bar{A}] \times [0, \bar{c}]$, $\bar{c} \geq 0$, $0 < \bar{A} - \bar{c} < \infty$.[6] Suppose $a_k = A_i$ for all $k \in \mathcal{N}_i$ in (1). Thus, both product quality and unit cost apply to any variety a firm offers. This is meant to capture that, for instance, technological characteristics are of public good nature from the perspective of a single firm (e.g., Caves, 1971).

After deciding, based on firm characteristics, whether or not to enter the market, there are two stages, in which firms make decisions non-cooperatively and simultaneously. At stage 1, firms choose their number of products N_i ("product range"). Let $C(N_i)$ denote the costs of firm i to introduce $N_i \in [1, \bar{N}]$ products in the market, where $C : [1, \bar{N}] \rightarrow R_+$ is an increasing, twice continuously differentiable and convex function, $\bar{N} < \infty$.[7] For instance, one may think of C as costs for marketing or designing products.[8] The I-tuple $\mathbf{N} = (N_1, N_2, \dots, N_I)$ is called a "configuration of product ranges". At stage 2, firms enter Cournot competition. This timing of events follows some existing literature on multiproduct firms (e.g., Raubitschek, 1987; Sutton,

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[5] Generally, multiproduct models with asymmetric firms are rare. A notable exception is Champsaur and Rochet (1989), who propose a duopoly model of asymmetric multiproduct firms with vertical product differentiation.

[6] Introducing asymmetry of firms under free entry in this way heavily draws on recent contributions of Anderson and de Palma (2001) and Melitz (2003).

[7] As will become apparent, restricting the choice set of firms at stage 1 to the *closed* interval $[1, \bar{N}]$, rather than to $[1, \infty)$, ensures existence of equilibrium. It is a weak assumption as $\bar{N}$ can be arbitrarily large.

[8] Assuming convexity of $C(\cdot)$ does not deny that there may be economies of scope (or "subadditive costs") in marketing, designing or manufacturing multiple products within a firm (see, e.g., Baumol, 1977). However, one may think of increasing (Coasian) bureaucracy costs of product proliferation as a counteracting force. In fact, all that is needed is that $C(\cdot)$ is not "too concave" in the relevant range.

1998; Ottaviano and Thisse, 1999). However, in contrast to this literature, the present set up allows for asymmetry of firms ex ante.

3. Equilibrium analysis

In this section, the equilibrium of the Cournot model is analyzed.

3.1. Cournot competition (stage 2)

First, consider the decision problem of firms at stage 2, for a given configuration **N**. Taking output levels of rival firms as given, each firm $i \in \mathcal{I}$ solves

$$\max_{x_k \geq 0,\ k \in \mathcal{N}_i} \pi_i = \sum_{k \in \mathcal{N}_i} (p_k - c_i)x_k \quad \text{s.t. (1) and } a_k = A_i \ \forall k \in \mathcal{N}_i. \tag{2}$$

The first result shows that stage 2 equilibrium profits depend on quality–cost margins $\alpha_i \equiv A_i - c_i$, $i \in \mathcal{I}$. We denote $\boldsymbol{\alpha} = (\alpha_1, \alpha_2, ..., \alpha_I)$. Moreover, a multi-product firm i produces equal output levels for all varieties $k \in \mathcal{N}_i$ which it offers.

Proposition 1. *Equilibrium at stage 2 in Cournot competition under (1). In an interior Cournot–Nash equilibrium at stage 2, for all $k \in \mathcal{N}_i$, firm $i \in \mathcal{I}$ produces output level*

$$x_k = \frac{\Lambda_i}{(1 + \sum_i \Gamma_i)[2(\beta - \gamma) + \gamma N_i]} \equiv X_i(\mathbf{N}, \boldsymbol{\alpha}, \beta, \gamma) \tag{3}$$

and earns profits

$$\pi_i = N_i(\beta - \gamma + \gamma N_i)X_i(\mathbf{N}, \boldsymbol{\alpha}, \beta, \gamma)^2 \equiv \Pi_i(\mathbf{N}, \boldsymbol{\alpha}, \beta, \gamma), \tag{4}$$

where $\Gamma_i \equiv \gamma N_i / [2(\beta - \gamma) + \gamma N_i] \in (0, 1)$ and $\Lambda_i \equiv \alpha_i(1 + \sum_{j \neq i} \Gamma_j) - \sum_{j \neq i} \alpha_j \Gamma_j$.

All proofs are relegated to the Appendix. The following corollary characterizes profit functions Π_i in equilibrium at stage 2.

Corollary 1. *For all $i,j \in \mathcal{I}$, $j \neq i$, we have (i) $\partial \Pi_i / \partial N_i > 0$ and $\partial^2 \Pi_i / \partial N_i^2 < 0$, (ii) $\partial \Pi_i / \partial N_j < 0$, (iii) $\partial \Pi_i / \partial \alpha_i > 0$ and $\partial \Pi_i / \partial \alpha_j < 0$, (iv) $\partial^2 \Pi_i / \partial N_i \partial \alpha_i > 0$ and $\partial^2 \Pi_i / \partial N_i \partial \alpha_j < 0$; (v) if $\alpha_i \leq \alpha_j$, then $\partial^2 \Pi_i / \partial N_i \partial N_j < 0$.*

To gain insight into Corollary 1, it is helpful to decompose Π_i into the product of total demand (or sales) of firm i in equilibrium at stage 2, $D_i(\mathbf{N}, \boldsymbol{\alpha}, \beta, \gamma) \equiv N_i X_i(\mathbf{N}, \boldsymbol{\alpha}, \beta, \gamma)$, and its price–cost difference ("mark-up"), $M_i(\mathbf{N}, \boldsymbol{\alpha}, \beta, \gamma) \equiv (\beta - \gamma + \gamma N_i)X_i(\mathbf{N}, \boldsymbol{\alpha}, \beta, \gamma)$.[9] That is, $\Pi_i = D_i M_i$, implying $\partial \Pi_i / \partial N_j = M_i(\partial D_i / \partial N_j) + D_i(\partial M_i / \partial N_j)$ and

$$\frac{\partial^2 \Pi_i}{\partial N_j \partial N_i} = \frac{\partial^2 D_i}{\partial N_j \partial N_i} M_i + \frac{\partial D_i}{\partial N_j} \frac{\partial M_i}{\partial N_i} + \frac{\partial D_i}{\partial N_i} \frac{\partial M_i}{\partial N_j} + D_i \frac{\partial^2 M_i}{\partial N_j \partial N_i}, \tag{5}$$

$i,j \in \mathcal{I}$. Total sales D_i of a firm are used as measure of firm size throughout the paper. The properties of D_i and M_i as functions of $(\mathbf{N}, \boldsymbol{\alpha})$, which are referred to in the following discussion of Corollary 1, are formally derived in a supplement available online.

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[9] I am grateful to Armin Schmutzler for this suggestion.

First, the impact of an increase in product range N_i on both equilibrium demand, D_i, and on equilibrium mark-up, M_i, are positive.[10] Thus, $\partial\prod_i/\partial N_i > 0$, as stated in part (i) of Corollary 1.[11] Moreover, strict concavity of $\prod_i$ as function of N_i means that a firm's incentive to launch additional varieties is weaker, the more diversified the firm is. This result is driven by the fact that the marginal gain of an increase in N_i regarding both D_i and M_i is decreasing, i.e., $\partial^2 D_i/\partial N_i^2 < 0$ and $\partial^2 M_i/\partial N_i^2 < 0$. It reflects competition within a firm's own product line. Part (ii) of Corollary 1 means that equilibrium profits at stage 2 decline if any rival offers additional products, which reflects a conventional "business-stealing effect". In fact, an increase in N_j reduces both D_i and M_i when $i \neq j$. Part (iii) says that, not surprisingly, $\prod_i$ increases with its own quality–cost margin, α_i, but decreases with that of other firms, α_j, $j \neq i$, holding the configuration of product ranges **N** constant. Again, the effects regarding both D_i and M_i go in the same direction. According to part (iv), the profit gain of firm i from introducing an additional variety increases with α_i, but decreases with quality–cost margins of rivals, α_j, $j \neq i$, all other things equal. An increase in α_i raises the impact of an increase in product range N_i on both sales D_i and mark-up M_i (i.e., $\partial^2 D_i/\partial N_i \partial \alpha_i > 0$ and $\partial^2 M_i/\partial N_i \partial \alpha_i > 0$), whereas an increase in α_j, $j \neq i$, has the opposite effect on $\partial D_i/\partial N_i$ and $\partial M_i/\partial N_i$, respectively.

Finally, consider the impact of an increase in a rival's product range N_j on the incentive of a firm $i \neq j$ to launch new varieties (i.e., how $\partial\prod_i/\partial N_i$ changes with N_j, $j \neq i$). From the previous discussion of parts (i) and (ii), for $j \neq i$, one can conclude that the second and third summand of the right-hand side of (5) are both negative. However, one can also show that the first and last summand have ambiguous sign, i.e., an increase in N_j, when $j \neq i$, may accentuate or weaken either effect, $\partial D_i/\partial N_i$ and $\partial M_i/\partial N_i$. Part (v) of Corollary 1 says that the profit gain of a firm i from increasing product diversification is reduced by an increase in a rival's product range N_j, $j \neq i$, if $\alpha_i \leq \alpha_j$. In this case, the optimal response at stage 1 to an increase in a rival's product number is to decrease the own number of varieties, i.e., product ranges of firms are strategic substitutes. $\partial^2\prod_i/\partial N_i \partial N_j \geq 0$ may occur, however, if $\alpha_i > \alpha_j$.

3.2. Firms' choice of number of products (stage 1)

The profit maximization problem for each firm $i \in \mathcal{I}$ at stage 1 is to solve

$$\max_{N_i \in [1,\bar{N}]} \Psi_i(\mathbf{N},\boldsymbol{\alpha},\beta,\gamma) \equiv \Pi_i(\mathbf{N},\boldsymbol{\alpha},\beta,\gamma) - C(N_i). \tag{6}$$

Applying a classical result (Debreu, 1952), as profit functions are continuous on $N_i \in [1, \bar{N}]$ and $\prod_i$ is strictly concave as function of N_i (part (i) of Corollary 1), existence of equilibrium is ensured.

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[10] The latter effect may be somewhat surprising at first glance, but can easily be understood as follows. Note that $p_k - c_i = \alpha_i - (\beta-\gamma)X_i - \gamma Q [=M_i]$ for all $k \in \mathcal{N}_i$, $i \in \mathcal{I}$ where $Q \equiv \sum_i N_i X_i$ equals total sales in the industry. On the one hand, it is easy to check that an increase in N_i raises Q in stage 2 equilibrium, all other things equal (use Eq. (A.4) in the Appendix). This has a negative effect on M_i. On the other hand, however, firm i reduces equilibrium output per variety (X_i) when increasing N_i, which has a positive effect on M_i. The second effect dominates the first one under Cournot competition. (This does typically not hold under Bertrand competition; see Section 4.2.).

[11] An important assumption for this is $\beta > \gamma$, i.e., varieties are imperfect substitutes. In contrast, for $\gamma \to \beta$, the limiting profit function of a firm i at stage 2 is given by $\lim_{\gamma \to \beta}\prod_i = [(I\alpha_i - \sum_{j \neq i}\alpha_j)/(1+I)]^2/\gamma$, according to (3) and (4). Obviously, it does not pay for firms to supply more than one variety in this limit case.

Let $N_i{*}(\alpha, \beta, \gamma)$ be an equilibrium product range offered by firm $i \in \mathcal{I}$. Using (6), an equilibrium configuration of product ranges, $\mathbf{N}^*$, is (provided that $N_i^* < \bar{N}$ for all i) given by first-order conditions

$$\frac{\partial \, \Pi_i(\mathbf{N}^*, \alpha, \beta, \gamma)}{\alpha N_i} \leq C'(N_i^*), \qquad i \in \mathcal{I}, \tag{7}$$

with strict equality if $N_i^* > 1$.

3.3. Entry

As will become apparent (see Proposition 4), not surprisingly, firms with higher quality–cost margins earn higher stage 1 equilibrium profits, $\Psi_i^*(\alpha, \beta, \gamma) \equiv \Psi_i(\mathbf{N}^*, \alpha, \beta, \gamma)$. Moreover, since $\partial \Pi_i / \partial N_j < 0$ for $j \neq i$ (part (ii) of Corollary 1), it is immediate that entry of an additional firm lowers stage 1 profits. Thus, there exists a "long-run" equilibrium (with free entry of firms) in which firms in the market are those with the highest quality–cost margins. That is, there is a unique cut-off point for quality–cost margins such that $\Psi_i^*(\alpha, \cdot) \geq F$ for entering firms and all other firms from the pool of potential entrants rationally anticipate that they will not be able to enter.[12]

3.4. Diversification of symmetric firms

Determinants of average diversification is examined next. Some empirical studies look at the determinants of average diversification (e.g. Gorecki, 1975; Baldwin et al., 2000). As asymmetry of firms is not crucial for this issue, for simplicity, suppose $\alpha_i = \alpha$ for all $i \in \mathcal{I}$, i.e., $\boldsymbol{\alpha} = (\alpha, ..., \alpha)$, and focus on a symmetric equilibrium at stage 1. Thus, we have $N_i^*(\boldsymbol{\alpha}, \beta, \gamma) = N^*$ for all $i \in \mathcal{I}$. In the following, the impact of an increase in both quality–cost margin α and the "degree of substitutability", measured by γ, on equilibrium product range, N^*, is considered for a given set of firms (i.e., when there are barriers to entry).[13]

Proposition 2. *Diversification of symmetric firms. For a given set of symmetric firms, an increase in α raises N^*, whereas an increase in γ lowers N^*.*

Differences in α across industries may be thought of inter-industry differences in technological characteristics. Thus, Proposition 2 is consistent with the empirical result of Gorecki (1975) that industries which are characterized by better technological know-how tend to be more diversified.[14] Moreover, quite intuitively, better substitutability of products lowers the incentive of firms to launch new varieties.

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[12] This reasoning is analogous to Anderson and de Palma (2001; Proposition 3.1), who consider a logit model with single-product firms. As Anderson and de Palma (2001, p. 124) point out, however: "This will not be the only equilibrium. It may be possible that some other set of firms is in the market but yet some excluded firm with a higher quality–cost cannot profitably enter due to the presence of established firms even though it could make more money were it to replace the latter". Fortunately, as will become apparent, results on the size–diversification relationship analyzed in Section 3.5 hold in any free-entry equilibrium.

[13] Unfortunately, for the long-run equilibrium, this analysis becomes highly intractable. The exclusive focus in this subsection therefore is on the case with entry barriers.

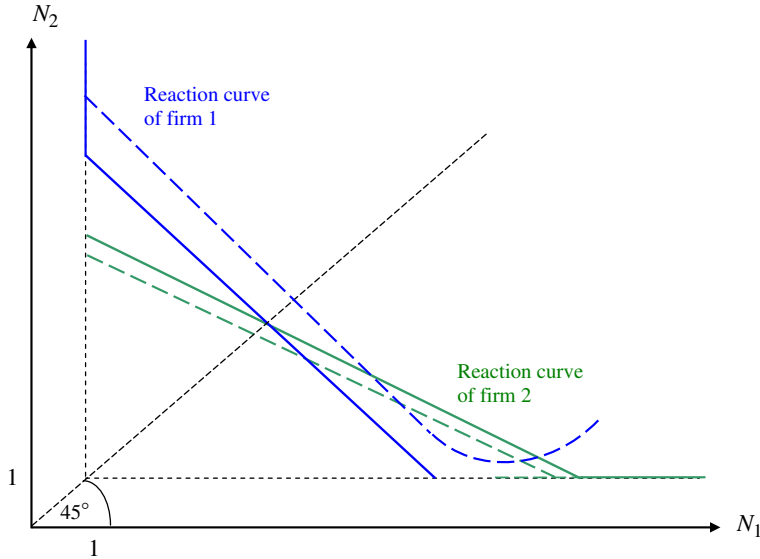[14] Gorecki (1975) uses R&D-intensity in an industry to proxy its technological know-how.

Fig. 1. Comparison of $\alpha_1 = \alpha_2$ (solid lines) and $\alpha_1 > \alpha_2$ (dashed lines).

3.5. Firm size and diversification

To examine the role of quality–cost margins for the relationship between firm size and product diversification, we first turn to the question how differences in equilibrium product ranges among firms (i.e., equilibrium diversification) depend on differences in quality–cost margins.

Proposition 3. *Diversification of asymmetric firms. Suppose that for all $i, j \in \mathcal{I}, j \neq i$,*

$$\left| \frac{\partial^2 \, \Pi_i(\mathbf{N}, \cdot)}{\partial N_i^2} \right| > \left| \frac{\partial^2 \, \Pi_i(\mathbf{N}, \cdot)}{\partial N_i \partial N_j} \right| \text{ when } \alpha_i = \alpha_j. \tag{8}$$

Then $\alpha_i > \alpha_j$ implies $N_i^* > N_j^*$.

To gain insight into this result, let us first look at the duopoly case, $\mathcal{I} \in \{1,2\}$. According to parts (i) and (v) of Corollary 1 and first-order conditions (7), if $\alpha_1 = \alpha_2$, reaction functions are downward sloping in $N_1 - N_2$ space, as illustrated in Fig. 1. That is, product ranges are strategic substitutes. From (7), it is also easy to see that under condition (8), if $\alpha_1 = \alpha_2$, reaction function of firm 1 is steeper than that of firm 2 in Fig. 1; this ensures both uniqueness of equilibrium and $N_1^* = N_2^*$. According to part (iv) of Corollary 1, an increase in α_1 or a decrease in α_2 raises the marginal gain of firm 1 to extend its product range and reduces the marginal gain of firm 2. Thus, when $\alpha_1 > \alpha_2$, any intersection of reaction curves lies South-East of the equilibrium for $\alpha_1 = \alpha_2$, as illustrated by the dashed lines in Fig. 1. The reaction function of firm 2 is still downward sloping when $\alpha_1 > \alpha_2$, according to part (v) of Corollary 1. This may or may not be true for firm 1. But irrespective of whether the new equilibrium is still unique, it is apparent that $\alpha_1 > \alpha_2$ implies $N_1^* > N_2^*$ in the duopoly case.[15]

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[15] In principle, it is possible that there is no intersection of reaction curves when $\alpha_1 > \alpha_2$. In this case, $N_1^* = \bar{N} > 1 = N_2^*$.

This does not necessarily hold when condition (8) is violated. When $\alpha_1 = \alpha_2$ and reaction function of firm 2 would be steeper than that of firm 1, there would be three equilibria (one symmetric, one equilibrium where $N_1^* = 1$ and one equilibrium with $N_2^* = 1$). Thus, when $\alpha_1 > \alpha_2$, an intersection of reaction curves where $N_i^* > 1$ for $i = 1, 2$ would imply $N_1^* < N_2^*$. But as can be deduced from the expressions for $\partial^2 \prod_i / \partial N_i^2$ and $\partial^2 \prod_i / \partial N_i \partial N_j$ given in the Proof of Corollary 1 (see Appendix), under rather weak restrictions condition (8) does hold.

What happens when there are other firms? As we are concerned with pairwise comparisons of firms' product ranges, we can analyze the game between any pair of firms by holding fixed the actions of all other firms at equilibrium values.[16] Then, under (8), the firm with a higher quality–cost margin must again have a larger product line.[17]

We are now ready to infer the relationship between firm size and diversification in equilibrium (recall $D_i = N_i X_i$). Equilibrium firm size is measured by total sales of a firm in equilibrium, i.e., by equilibrium demand $D_i^*(\boldsymbol{\alpha}, \beta, \gamma) \equiv D_i(\mathbf{N}^*, \boldsymbol{\alpha}, \beta, \gamma)$, $i \in \mathcal{I}$. The next result is basically a corollary of Proposition 3.

Proposition 4. *Firm size and diversification in the Cournot model. Suppose that (8) holds. Then $\alpha_i > \alpha_j$ implies $\Psi_i^* > \Psi_j^*$ and $D_i^* > D_j^*$ for all $i, j \in \mathcal{I}$. Thus, (i) there exists a long-run equilibrium in which firms with the highest quality–cost margins enter, and (ii) firm size and product diversification are positively related.*

Proposition 4 says that firms with higher quality–cost margins have both higher equilibrium profits and higher firm sizes. The first property implies that there exists an equilibrium where firms enter the economy if and only if their quality–cost margin is sufficiently high (part (i) of Proposition 4). The second property (positive relationship between quality–cost margin and firm size) is consistent with evidence that productivity of a firm is positively related to its size (Baily et al., 1992; Roberts and Supina, 2000). Importantly, combining this result with Proposition 3, it follows that larger firms have more diversified product lines (part (ii) of Proposition 4). The analysis suggests that this stylized fact is driven by differences in quality–cost margins across firms.

4. Size and diversification in alternative models

This section examines whether the properties of the linear-demand model under Cournot competition which give rise to a positive size–diversification relationship also hold under modifications of the nature of competition or the structure of demand. First, Bertrand competition in the nested multinomial logit model (e.g., Anderson and de Palma, 1992, 2001; Anderson et al., 1992) is analyzed. Second, the case of Bertrand competition with linear demand (1) is briefly discussed and compared to the Cournot case.

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[16] As Athey and Schmutzler (2001) point out, important properties to generalize beyond duopoly in such a way (applied to an investment game in their paper) are "exchangeability" of profits as functions of $(\mathbf{N}, \boldsymbol{\alpha})$ and "conditional uniqueness". Exchangeability means that if we exchange both (N_i, α_i) and (N_j, α_j) of two firms i and j (while holding characteristics of all other firms constant), then profits of firms i and j exchange, without affecting other firms' profits. This trivially holds in the present model. Conditional uniqueness essentially means that if we look at two firms and hold other firms' actions fixed, equilibrium must be unique. Condition (8) ensures that this is the case when we consider two symmetric firms.

[17] Suppose that choice sets at stage 1 are restricted to positive integers, i.e. $N_i \in \{1, 2, ..., \bar{N}\}$, and a pure-strategy equilibrium exists. In this case, one can show that $\alpha_i > \alpha_j$ implies $N_i^* \geq N_j^*$.

4.1. A nested logit approach

Consider the nested multinomial logit model by Anderson and de Palma (1992). The model may be described as follows. Let P_{ik} be the probability that a consumer chooses variety $k \in \mathcal{K}$ when supplied by firm $i \in \mathcal{I}$. Normalizing the number of consumers to unity, the expected demand for this product, x_{ik}, is thus given by $x_{ik} = P_{ik}$. Suppose consumers first choose a (multiproduct) firm i and subsequently choose amongst the set $\mathcal{N}_i$ of products offered by i. Let P_i denote the probability that firm i is selected and $P_{k|i}$ the probability that good $k \in \mathcal{N}_i$ is selected, conditional on firm i being chosen. Thus, $P_{ik} = P_i P_{k|i}$. Suppose that the structure of preferences exactly follows Anderson and de Palma (1992, p. 263f.) and let $A_i > 0$ again be the common quality measure of varieties $k \in \mathcal{N}_i$. That is, we obtain demand functions $x_{ik} = P_i P_{k|i}$ with

$$P_i = \frac{\exp\left(\frac{v}{\mu} \ln \sum_{l \in \mathcal{N}_i} \exp\left[\frac{A_i - p_{il}}{v}\right]\right)}{\sum_{j \in \mathcal{I}} \exp\left(\frac{v}{\mu} \ln \sum_{l \in \mathcal{N}_j} \exp\left[\frac{A_j - p_{jl}}{v}\right]\right)}, \tag{9}$$

$$P_{k|i} = \frac{\exp\left[\frac{A_i - p_{ik}}{v}\right]}{\sum_{l \in \mathcal{N}_i} \exp\left[\frac{A_i - p_{il}}{v}\right]}, \qquad k \in \mathcal{N}_i, \tag{10}$$

where p_{ik} is the price of variety $k \in \mathcal{N}_i$ and $\mu \geq v \geq 0$.[18] Suppose firms compete in prices at stage 2. Assumptions on technology are maintained from the previous analysis and, again, $\alpha_i = A_i - c_i$, $i \in \mathcal{I}$.

For tractability reasons, the exclusive focus is on the duopoly case. The following result for stage 2 equilibrium holds.

Proposition 5. *Equilibrium at stage 2 in nested logit model. Let $\mathcal{I} = \{1,2\}$. At stage 2 equilibrium, output levels and mark-ups are the same within a firm's product line and total output of firm $i \in \mathcal{I}$ is given by $D_i = P_i$. Moreover, gross profits of firm i are given by $\prod_i = \mu \Upsilon_i$, where Υ_i solves $\Upsilon_i = (N_i/N_j)^{v/\mu} \exp[(\alpha_i - \alpha_j)/\mu + 1/\Upsilon_i - \Upsilon_i]$, $j \neq i$.*

Again, the structure of demand leads to symmetry within a firm's product line. Total output of a firm (equilibrium sales), D_i, simply equals the probability P_i that firm i is chosen (given a unit mass of consumers). Moreover, Proposition 5 implies:

Corollary 2. *For $\mathcal{I} = \{1,2\}$, $\partial \prod_1/\partial N_1 > 0$, $\partial \prod_1/\partial N_2 < 0$, $\partial \prod_1/\partial \alpha_1 > 0$, $\partial \prod_1/\partial \alpha_2 < 0$, $\partial^2 \prod_1/\partial N_1 \partial \alpha_1 > 0$, $\partial^2 \prod_1/\partial N_1 \partial \alpha_2 < 0$, $\partial^2 \prod_1/\partial N_1 \partial N_2 < 0$ and $\partial^2 \prod_1/\partial N_1^2 < 0$. (Firm 2 is analogous.)*

Hence, the properties of profit functions in the nested logit model are similar to those of the Cournot model with linear demand (Corollary 1). Moreover, it turns out that in $N_1 - N_2$ space, the reaction function of firm 1 at stage 1 (choice of product range) is always steeper than that of firm 2, i.e., equilibrium is unique (see Fig. 1). Hence, applying Corollary 2 gives rise to

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[18] When $\mu > (=) v$, goods are better (equal) substitutes within a firm than across firms. Moreover, v measures the degree of intra-firm heterogeneity of goods. If $v \to 0$, goods become perfectly substitutable. (For a more detailed discussion of these issues, see Anderson and de Palma, 1992, p. 263.).

Proposition 6. *Firm size and diversification in the nested logit model. For $\mathcal{I} = \{1,2\}$, $\alpha_1 > \alpha_2$ implies $N_1^* > N_2^*$ and $D_1^* > D_2^*$, i.e., the firm with the higher quality–cost margin has both larger product range and larger size.*

4.2. Bertrand case with linear demand

Under Bertrand competition with linear demand (1), equilibrium profits at stage 2, $\prod_i$, can be derived analogously to Proposition 1. (The formal analysis to this subsection is relegated to the supplement on the Journal's editorial web site.) Decomposing profits again into sales and mark-up, $\prod_i = D_i M_i$, reveals that equilibrium mark-up M_i is typically decreasing in N_i. That is, launching additional varieties typically forces a firm to charge lower mark-ups to balance against the increased competition induced by availability of new varieties. For instance, in an interior Bertrand–Nash equilibrium with two firms, one finds that $\partial M_i/\partial N_i < 0$ if $\alpha_i \leq \alpha_j$ or if $|\alpha_i - \alpha_j|$ is sufficiently small. This is in contrast to the Cournot case, where $\partial M_i/\partial N_i > 0$ always holds (see Section 3.1). It reflects the well-known fact that the intensity of competition under Bertrand competition is higher than under Cournot competition.

Consequently, and in contrast to the results in Corollary 1 and 2, profits $\prod_i$ may not be increasing as a function of own product range N_i everywhere. In particular, this may occur if the firm's quality–cost margin is relatively low (compared to other firms) or if substitutability among products, γ, is high. Some other properties of $\prod_i$ derived for the Cournot model and the nested logit model fail to hold in general. What can be said, however, is the following. Focussing on the duopoly case for simplicity, in the neighborhood of a symmetric equilibrium, we have $\partial^2 \prod_1/\partial N_1 \partial \alpha_1 > 0$ and $\partial^2 \prod_1/\partial N_1 \partial \alpha_2 < 0$. This suggests that the firm with the higher quality–cost margin again offers a larger product range, provided that reaction functions are well-behaved as in Fig. 1 (which may be the case, according to numerical analysis). Moreover, one can show that if $\alpha_1 > \alpha_2$ and $N_1^* > N_2^*$, then $D_1^* > D_2^*$, giving rise to a positive size–diversification relationship.

5. Concluding remarks

This paper has analyzed an oligopoly framework with asymmetric multiproduct firms, which is able to address the empirical regularity that larger firms offer more diversified product lines. The analysis suggests that heterogeneity of enterprises with respect to technological characteristics is a driving force behind a positive relationship between firm size, measured by total sales, and product diversification. Moreover, it has been shown that quality–cost margins also play a crucial role for average product diversification at the industry level, as does the substitutability of goods.

Admittedly, the focus of the present analysis on the number of products as a measure of product diversification is quite narrow. For instance, Gollop and Monahan (1991) construct a diversification index which, in addition to the number of products supplied by an enterprise, also accounts for the distribution of sales from these products within a firm and differences in the heterogeneity of products. However, applying this index to a large data set of U.S. manufacturing firms and establishments, they find that the "number component is the dominant force" in explaining corporate diversification (p. 327). This gives some justification for focussing the theoretical analysis on the number of products, exogenously fixing the degree of product differentiation, and in turn leading to a uniform sales distribution within a firm.

The proposed oligopoly framework with asymmetric multiproduct firms may be employed to examine other interesting questions. To name one, the issues of profitability and desirability of horizontal mergers with multiproduct firms remain an area open to further analysis, which can be addressed in this framework.

Acknowledgements

Appendix A

Proof of Proposition 1. First, note that $\pi_i = \sum_{k\in\mathcal{N}_i}(p_k - c_i)x_k$ implies

$$\frac{\partial \pi_i}{\partial x_k} = p_k - c_i + \sum_{l\in\mathcal{N}_i}\frac{\partial p_l}{\partial x_k}x_l, \tag{A.1}$$

where $\partial p_l/\partial x_l = -\beta$ and $\partial p_l/\partial x_k = -\gamma$ for $l\neq k$, according to demand structure (1). Thus, optimal behavior of firm $i\in\mathcal{I}$ at stage 2 is given by the following set of first-order conditions (presuming an interior solution): $\alpha_i - 2\beta x_k - \gamma\sum_{l\in\mathcal{K}\setminus\{k\}}x_l - \gamma\sum_{l\in\mathcal{N}_i\setminus\{k\}}x_l = 0, k\in\mathcal{N}_i$, where $a_k = A_i$ for $k\in\mathcal{N}_i$ and $\alpha_i = A_i - c_i$ has been used. Adding and subtracting $2\gamma x_k$ implies

$$\alpha_i - 2(\beta - \gamma)x_k - \gamma Q - \gamma\sum_{l\in\mathcal{N}_i}x_l = 0 \tag{A.2}$$

where $Q \equiv \sum_{l\in\mathcal{K}}x_l$ is total output in the market. Thus, $x_k = X_i$ for all $k\in\mathcal{N}_i$, which implies $\sum_{l\in\mathcal{N}_i}x_l = N_iX_i$. Hence, using (A.2),

$$X_i = \frac{\alpha_i - \gamma Q}{2(\beta - \gamma) + \gamma N_i}. \tag{A.3}$$

Also note that $Q = \sum_i N_iX_i$. Multiplying both sides of (A.3) by N_i and summing over all $i\in\mathcal{I}$, one thus obtains

$$\gamma Q = \frac{\sum_i \alpha_i\Gamma_i}{1 + \sum_i \Gamma_i}, \tag{A.4}$$

where $\Gamma_i = \gamma N_i/[2(\beta - \gamma) + \gamma N_i]$ has been used. (A.4) implies

$$\alpha_i - \gamma Q = \frac{\Lambda_i}{1 + \sum_i \Gamma_i}, \tag{A.5}$$

where Λ_i is defined in Proposition 1. Combining (A.3) and (A.5) yields (3).

To obtain (4), first, note that $p_k - c_i = \alpha_i - \gamma Q - (\beta - \gamma)x_k$ for all $k\in\mathcal{N}_i$, according to (1), $a_k = A_i$ for $k\in\mathcal{N}_i$ and $\alpha_i = A_i - c_i$. Since $x_k = X_i$ for all $k\in\mathcal{N}_i$ and $\alpha_i - \gamma Q = [2(\beta - \gamma) + \gamma N_i]X_i$, according to (A.3), we obtain equilibrium price–cost differences $p_k - c_i = (\beta - \gamma + \gamma N_i)X_i[=M_i]$ for all $k\in\mathcal{N}_i$. Finally, noting that $\prod_i = N_iX_iM_i$ confirms (4). This concludes the proof. $\square$

Proof of Corollary 1. First, let us write $\sum_{h\in\mathcal{I}}\Gamma_h = 1 + \Phi_{-i} + \Gamma_i$, where $\Phi_{-i} \equiv \sum_{h\neq i}\Gamma_h$. Thus, using $\Gamma_i = \gamma N_i/[2(\beta-\gamma)+\gamma N_i]$, we have

$$X_i = \frac{\Lambda_i}{(1 + \Phi_{-i})(2(\beta-\gamma)+\gamma N_i)+\gamma N_i}, \tag{A.6}$$

according to (3). By substituting (A.6) into (4), we obtain

$$\Pi_i = \frac{N_i(\beta-\gamma+\gamma N_i)\Lambda_i^2}{[(1+\Phi_{-i})(2(\beta-\gamma)+\gamma N_i)+\gamma N_i]^2}. \tag{A.7}$$

Tedious derivations reveal that[19]

$$\frac{\partial\Pi_i}{\partial N_i} = \frac{(\beta-\gamma)[2(\beta-\gamma+\gamma N_i)+(2(\beta-\gamma)+3\gamma N_i)\Phi_{-i}]\Lambda_i^2}{[(1+\Phi_{-i})(2(\beta-\gamma)+\gamma N_i)+\gamma N_i]^3} > 0, \tag{A.8}$$

$$\frac{\partial^2\Pi_i}{\partial N_i^2} = \frac{-2\gamma(\beta-\gamma)\Lambda_i^2}{[(1+\Phi_{-i})(2(\beta-\gamma)+\gamma N_i)+\gamma N_i]^4} \times [\beta-\gamma+\gamma N_i+(\beta-\gamma+5\gamma N_i)\Phi_{-i}$$
$$+ 3\gamma N_i\Phi_{-i}^2] < 0. \tag{A.9}$$

Moreover, for $j\neq i$,

$$\frac{\partial\Pi_i}{\partial N_j} = \frac{-4\gamma(\beta-\gamma)N_i(\beta-\gamma+\gamma N_i)(2(\beta-\gamma)+\gamma N_i)\Lambda_i\Lambda_j}{[2(\beta-\gamma)+\gamma N_j]^2[(1+\Phi_{-i})(2(\beta-\gamma)+\gamma N_i)+\gamma N_i]^3} < 0, \tag{A.10}$$

$$\frac{\partial^2\Pi_i}{\partial N_i\partial N_j} = -\frac{4\gamma(\beta-\gamma)^2\Lambda_i}{[(1+\Phi_{-i})(2(\beta-\gamma)+\gamma N_i)+\gamma N_i]^4[2(\beta-\gamma)+\gamma N_j]^2}$$
$$\times \{(\alpha_i-\alpha_j)[2(\beta-\gamma+\gamma N_i)+(2(\beta-\gamma)+3\gamma N_i)\Phi_{-i}] \times [2(\beta-\gamma+\gamma N_i)$$
$$+ (2(\beta-\gamma)+\gamma N_i)\Phi_{-i}] - \Lambda_i[(2(\beta-\gamma)+\gamma N_i)(2(\beta-\gamma)+3\gamma N_i)\Phi_{-i}$$
$$+ 4(\beta-\gamma)(\beta-\gamma+\gamma N_i)]\}. \tag{A.11}$$

(A.8) and (A.9) confirm part (i) of Corollary 1 and (A.10) confirms part (ii). (Recall that Λ_i, $\Lambda j>0$ in interior equilibrium.) Moreover, note that for all $i,j\in\mathcal{I}$, $j\neq i$, $\partial\Lambda_i/\partial\alpha_i > 0$ and $\partial\Lambda_i/\partial\alpha_j < 0$ $\left(\text{recall } \Lambda_i = \alpha_i\left(1+\sum_{j\neq i}\Gamma_j\right)-\sum_{j\neq i}\alpha_j\Gamma_j\right)$. Using this, parts (iii) and (iv) follow from (A.7) and (A.8), respectively. Part (v) follows from (A.11) and the definition of Λ_i. This concludes the proof. □

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[19] Detailed derivations of (A.8)–(A.11) can be found online.

Proof of Proposition 2. First, let $W(N, \alpha, \beta, \gamma) \equiv \partial \prod_i(N, ..., N, \alpha, ..., \alpha, \beta, \gamma)/\partial N_i$ and consider the following

Lemma A.1. $\partial W/\partial \alpha > 0$ and $\partial W/\partial \gamma < 0$.

Proof. Noting that $\Lambda_i = \alpha$ under symmetry of firms, $\partial W/\partial \alpha > 0$ is immediately implied by (A.8). $\partial W/\partial \gamma < 0$ is shown in the supplement on the Journal's editorial web site. $\square$

According to (7) and the definition of W, the equilibrium product range, N^*, is given by $W(N^*, \alpha, \beta, \gamma) - C'(N^*) = 0$. Applying the implicit function theorem, one obtains $\partial N^*/\partial \xi = (\partial W/\partial \xi)/\Delta$ for $\xi \in \{\alpha, \gamma\}$, where $\Delta \equiv \partial^2 \prod_i/\partial N_i^2 + \sum_{j \neq i} \partial^2 \prod_i/\partial N_i \partial N_j - C''(N^*) < 0$. Thus, $\Delta < 0$, according to $C''(\cdot) \geq 0$ as well as parts (i) and (v) of Corollary 1. Proposition 2 then follows from Lemma A.1. $\square$

Proof of Proposition 4. It is first proven that $\alpha_i > \alpha_j$ implies $\Psi_i^* > \Psi_j^*$. The proof is by contradiction. Note that $\alpha_i > \alpha_j$ implies $\Psi_i(\mathbf{N}, \boldsymbol{\alpha}, \cdot) > \Psi_j(\mathbf{N}, \boldsymbol{\alpha}, \cdot)$ if $N_i = N_j$, according to part (iii) of Corollary 1, and recall $\partial \prod_i/\partial N_j < 0$ for $j \neq i$ from part (ii) of Corollary 1. Now suppose $\Psi_i^*(\boldsymbol{\alpha}, \cdot) \leq \Psi_j^*(\boldsymbol{\alpha}, \cdot)$ if $\alpha_i > \alpha_j$ and recall that under (8), we have $N_i^* > N_j^*$ if $\alpha_i > \alpha_j$ (Proposition 3). Also suppose firm i decreases its product range from N_i^* to $N_i = N_j^*$, which increases profits of firm j (as $\partial \prod_j/\partial N_i < 0$ for $i \neq j$). Moreover, profits of firm i would now be higher than those of j (as seen above). Thus, profits of i must have increased. But this means that no situation with $\Psi_i*(\boldsymbol{\alpha}, \cdot) \leq \Psi_j^*(\boldsymbol{\alpha}, \cdot)$ if $\alpha_i > \alpha_j$ can occur.

To show that $\alpha_i > \alpha_j$ implies $\Psi_i^* > \Psi_j^*$, define $\Gamma_i^* \equiv \gamma N_i^*/[2(\beta - \gamma) + \gamma N_i^*]$ and

$$\Lambda_{i*} \equiv \alpha_i \left(1 + \sum_{j \neq i} \Gamma_j\right) - \sum_{j \neq i} \alpha_j \Gamma_j, \tag{A.12}$$

$i \in \mathcal{I}$. Thus, one can write

$$D_i(\boldsymbol{\alpha}, \cdot) = N_i X_i(\mathbf{N}^*, \boldsymbol{\alpha}, \cdot) = \frac{N_i \Lambda_i}{(1 + \sum_i \Gamma_i)[2(\beta - \gamma) + \gamma N_i]}, \tag{A.13}$$

according to (3). Hence, we have $D_i^*(\boldsymbol{\alpha}, \cdot) > D_j^*(\boldsymbol{\alpha}, \cdot)$ if and only if

$$\frac{N_i \Lambda_i}{2(\beta - \gamma) + \gamma N_i} > \frac{N_j \Lambda_j}{2(\beta - \gamma) + \gamma N_j}. \tag{A.14}$$

Recall from Proposition 3 that $N_i^* > N_j^*$ if $\alpha_i > \alpha_j$. Thus, using (A.14), the result is confirmed if, for instance, $\alpha_i > \alpha_j$ implies $\Lambda_i^* > \Lambda_j^*$. To see that this is indeed the case, first, rewrite (A.12) as

$$\Lambda_i = \alpha_i \left(1 + \sum_{h \neq i,j} \Gamma_h\right) + (\alpha_i - \alpha_j) \Gamma_j - \sum_{h \neq i,j} \alpha_h \Gamma_h, \tag{A.15}$$

$i, j \in \mathcal{I}$. (A.15) then implies that

$$\Lambda_i - \Lambda_j = (\alpha_i - \alpha_j) \left(1 + \sum_{h \neq i,j} \Gamma_h\right) + (\alpha_i - \alpha_j) \Gamma_j - (\alpha_j - \alpha_i) \Gamma_i$$

$$= (\alpha_i - \alpha_j) \left(1 + \sum_{i \in \mathcal{I}} \Gamma_i\right), \tag{A.16}$$

i.e., $\Lambda_i^* > \Lambda_j^*$ if $\alpha_i > \alpha_j$. This concludes the proof. $\square$

Proof of Proposition 5. Noting that $x_{il}=P_iP_{l|i}$ for all $l\in\mathcal{N}_i$, at stage 2 firm i solves $\max_{p_{il},l\in\mathcal{N}_i}\sum_{l\in\mathcal{N}_i}(p_{il}-c_i)P_iP_{l|i}$ s.t. (9) and (10). The first-order condition with respect to p_{ik} yields:

$$P_iP_{k|i} + \sum_{l\in\mathcal{N}_i}(p_{il}-c_i)\frac{\partial P_i}{\partial p_{ik}}P_{l|i} + \sum_{l\in\mathcal{N}_i}(p_{il}-c_i)P_i\frac{\partial P_{l|i}}{\partial p_{ik}} = 0. \tag{A.17}$$

Using (10), it is straightforward to show that $\partial P_{k|i}/\partial p_{ik}=-P_{k|i}(1-P_{k|i})/v$ and, for $l\neq k$, $\partial P_{l|i}/\partial p_{ik}=P_{k|i}P_{l|i}/v$; moreover, $\partial P_i/\partial p_{ik}=-P_i(1-P_i)P_{k|i}/\mu$, according to (9), $k\in\mathcal{N}_i$. Substituting these expressions into (A.17), one obtains[20]

$$p_{ik}-c_i = v + \left[1-\frac{\mu}{v}(1-P_i)\right]\sum_{l\in\mathcal{N}_i}(p_{il}-c_i)P_{l|i}. \tag{A.18}$$

We seek for a price equilibrium in which $p_{ik}-c_i$ is the same for all $k\in\mathcal{N}_i$. Using $\sum_{l\in\mathcal{N}_i}P_{l|i}=1$, (A.18) implies that mark-up $p_{ik}-c_i=\mu/(1-P_i)$ in such an equilibrium. Thus, $p_{ik}=c_i+\mu/(1-P_i)$, which implies $A_i-p_{ik}=\alpha_i-\mu/(1-P_i)\equiv\zeta_i$ for all $k\in\mathcal{N}_i$. Using this fact, $P_{k|i}=1/N_i$ for all $k\in\mathcal{N}_i$, according to (10). Thus, recalling $x_{ik}=P_iP_{k|i}$, we have $x_{ik}=P_i/N_i$ for all $k\in\mathcal{N}_i$. This confirms that both output levels and mark-ups are the same within a firm's product line. Moreover, obviously, $D_i=P_i$.

We next turn to derive $\prod_1$. (The derivation of $\prod_2$ is analogous.) Substituting $A_i-p_{ik}=\zeta_i$ into (9), one obtains in duopoly:

$$P_1 = \frac{(N_1)^{v/\mu}}{(N_1)^{v/\mu} + (N_2)^{v/\mu}\exp\left[\frac{\zeta_2-\zeta_1}{\mu}\right]}. \tag{A.19}$$

Also note that $p_{1k}-c_1=\mu/(1-P_1)$ and $P_{k|1}=1/N_1$ for all $k\in\mathcal{N}_1$ imply $\sum_{l\in\mathcal{N}_1}(p_{1l}-c_1)P_1P_{l|1}=\mu P_1/(1-P_1)[=\prod_1]$ for stage 2 equilibrium profits of firm 1. Thus, using (A.19),

$$\prod_1 = \mu\left(\frac{N_1}{N_2}\right)^{v/\mu}\exp\chi, \tag{A.20}$$

where $\chi\equiv(\zeta_1-\zeta_2)/\mu$. Moreover, recalling $\zeta_i=\alpha_i-\mu/(1-P_i)$ and using $P_2=1-P_1$, we have $\chi=(\alpha_1-\alpha_2)/\mu-1/(1-P_1)+1/P_1$. We can now confirm that $\Upsilon_1=P_1/(1-P_1)$ (and thus $\prod_1=\mu\Upsilon_1>0$) with Υ_1 as defined in Proposition 5. To see this, first, note that $\Upsilon_1=P_1/(1-P_1)$ implies $1/P_1=1/\Upsilon_1+1$ and $1/(1-P_1)=1+\Upsilon_1$. Thus,

$$\chi = \frac{\alpha_1-\alpha_2}{\mu}+\frac{1}{\Upsilon_1}-\Upsilon_1. \tag{A.21}$$

In view of (A.20), this confirms $\prod_1=\mu\Upsilon_1$ with Υ_1 as defined in Proposition 5. $\square$

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[20] Comparison to Anderson and de Palma (1992, p. 272) reveals that, in the duopoly case, one can follow their proof to establish existence of a unique price equilibrium at stage 2.

Proof of Corollary 2. Using $\Upsilon_1 = (N_1/N_2)^{\nu/\mu} \exp \chi$, we obtain $\partial \Upsilon_1/\partial N_1 = \Upsilon_1[(\nu/\mu)/N_1 + \partial \chi/\partial N_1]$ and $\partial \Upsilon_1/\partial N_2 = \Upsilon_1[-(\nu/\mu)/N_2 + \partial \chi/\partial N_2]$, where $\partial \chi/\partial N_i = -(1/\Upsilon_1^2 + 1)\partial \Upsilon_1/\partial N_1$, $i \in \{1, 2\}$, according to (A.21). Combining these results and solving for $\partial \Upsilon_1/\partial N_1$ and $\partial \Upsilon_1/\partial N_2$ yields

$$\frac{\partial \Upsilon_1}{\partial N_1} = \frac{(\nu/\mu)\Upsilon_1}{N_1\left(1 + \Upsilon_1 + \frac{1}{\Upsilon_1}\right)} > 0 \quad \text{and} \quad \frac{\partial \Upsilon_1}{\partial N_2} = -\frac{N_1}{N_2}\frac{\partial \Upsilon_1}{\partial N_1} < 0, \tag{A.22}$$

respectively. Similarly, $\partial \Upsilon_1/\partial \alpha_i = \Upsilon_1 \partial \chi/\partial \alpha_i$, $i \in \{1, 2\}$, where $\partial \chi/\partial \alpha_1 = 1/\mu - (1/\Upsilon_1^2 + 1)\partial \Upsilon_1/\partial \alpha_1$ and $\partial \chi/\partial \alpha_2 = -1/\mu - (1/\Upsilon_1^2 + 1)\partial \Upsilon_1/\partial \alpha_2$, according to (A.21). Hence,

$$\frac{\partial \Upsilon_1}{\partial \alpha_1} = \frac{\Upsilon_1/\mu}{1 + \Upsilon_1 + \frac{1}{\Upsilon_1}} > 0 \quad \text{and} \quad \frac{\partial \Upsilon_1}{\partial \alpha_2} = -\frac{\partial \Upsilon_1}{\partial \alpha_1}. \tag{A.23}$$

Thus, as $\prod_1 = \mu \Upsilon_1$ and $\Upsilon_1 > 0$, $\partial \prod_1/\partial N_1 > 0$, $\partial \prod_1/\partial N_2 < 0$, $\partial \prod_1/\partial \alpha_1 > 0$, $\partial \prod_1/\partial \alpha_2 < 0$, according to (A.22) and (A.23). Moreover, using again (A.22) and $\prod_1 = \mu \Upsilon_1$, it is easy to show that

$$\frac{\partial^2 \prod_1}{\partial N_1 \partial \delta} = \frac{\nu\left(1 + \frac{2}{\Upsilon_1}\right)\frac{\partial \Upsilon_1}{\partial \delta}}{N_1\left(1 + \Upsilon_1 + \frac{1}{\Upsilon_1}\right)^2}, \tag{A.24}$$

$\delta \in \{\alpha_1, \alpha_2, N_2\}$. Hence, as $\partial \Upsilon_1/\partial \alpha_1 > 0$, $\partial \Upsilon_1/\partial \alpha_2 < 0$ and $\partial \Upsilon_1/\partial N_2 < 0$, one finds $\partial^2 \prod_1/\partial N_1 \partial \alpha_1 > 0$, $\partial^2 \prod_1/\partial N_1 \partial \alpha_2 < 0$ and $\partial^2 \prod_1/\partial N_1 \partial N_2 < 0$, respectively. Finally, to confirm strict concavity of $\prod_1$ as a function of N_1, note that

$$\frac{\partial^2 \prod_1}{\partial N_1^2} = \underbrace{-\frac{\nu \Upsilon_1}{N_1\left(1 + \Upsilon_1 + \frac{1}{\Upsilon_1}\right)}\frac{1}{N_1}}_{=\mu(\partial \Upsilon_1/\partial N_1)} + \mu\frac{\partial \Upsilon_1}{\partial N_1}\frac{\frac{\nu}{\mu}\left(1 + \frac{2}{\Upsilon_1}\right)}{N_1\left(1 + \Upsilon_1 + \frac{1}{\Upsilon_1}\right)^2}, \tag{A.25}$$

according to $\prod_1 = \mu \Upsilon_1$ and (A.22). Hence,

$$\frac{\partial^2 \prod_1}{\partial N_1^2} = -\frac{\mu\frac{\partial \Upsilon_1}{\partial N_1}}{N_1\left(1 + \Upsilon_1 + \frac{1}{\Upsilon_1}\right)^2}\left[\Upsilon_1(2 + \Upsilon_1) + 2 + \frac{1}{\Upsilon_1^2} + \left(1 - \frac{\nu}{\mu}\right)\left(1 + \frac{2}{\Upsilon_1}\right)\right] < 0 \tag{A.26}$$

(recall $\nu \leq \mu$). This confirms Corollary 2. $\quad \square$

Proof of Proposition 6. The result is proven in two steps. First, it is shown that $\alpha_1 > \alpha_2$ implies $N_1^* > N_2^*$, and second, that $\alpha_1 > \alpha_2$ implies $D_1^* > D_2^*$.

Step 1: Corollary 2 implies that reaction functions at stage 1 are downward-sloping in $N_1 - N_2$ space. Moreover, an increase in α_i or a decrease in α_j shifts the reaction curve of firm i outward and that of firm $j \neq i$ inward. Hence, $\alpha_1 > \alpha_2$ implies $N_1^* > N_2^*$ if the reaction function of firm 1 is steeper than that of firm 2 (compare Fig. 1). Applying the implicit function theorem to (7) and using $C'' \geq 0$, a sufficient condition for this is

$$\frac{\partial^2 \prod_1}{\partial N_1^2}\frac{\partial^2 \prod_2}{\partial N_2^2} > \frac{\partial^2 \prod_1}{\partial N_1 \partial N_2}\frac{\partial^2 \prod_2}{\partial N_1 \partial N_2}. \tag{A.27}$$

Substituting (A.24) and (A.26) into (A.27) as well as using both (A.22) and $v \leq \mu$ reveals that (A.27) holds if

$$
\left[\Upsilon_1(2 + \Upsilon_1) + 2 + \frac{1}{\Upsilon_1^2} \right] \left[\Upsilon_2(2 + \Upsilon_2) + 2 + \frac{1}{\Upsilon_2^2} \right]
$$
$$
> \left(1 + \frac{2}{\Upsilon_1} \right) \left(1 + \frac{2}{\Upsilon_2} \right). \tag{A.28}
$$

It is easy to show that, for any (Υ_1, Υ_2), (A.28) is fulfilled. This concludes step 1.

Step 2: To see that $\alpha_1 > \alpha_2$ also implies $D_1^* > D_2^*$, recall that Υ_i solves $\Upsilon_i = (N_i/N_j)^{v/\mu} \exp [(\alpha_i - \alpha_j)/\mu + 1/\Upsilon_i - \Upsilon_i]$, $j \neq i$ (Proposition 5). Thus, obviously, $\Upsilon_1 > \Upsilon_2$ if $\alpha_1 > \alpha_2$ and $N_1 > N_2$. But we already know that $N_1^* > N_2^*$ if $\alpha_1 > \alpha_2$. Hence, in equilibrium, $\Upsilon_1 > \Upsilon_2$ if $\alpha_1 > \alpha_2$. Finally, recall that $D_i = P_i$ and $\Upsilon_1 = P_1/(1 - P_1)$, i.e., $P_1 = \Upsilon_1/(1 + \Upsilon_1)$. This confirms $D_1^* > D_2^*$ if $\alpha_1 > \alpha_2$, concluding the proof. $\square$

Appendix B. Supplementary data

Supplementary data associated with this article can be found in the online version at doi:10.1016/j.ijindorg.2005.12.003.

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