# Non-Routine Tasks, Restructuring of Firms, and Wage Inequality Within and Between Skill-Groups 

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#### Abstract

This paper argues that endogenous restructuring processes within firms towards analytical and interactive non-routine tasks (like problem-solving and organizational activities, respectively), triggered by advances in information and communication technologies (ICT) and rising supply of educated workers, are associated with an increase of wage inequality within education groups. We show that this may be accompanied by a decline or stagnation of between-group wage dispersion. The mechanisms proposed in this research are not only consistent with the evolution of the distribution of wages in advanced countries, but also with the evolution of task composition in firms and a frequently confirmed complementarity between skillupgrading, new technologies and knowledge-based work organization.


Keywords: non-routine tasks, skill supply, technological progress, unobserved abilities, within-group wage inequality.

JEL Classification: D20, J31.

## 1 Introduction

The dramatic changes in the distribution of wages in many advanced countries during the last few decades have stimulated an intensive debate in economic research. Most of the literature has focussed on a rise in wage inequality between education groups. ${ }^{1}$ Interestingly, however,

[^0]an increase in the college premium has been largely confined to the US and UK (from the 1980s onwards until recently), whereas inequality within education groups (sometimes referred to as residual wage inequality) has risen substantially also in other advanced countries (see, e.g., Fitzenberger, 1999, and Fitzenberger et al., 2001, for Germany). ${ }^{2}$ Moreover, most empirical studies conclude that even for the US the rise in within-group wage inequality accounts for more than half of the rise in total wage inequality (e.g., Juhn et al., 1993; Gottschalk and Smeeding, 1997; Katz and Autor, 1999).

In the first theoretical contribution which focuses on within-group wage inequality, Galor and Moav (2000) argue that an increase in the rate of technological progress, by adversely affecting the relative productivity of low-skilled labor, raises educational attainment of workers with relatively low learning abilities. As a result, wage inequality within skilled and unskilled labor increases, in addition to rising between-group inequality. Gould et al. (2001) offer an alternative explanation for rising residual wage inequality by arguing that increasing uncertainty in the rate of technological change across sectors disproportionately affects the learning requirements of unskilled labor, thereby raising demand for education as insurance. In a similar vein, Aghion (2002) and Aghion et al. (2002) argue that within-group wage inequality rises with the speed of diffusion of new general purpose technologies, by showing that inequality may arise even among (with respect to their abilities) identical workers with different opportunities to adapt to the most recent vintages of machines.

In contrast, this paper argues that endogenous restructuring processes within firms towards analytical and interactive non-routine tasks (like problem-solving and organizational activities, respectively), triggered by advances in information and communication technologies (ICT) and rising supply of educated workers, are associated with rising wage inequality within education groups. Moreover, we show that, at the same time, between-group wage dispersion may fall, as especially observed in Continental Europe.

[^1]Our goal is to provide a theory based on the behavioral response of firms in adapting to changes in technology and factor supply conditions. ${ }^{3}$ The proposed framework is designed to shed light on the microeconomic underlying forces for changes in the wage distribution within and across education groups. The model rests on the following hypotheses. First, reallocating production workers from routine to non-routine tasks is productivity-enhancing, but requires a larger degree of (informal) training by non-production workers (encompassing "support" and "supervision" tasks). For instance, this includes the regular updating of workers about changes in work procedures, the organizational structure and employers' goals. Second, non-routine tasks require a wider spectrum of abilities like analytical skills, adaptability to new environments, management skills, the ability to communicate with coworkers (and other social skills) - than routine tasks. For instance, autonomous decision-making and problem-solving presumes interaction among workers, performance and coordination of multiple tasks and the need to gather relevant information from coworkers (e.g., Lindbeck and Snower, 1996, 2000). Moreover, providing support and information to production workers itself requires social skills of non-production workers. Abilities to perform these tasks effectively are typically unobservable for empirical researchers, and are possibly unrelated to formal education levels, thus providing a natural ingredient for a theory of residual inequality. Third, we hypothesize that non-production (e.g., organizational) activities are intensive in educated labor.

The idea of the present paper is to propose a unified framework which identifies mechanisms for the decision by firms how to allocate workers to routine and non-routine tasks, and thereby to utilize analytical and social abilities of workers. Endogenizing the allocation of workers into routine and non-routine tasks and taking into account their connection to unobservable abilities allows us to address within-group wage inequality as found in standard estimations of Mincer equations. Distinguishing, in addition, between education levels serves to examine the interaction between within-group and between-group wage inequality in a model with endogenous task composition.

As an immediate consequence of taking into account withingroup and between-group heterogeneity of workers as well as the

[^2]heterogeneity of tasks in a unified framework, however, the analysis becomes quite complex. But in our view, these cost in terms of exposition has to be weighted against the benefit of obtaining an (admittedly still very stylized) theory in which not only the outcome (wage inequality) but also the mechanism (restructuring and task composition) is empirically testable. In other words, the advantage of our approach is to bring light into the "black box" of production process.

In the next section, we discuss supporting evidence of our analysis on the evolution of task composition in firms and a frequently confirmed complementarity between skill-upgrading, new technologies and know-ledge-based work organization. Section 3 sets up the model. Section 4 analyzes the equilibrium and provides comparative-static results. Section 5 summarizes and briefly argues that our model may shed light into the differences in the evolution of wage inequality patterns between the U.S. and Continental Europe. All proofs are relegated to an Appendix.

## 2 Evidence on Task Composition and Training

As argued above, the goal of our analysis is to simultaneously address wage inequality within and between education groups together with observed restructuring of firms towards non-routine tasks.

A growing body of empirical evidence attempts to obtain insight into the nature of the apparent "skill-biased technological change" in the developed world. For instance, autonomous problem-solving and assignment of more responsibility to workers have been major innovations in work organization practices (e.g., OECD, 1999, chap. 4). Brynjolfsson and Hitt (2000) and Bresnahan et al. (2002) for the U.S., and Falk (2002) for Germany find a strong positive relationship between computerization, organizational change and training provision. Moreover, evidence by Autor et al. (2003) for the U.S. and Spitz (2004) for West Germany suggests that computerization has caused a significant shift in the job composition from routine to non-routine tasks. ${ }^{4}$ Their results lend considerable support for the hypothesis that computer

[^3]technology is complementary to workers with relatively high analytical and interactive abilities. Consistent with these kinds of evidence, our model suggests that ICT has induced firms to incur training expenditures to restructure towards non-routine tasks, and that this restructuring favors workers with analytical and social skills. ${ }^{5}$ It is important to note that this seems to concern all education levels. For instance, Spitz (2004, table 6) finds that between 1979 and 1998/99 the share of analytical and interactive tasks have increased even for low-educated workers from 7.6 to 21.4 percent. For medium-educated and higheducated workers, the change has been from 12.8 to 40.2 percent and from 35.6 to 73 percent, respectively. Apart from the role of ICT, Caroli and van Reenen (2001) present evidence from both France and UK which is consistent with the impact of an increase in the educated workforce in our model. Their findings suggest a strongly positive effect of changes in the relative supply of skilled labor (proxied by regional skill price differentials) on restructuring of firms towards such know-ledge-based organizational forms. Finally, Gould (2002) provides evidence on a surge of the demand for general skills like analytical and social abilities within all broad occupations, along with rising residual wage inequality.

## 3 The Model

We suppose that individual skills differ in two dimensions: first, in formal education levels and, second, in abilities like adaptability to new environments as well as analytical, management and social (e.g., communication) skills, which are typically unobservable for empirical researchers.

Formally, suppose that there are two types of education levels, highly educated ( $\mathrm{H}-$ ) and less educated ( $L-$ ) labor. Both are inelastically supplied in segmented and perfect labor markets. Labor supply is denoted

[^4]by $H$ and $L$, respectively. ${ }^{6}$ To allow for within-group heterogeneity, e.g., capturing individual differences in analytical and social skills, first, let $L-$ individuals differ in ability $\beta \in B=\left\{\beta^{1}, \ldots, \beta^{K}\right\}, 0 \leq \beta^{1}<\cdots<$ $\beta^{K}<\infty$. The supply of type $\beta$ is denoted by $l^{S}(\beta)$, i.e., $\sum_{\beta \in B} l^{S}(\beta)=L$. Second, let $H$-workers differ in ability $\gamma \in \Gamma=\left\{\gamma^{1}, \ldots, \gamma^{J}\right\}$, $0 \leq \gamma^{1}<\cdots<\gamma^{J}<\infty$. The supply of type $\gamma$ is denoted by $h^{S}(\gamma)$, i.e., $\sum_{\gamma \in \Gamma} h^{S}(\gamma)=H .{ }^{7}$

At the firm level, we distinguish between routine and non-routine tasks in the production process of firms. The characterization of nonroutine tasks rests on two elements. First, workers assigned to non-routine tasks are more productive than those in routine tasks but costly in terms of non-production (e.g., managerial) labor. For instance, performing non-routine production tasks requires steady provision of information by non-production workers about production processes, products, employers' goals, work procedures, customer feedbacks, legal regulations etc. We follow Porter (1986) in using the term "support activities" for this kind of informal training provision. Thus, we refer to workers assigned to non-routine tasks as "supported" workers. Second, we assume that abilities like analytical and social skills play a role only

[^5]for non-routine tasks, which captures in an extreme form that they are more important for non-routine than for routine tasks. ${ }^{8}$

Formally, let there be a unit mass of firms which produce a homogenous good in a perfect market. Output $y_{i}$ of firm $i$ is produced according to the linearly homogeneous function

$$
\begin{equation*}
y_{i}=F\left(\tilde{h}_{i}, \tilde{l}_{i}\right) \equiv \tilde{l}_{i} f\left(\kappa_{i}\right), \kappa_{i} \equiv \tilde{h}_{i} / \tilde{l}_{i} \tag{1}
\end{equation*}
$$

(i.e., $f(\cdot) \equiv F(\cdot, 1)$ ), where $\tilde{h}_{i}$ and $\tilde{l}_{i}$ denote efficiency units of $H-$ and $L$-labor in production, respectively, i.e., $\kappa_{i}$ is the education-intensity of production labor. $f(\cdot)$ is a strictly increasing and strictly concave function which fulfills the boundary conditions $\lim _{\kappa \rightarrow \infty} f^{\prime}(\kappa)=0$ and $\lim _{\kappa \rightarrow 0^{+}} f^{\prime}(\kappa)=\infty . \tilde{h}_{i}$ and $\tilde{l}_{i}$ depend on the respective number of workers $\underset{\substack{k \rightarrow 0^{+} \\ \text {assigned }}}{ }$ to routine and non-routine tasks, denoted by $h_{i}^{1}, l_{i}^{1}$ and $h_{i}^{2}, l_{i}^{2}$, respectively, and the within-group ability distribution of workers assigned to non-routine tasks in firm $i$. Let $\hat{l}_{i}(\beta)$ be the number of $L$-workers in firm $i$ with ability $\beta$ when assigned to non-routine tasks, i.e.,

$$
\begin{equation*}
l_{i}^{2}=\sum_{\beta \in B} \hat{l}_{i}(\beta) . \tag{2}
\end{equation*}
$$

To simplify the analysis, suppose that $H$-workers are equally productive when assigned to non-routinized production activities. That is, $\gamma$ refers to the ability of $H$-workers to provide support to production workers (when assigned as non-production worker), as introduced shortly. ${ }^{9}$ Total efficiency units of production labor within firm $i$ may then be written as

$$
\begin{equation*}
\tilde{h}_{i}=h_{i}^{1}+\tilde{h}_{i}^{2} \text { and } \tilde{l}_{i}=l_{i}^{1}+\tilde{l}_{i}^{2} \tag{3}
\end{equation*}
$$

[^6]respectively, where
\[

$$
\begin{equation*}
\tilde{h}_{i}^{2}=a h_{i}^{2} \quad \text { and } \quad \tilde{l}_{i}^{2}=b \sum_{\beta \in B} \beta \hat{l}_{i}(\beta) \tag{4}
\end{equation*}
$$

\]

are the efficiency units in non-routine tasks of $H-$ and $L$-labor, respectively. Parameters $a$ and $b$ are related to productivity differences between non-routine and routine labor. To capture that supported workers $\left(h^{2}, l^{2}\right)$ have higher productivity than those who are assigned to routine tasks $\left(h^{1}, l^{1}\right)$ except, possibly, the least able $L$-workers, suppose $a>1$ and $b \beta>1$ for all $\beta \in\left\{\beta^{2}, \ldots, \beta^{K}\right\} .{ }^{10}$

A final building block of our model is the "support technology", reflecting the organizational (support/training) effort necessary to raise productivity of production workers. To capture that these activities are intensive in educated labor, suppose for simplicity that only $H$-workers can be assigned to these (non-routine) non-production tasks. Specifically, in order to raise productivity of $h_{i}^{2}$ and $l_{i}^{2}$ workers (assigned to nonroutine tasks) with high and low education, respectively, firm $i$ needs to employ

$$
\begin{equation*}
\tilde{m}_{i}=G\left(h_{i}^{2}, l_{i}^{2}\right) \equiv l_{i}^{2} g\left(\chi_{i}\right), \quad \chi_{i} \equiv h_{i}^{2} / l_{i}^{2} \tag{5}
\end{equation*}
$$

efficiency units of non-production $(H-)$ labor, where $G$ is a linearly homogeneous function. Note that the intensive form in (5) requires $l_{i}^{2}>0$. We exclusively focus on this case in the following (in order to avoid only mildly interesting borderline cases). $\chi_{i}$ is the educationintensity of supported labor in firm $i$. Let $g(\cdot)$ be a strictly increasing and strictly convex function. ${ }^{11}$ That is, the support technology exhibits complementarities among both types of labor (i.e., $G_{12}<0$ ), in analogy to the standard assumption that $H-$ and $L$-labor are complements in the production technology $F$. (Note that $g^{\prime \prime}(\cdot)>0$ is equivalent to $G_{12}<0$ under linear homogeneity of $G$.) Let $m_{i}$ be the total amount of non-

[^7]production labor and $\hat{m}_{i}(\gamma)$ be the amount of type $\gamma$ employed as nonproduction worker in firm $i$, respectively, i.e.,
\[

$$
\begin{equation*}
m_{i}=\sum_{\gamma \in \Gamma} \hat{m}_{i}(\gamma) \tag{6}
\end{equation*}
$$

\]

For a given amount $m_{i}$, efficiency units $\tilde{m}_{i}$ depend on the ability distribution of $\gamma$ within firm $i$ (e.g., reflecting differences in managerial ability) according to

$$
\begin{equation*}
\tilde{m}_{i}=\sum_{\gamma \in \Gamma} \gamma \hat{m}_{i}(\gamma) . \tag{7}
\end{equation*}
$$

Finally, suppose that wage costs for non-production labor are variable (rather than fixed) costs. ${ }^{12}$ This reflects the idea that providing support to production workers assigned to non-routine tasks is an ongoing necessity, in contrast to one-shot formal training programmes which improve workers' human capital stock.

In sum, the model exhibits the minimum structure needed to examine the decision of firms how to allocate workers with different skills to different tasks, taking into account that non-production labor plays a major role in the organization of firms and allowing for a second dimension of skill in addition to formal education. Stated differently, the theoretical innovation of the model is to allow for a distinction between a "Tayloristic" and a more "Holistic" organizational structure in firms, where the latter is characterized by non-routine tasks which are more productive, but also more costly (in terms of informal training provision) than routine tasks.

[^8]
## 4 Equilibrium Analysis

This section provides the equilibrium analysis. After setting up the equilibrium conditions, we derive comparative-static results regarding technology parameters $a, b$, and changes in the supply of labor, $H, L$. This allows us to analyze the behavioral response of firms to technology and factor supply conditions on the task composition of firms and their impact on wage inequality within and between education groups.

First, it is plausible to argue that the introduction of ICT and advances in (human resource) management techniques can be reflected by an increase in $a$ and $b$ (see (4)). For instance, new ICT reduces the cost of lateral communication among workers and increases the ability to process information (e.g., Radner, 1993; Lindbeck and Snower, 1996, 2000). Second, as well known, the share of educated workers has considerably increased in most advanced countries over the last decades, which is reflected by an increase in $\phi \equiv H / L$ in our model. ${ }^{13}$ Note that, at least for the 1970 s, these education supply shifts were not a response to an increasing college premium, which actually decreased in the 1970s in the U.S. and remained fairly stable in Continental Europe during the last few decades. Rather, consistent with our modelling strategy, it seems plausible to interpret these shifts as being related to changes in exogenous factors like the significant increases in financial aid for college students in the U.S. and a surge in public education facilities in Continental Europe. We exclusively focus on the case in which the composition of analytical and social (i.e., typically unobservable) abilities remain unchanged if labor supply changes. That is, $h^{S}(\gamma) / H, \gamma \in \Gamma$, and $l^{S}(\beta) / L, \beta \in B$, remain constant if $H$ or $L$ changes. This assumption is consistent with the idea that unobservable ability and education are not (perfectly) related to each other. ${ }^{14}$

[^9]
### 4.1. Equilibrium Conditions

Let $w_{h}^{1}$ and $w_{l}^{1}$ denote the wage rates of unsupported $H-$ and $L$-labor assigned to routinized production tasks, respectively, $w_{h}^{2}$ the wage rate of supported $H$-labor in modern production, and $w_{m}(\gamma)$ and $w_{l}(\beta)$ the wage rates of supporting labor of type $\gamma$ and supported $L$-labor of type $\beta$, respectively. According to (1)-(5) and (7), the decision problem of firm $i$ is given by

$$
\begin{align*}
& \max _{h_{i}^{1}, l_{i}^{1}, h_{i}^{2}, \hat{l}_{i}(\beta), \beta \in B, \hat{m}_{i}(\gamma), \gamma \in \Gamma} F\left(h_{i}^{1}+a h_{i}^{2}, l_{i}^{1}+b \sum_{\beta \in B} \beta \hat{l}_{i}(\beta)\right) \\
& -w_{h}^{1} h_{i}^{1}-w_{l}^{1} l_{i}^{1}-w_{h}^{2} h_{i}^{2}-\sum_{\beta \in B} w_{l}(\beta) \hat{l}_{i}(\beta) \\
& -\sum_{\gamma \in \Gamma} w_{m}(\gamma) \hat{m}_{i}(\gamma) \text { s.t. } \sum_{\gamma \in \Gamma} \gamma \hat{m}_{i}(\gamma)=G\left(h_{i}^{2}, \sum_{\beta \in B} \hat{l}_{i}(\beta)\right), \tag{8}
\end{align*}
$$

and subject to nonnegativity constraints. ${ }^{15}$ If $l_{i}^{2}>0$, the first-order conditions from optimization problem (8) can be written as

$$
\begin{align*}
f^{\prime}\left(\kappa_{i}\right) & =w_{h}^{1},  \tag{9}\\
a f^{\prime}\left(\kappa_{i}\right) & \leq w_{h}^{2}+\lambda_{i} g^{\prime}\left(\chi_{i}\right),  \tag{10}\\
f\left(\kappa_{i}\right)-\kappa_{i} f^{\prime}\left(\kappa_{i}\right) & =w_{l}^{1},  \tag{11}\\
b \beta\left(f\left(\kappa_{i}\right)-\kappa_{i} f^{\prime}\left(\kappa_{i}\right)\right) & \leq w_{l}(\beta)+\lambda_{i}\left(g\left(\chi_{i}\right)-\chi_{i} g^{\prime}\left(\chi_{i}\right)\right), \quad \beta \in B,  \tag{12}\\
\lambda_{i} \gamma & \leq w_{m}(\gamma), \quad \gamma \in \Gamma, \tag{13}
\end{align*}
$$

holding with equality if the relevant non-negativity constraint is binding. ${ }^{16}$ $\lambda_{i}$ denotes the Lagrange multiplier associated with the constraint in (8). The left-hand sides of (9)-(12) are the marginal products of the respective types of labor, whereas the right-hand sides are the marginal costs (which, for supported labor, also contain the costs of non-production labor).

[^10]We focus on a symmetric equilibrium (and thus omit the firm index $i$ from now on). ${ }^{17}$ Full employment in equilibrium with a unit mass of firms then implies

$$
\begin{equation*}
h^{1}+h^{2}+m=H, \quad l^{1}+l^{2}=L . \tag{14}
\end{equation*}
$$

Since $H$-labor assigned to production tasks is homogeneous and the wage costs for non-production $H$-workers are entirely born by firms, we have $w_{h}^{1}=w_{h}^{2} \equiv w_{h}$ in equilibrium. ${ }^{18}$

Let the relative wage of labor in routine tasks, $\omega \equiv w_{h} / w_{l}^{1}$, be our measure of between-group wage inequality. ${ }^{19}$ According to (9) and (11),

$$
\begin{equation*}
\omega=\frac{f^{\prime}(\kappa)}{f(\kappa)-\kappa f^{\prime}(\kappa)} \equiv \Omega(\kappa) \tag{15}
\end{equation*}
$$

where $\Omega^{\prime}(\kappa)<0$. The following first result emerges.
Lemma 1: In equilibrium, there exist threshold ability levels $\tilde{\beta} \in B$ and $\tilde{\gamma} \in \Gamma$ such that the following holds.
(i) $\hat{l}(\beta)=l^{S}(\beta)$ for all $\beta>\tilde{\beta}, \hat{l}(\tilde{\beta})>0$, and $\hat{l}(\beta)=0$ for all $\beta<\tilde{\beta}$. Moreover, $w_{l}(\beta) / w_{l}^{1}=b(\beta-\tilde{\beta})+1$ for all $\beta \geq \tilde{\beta}$ if $\hat{l}(\tilde{\beta})<l^{S}(\tilde{\beta})$, whereas $\hat{l}(\tilde{\beta})=l^{S}(\tilde{\beta})$ implies $w_{l}(\tilde{\beta}) \geq w_{l}^{1}$ and, with $\gamma \geq \tilde{\gamma}$,

$$
\begin{equation*}
b \beta=\frac{w_{l}(\beta)}{w_{l}^{1}}+\frac{w_{m}(\gamma)}{w_{h}} \omega \frac{g(\chi)-\chi g^{\prime}(\chi)}{\gamma} \quad \text { for all } \beta \geq \tilde{\beta} . \tag{16}
\end{equation*}
$$

17 As often the case in models with identical firms, it is conceivable that asymmetric equilibria exist in addition to a symmetric one. However, note that $\kappa_{i}=\kappa$ is directly implied by (9) or (11), respectively. Also note that, due to the linear-homogeneity of functions $F$ and $G$, firms make zero profits in equilibrium.
18 Moreover, as will become apparent below, the symmetric equilibrium is unique.

19 We suppose $h^{1}>0$ and $l^{1}>0$ to focus on interior solutions (which is the empirically relevant case). In fact, if the majority of less educated workers still holds traditional jobs and the majority of educated labor are production workers (as plausible for the time periods which most empirical studies about the evolution of wage inequality have considered), $\omega$ represents the relative median wage of educated labor.
(ii) $\hat{m}(\gamma)=h^{S}(\gamma)$ for all $\gamma>\tilde{\gamma}, \hat{m}(\tilde{\gamma})>0$, and $\hat{m}(\gamma)=0$ for all $\gamma<\tilde{\gamma}$. Moreover, $w_{m}(\tilde{\gamma})=w_{h}$ and $w_{m}(\gamma) / w_{h}=\gamma / \tilde{\gamma}$ for all $\gamma \geq \tilde{\gamma}$ if $\hat{m}(\tilde{\gamma})<h^{S}(\tilde{\gamma})$, whereas $\hat{m}(\tilde{\gamma})=h^{S}(\tilde{\gamma})$ implies $w_{m}(\tilde{\gamma}) \geq w_{h}$ and

$$
\begin{equation*}
a=1+\frac{w_{m}(\gamma)}{w_{h}} \frac{g^{\prime}(\chi)}{\gamma} \quad \text { for all } \gamma \geq \tilde{\gamma} \tag{17}
\end{equation*}
$$

Proof: See Appendix.
Part (i) of Lemma 1 states that $L$-workers are supported (i.e., are assigned to non-routine tasks) up to a threshold level $\tilde{\beta}$ in the ability distribution of $\beta$, whereas those with ability lower than $\tilde{\beta}$ are assigned to routine tasks. $L$-workers with ability above threshold $\tilde{\beta}$ earn a wage premium (i.e., $w_{l}(\beta) / w_{l}^{1}=b(\beta-\tilde{\beta})+1>1$ for all $\beta>\tilde{\beta}$ ), whereas $w_{l}(\tilde{\beta})=w_{l}^{1}$ if $L$-workers of type $\tilde{\beta}$ are not a scarce resource (i.e., if $\left.\hat{l}(\tilde{\beta})<l^{S}(\tilde{\beta})\right)$. If $\hat{l}(\tilde{\beta})=l^{S}(\tilde{\beta})$, however, then $w_{l}(\tilde{\beta})>w_{l}^{1}$ typically holds (except of a knife-edge case). Similarly, $H$-workers with ability $\gamma \geq \tilde{\gamma}$ are assigned to non-production tasks, and always earn a wage premium (as $w_{m}(\gamma) / w_{h}=\gamma / \tilde{\gamma}$ for all $\gamma \geq \tilde{\gamma}$ ) if ability $\gamma$ exceeds threshold ability $\tilde{\gamma}$. (Also the remainder of part (ii) of Lemma 1 is analogous to part (i)).

Let within-group wage inequality be measured by relative wages of workers assigned to non-routinized jobs, $w_{m}(\gamma) / w_{h}, \gamma \geq \tilde{\gamma}$, and $w_{l}(\beta) / w_{l}^{1}$, $\beta \geq \tilde{\beta}$, respectively. ${ }^{20}$ Thus, (16) and (17) jointly give us a relationship between within-group wage inequality and between-group wage inequality (measured by $\omega$ ). In particular, within-group inequality and between-group inequality may be negatively related. The left-hand side of (16) is the productivity of a supported $L$-worker with ability $\beta$ relative to that of a $L$-worker in a routinized job, whereas the right-hand side is the respective relative cost. This relative cost consists of four (endogenous) components: within-group relative wage $w_{l}(\beta) / w_{l}^{1}$ of a $L$-worker of type $\beta$, within-group relative wage $w_{m}(\gamma) / w_{h}$ of a non-production $(H-)$ worker of type $\gamma$, between-group relative wage $\omega$, and the marginal physical cost

[^11]Table 1. Comparative-static results (marginal effects)
Scenario 1: $\hat{l}(\tilde{\beta})<l^{S}(\tilde{\beta})$ and $\hat{m}(\tilde{\gamma})<h^{S}(\tilde{\gamma})$

|  | $\frac{\hat{l}(\tilde{\beta})}{L}$ | $\frac{\hat{m}(\tilde{\gamma})}{H}$ | $\omega$ | $\frac{w_{l}(\beta)}{w_{\perp}^{1}}, \beta \geq \tilde{\beta}$ | $\frac{w_{m}(\gamma)}{w_{h}}, \gamma \geq \tilde{\gamma}$ |
| :--- | :--- | :--- | :--- | :--- | :--- |
| $\phi$ | + | + | 0 | 0 | 0 |
| $a$ | + | + | + | 0 | 0 |
| $b$ | $+, 0,-$ | $+, 0,-$ | + | + | 0 |

Scenario 2: $\hat{l}(\tilde{\beta})=l^{S}(\tilde{\beta})$ and $\hat{m}(\tilde{\gamma})<h^{S}(\tilde{\gamma})$

|  | $\frac{\tilde{m}(\tilde{\gamma})}{H}$ | $\omega$ | $\frac{w_{l}(\beta)}{w_{1}^{1}}, \beta \geq \tilde{\beta}$ | $\frac{w_{m}(\gamma)}{w_{h}}, \gamma \geq \tilde{\gamma}$ |
| :--- | :--- | :--- | :--- | :--- |
| $\phi$ | - | - | + | 0 |
| $a$ | + | - | + | 0 |
| b | 0 | + | $+, 0,-$ | 0 |

Scenario 3: $\hat{l}(\tilde{\beta})=l^{S}(\tilde{\beta})$ and $\hat{m}(\tilde{\gamma})=h^{S}(\tilde{\gamma})$

|  | $\omega$ | $\frac{w_{l}(\beta)}{w_{l}^{1}}, \beta \geq \tilde{\beta}$ | $\frac{w_{m}(\gamma)}{w_{h}}, \gamma \geq \tilde{\gamma}$ |
| :--- | :--- | :--- | :--- |
| $\phi$ | - | + | - |
| $a$ | - | $+, 0,-$ | + |
| $b$ | + | $+, 0,-$ | 0 |

Scenario 4: $\hat{l}(\tilde{\beta})<l^{S}(\tilde{\beta})$ and $\hat{m}(\tilde{\gamma})=h^{S}(\tilde{\gamma})$

|  | $\frac{\hat{i}(\tilde{\beta})}{L}$ | $\omega$ | $\frac{w_{l}(\beta)}{w_{L}^{1}}, \beta \geq \tilde{\beta} \frac{w_{m}(\gamma)}{w_{h}}, \gamma \geq \tilde{\gamma}$ |  |
| :--- | :--- | :--- | :--- | :--- |
| $\phi$ | + | - | 0 | + |
| $a$ | $+, 0,-$ | - | 0 | + |
| $b$ | $+, 0,-$ | + | + | $+, 0,-$ |

of a $H$-worker with ability $\gamma$ to support $L$-workers. ${ }^{21}$ Similarly, the left-hand side of (17) is the relative productivity of a supported $H$-worker,

21 Note that we decomposed $w_{m}(\gamma) / w_{l}^{1}$ into $\left(w_{m}(\gamma) / w_{h}\right) \times \omega$ (recall $\omega=$ $\left.w_{h} / w_{l}^{1}\right)$.
whereas the right-hand side is the respective relative cost. The latter consists of three components: the relative wage of a supported $H$-worker in production (which equals unity since $w_{h}^{1}=w_{h}^{2}=w_{h}$ ), within-group relative wage $w_{m}(\gamma) / w_{h}$ of a non-production worker of type $\gamma$, and the marginal physical cost of such a $H$-worker.

Lemma 1 also indicates that there are potential differences regarding within-group wage inequality between scenarios with $\hat{l}(\tilde{\beta})<l^{S}(\tilde{\beta})$ (i.e., not all $L$-workers with threshold ability $\tilde{\beta}$ are supported) or $\hat{m}(\tilde{\gamma})<h^{S}(\tilde{\gamma})$, and scenarios with $\hat{l}(\tilde{\beta})=l^{S}(\tilde{\beta})$ or $\hat{m}(\tilde{\gamma})=h^{S}(\tilde{\gamma})$, respectively. This plays an important role for the comparative-static analysis presented in the next subsection.

### 4.2. Comparative-statics

According to Lemma 1, there are four possible scenarios for which we can study the marginal impact of changes in relative labor supply $\phi=H / L$ (holding the composition of abilities constant) and technology shifts, reflected by changes in $a$ or $b$ :
(1) $\hat{l}(\tilde{\beta})<l^{S}(\tilde{\beta})$ and $\hat{m}(\tilde{\gamma})<h^{S}(\tilde{\gamma})$,
(2) $\hat{l}(\tilde{\beta})=l^{S}(\tilde{\beta})$ and $\hat{m}(\tilde{\gamma})<h^{S}(\tilde{\gamma})$,
(3) $\hat{l}(\tilde{\beta})=l^{S}(\tilde{\beta})$ and $\hat{m}(\tilde{\gamma})=h^{S}(\tilde{\gamma})$, and
(4) $\hat{l}(\tilde{\beta})<l^{S}(\tilde{\beta})$ and $\hat{m}(\tilde{\gamma})=h^{S}(\tilde{\gamma})$.

Lemma 2: Comparative-static results for scenarios 1-4 are as shown in Table 1.

Proof: See Appendix.
The remainder of this section discusses the intuition of Lemma 2 and derives implications. We start with changes in the relative supply of educated labor, $\phi$.

### 4.2.1 Relative Supply of Educated Labor

According to Table 1 , whenever $\hat{l}(\tilde{\beta})<l^{S}(\tilde{\beta})$ (i.e., scenarios 1 and 4), $\hat{l}(\tilde{\beta}) / L$ is strictly increasing in the relative supply of $H$-labor, $\phi=H / L$. That is, an increase in $\phi$ induces firms to restructure in the sense that a higher share of the $L$-labor force is assigned to non-Tayloristic jobs.

Moreover, for a certain range of relative supply $\phi$, firms may choose not to support workers from a low-ability group, even though all workers from an adjacent group with higher ability $\beta$ is already assigned to nonroutinized jobs. Before discussing the intuition of these results, note that this implies the following corollary.

Corollary 1: For any $k=2, \ldots, K$, there exist $\phi_{1}^{k}, \phi_{2}^{k}, \phi_{3}^{k}$, with $0<\phi_{1}^{k}<$ $\phi_{2}^{k}<\phi_{3}^{k}$ and $\phi_{1}^{k-1}=\phi_{3}^{k}$, such that
(i) $\tilde{\beta}=\beta^{k}$ for all $\phi \in\left(\phi_{1}^{k}, \phi_{3}^{k}\right]$ and $\tilde{\beta}<\beta^{k}$ for all $\phi>\phi_{3}^{k}$,
(ii) $\hat{l}(\tilde{\beta})<l^{S}(\tilde{\beta})$ for all $\phi \in\left(\phi_{1}^{k}, \phi_{2}^{k}\right)$ and $\hat{l}(\tilde{\beta})=l^{S}(\beta)$ for all $\phi \geq \phi_{2}^{k}$.

Thus, according to Lemma 1, Lemma 2 and Corollary 1, our measure of wage inequality within the group of $L$-workers for a particular ability type $\beta^{k}>\tilde{\beta}, w_{l}\left(\beta^{k}\right) / w_{l}^{1}$, evolves with increasing relative supply of $H$-workers, $\phi$, as shown in Fig. 1. The next result is thus immediately implied by the preceding ones.

Proposition 1 (Within-group wage inequality and education): (i) Wage inequality within the group of $L$-workers is a non-decreasing (and continuous) function of $\phi=H / L$, and strictly increasing in $\phi$ over some ranges. (ii) The impact of an increase in $\phi$ on wage inequality within the group of $H$-workers is ambiguous.

Let us start with a discussion of part (i) of Proposition 1. The crucial insight is that an increase in $\phi$ raises the incentive of firms to reallocate $L$-labor towards non-routinized production tasks, i.e., training of $L$-workers becomes more attractive. To see this, suppose this would not be the case. Then, an increase in $\phi$ unambiguously raises educationintensity of production labor, $\kappa=\tilde{h} / \tilde{l}$. Thus, all other things equal, between-group relative wage $\omega$ declines, according to (15). Ceteris paribus, this reduces the relative marginal costs to support $L$-workers (since this is intensive in $H$-labor), which is given by the right-hand side of (16). This gives firms an incentive to support a higher share of $L$-workers and thus raises the demand for $\beta$-abilities. Thus, for given within-group wage inequality (i.e., as long as $\hat{l}(\tilde{\beta}) / L<l^{S}(\tilde{\beta}) / L$, given threshold ability level $\tilde{\beta}), \hat{l}(\tilde{\beta}) / L$ will rise (scenarios 1 and 4 in Table 1). However, as soon as $\hat{l}(\tilde{\beta})=l^{S}(\tilde{\beta})$ is reached (at some $\phi_{2}$ in Fig. 1), a further marginal increase in $\phi$ does not make support of workers with lower ability than,


Fig. 1 Wage dispersion within the group of $L$-workers
say, $\beta^{k}=\tilde{\beta}$ attractive (scenarios 2 and 3 ). ${ }^{22}$ In this case, rising demand for all types $\beta \geq \beta^{k}$ meets "fixed supply" such that $w_{l}(\tilde{\beta}) / w_{l}^{1}$ rises with $\phi$. At some level of wage inequality $w_{l}(\tilde{\beta}) / w_{l}^{1}$, assigning $L$-labor with ability $\beta_{\tilde{\beta}}^{k-1}<\beta^{k}$ to non-routine tasks becomes attractive as $\phi$ increases so that $\tilde{\beta}$ falls to $\beta^{k-1} .{ }^{23}$ (Then we again start from scenario 1 or 4.)

Interestingly, regarding part (ii) of Proposition 1, the impact of an increase in $\phi$ on $\hat{m}(\gamma) / H$ and $w_{m}(\gamma) / w_{h}$, respectively, is less clear. This is due to our assumptions that non-routine tasks require educated labor for support and $H$-individuals differ in the ability to provide such support. To see this, first, note that between-group relative wage $\omega$ does not enter equation (17), which equates the relative benefit and relative costs of assigning $H$-workers to non-routine rather than routine production tasks. According to the previous discussion, an increase in $\phi$ raises the incentive to assign $L$-labor towards non-routinized jobs. This increases the nonproduction labor requirement for provision of support. Thus, if $\hat{l}(\tilde{\beta}) / L$ gradually increases with $\phi$, then there are two possibilities. Either

[^12]23 At this point, $w_{l}\left(\beta^{k}\right) / w_{l}^{1}=b\left(\beta^{k}-\beta^{k-1}\right)+1$, according to Lemma 1 .
$\hat{m}(\tilde{\gamma}) / H$ rises without raising $w_{m}(\gamma) / w_{h}, \gamma \geq \tilde{\gamma}$, which occurs as long as $\hat{m}(\tilde{\gamma}) / H<h^{S}(\tilde{\gamma}) / H$ (i.e., $H$-workers of type $\tilde{\gamma}$ are not scarce yet) or $w_{m}(\gamma) / w_{h}$ rises for all $\gamma \geq \tilde{\gamma}$. (These possibilities refer to scenarios 1 or 4 , respectively.) Moreover, as long as $\hat{l}(\tilde{\beta}) / L$ increases with $\phi$ for given $\tilde{\beta}$, threshold ability level $\tilde{\gamma}$ may fall after an increase in $\phi$ from, say, type $\gamma^{j}$ to $\gamma^{j-1}$. To the contrary, if $\hat{l}(\tilde{\beta})=l^{S}(\tilde{\beta})$, then an increase in $\phi$ does not raise demand for $\gamma$-abilities. On the one hand, if $\hat{m}(\tilde{\gamma}) / H<h^{S}(\tilde{\gamma}) / H$, then an even lower share of non-production workers is needed to support the same fraction $l^{2} / L$ of $L$-labor after a marginal increase in $\phi$, i.e., $\hat{m}(\tilde{\gamma}) / H$ declines (scenario 2). On the other hand, if $\hat{m}(\tilde{\gamma})=h^{S}(\tilde{\gamma})$, the reduction in demand for $\gamma$-abilities after a marginal increase in $\phi$ is reflected by a decline in $w_{m}(\gamma) / w_{h}, \gamma \geq \tilde{\gamma}$ (scenario 3). However, whenever $m / H$ does not decline after a (marginal or non-marginal) increase in $\phi$, then wage inequality within the group of $H$-labor does not decrease and, possibly, increases.

In sum, Proposition 1 is consistent with the empirical findings discussed in Sects. 1 and 2 that both overall within-group wage inequality and the share of workers assigned to non-Tayloristic jobs have risen, that these developments were accompanied by increased training provision within firms and that the observed increase in the relative supply of educated labor seemed to have a significant impact on restructuring of firms towards knowledge-based organizational forms.

At the same time, however, many countries have experienced stagnating or even declining between-group wage inequality. ${ }^{24}$ To address this fact, we now investigate the impact of an increase in $\phi=H / L$ on $\omega=w_{h} / w_{l}^{1}$.
Proposition 2 (Between-group wage inequality and education): Betweengroup wage inequality is a non-increasing function of $\phi=H / L$.

For the intuition of the $\phi$-effect on $\omega=w_{h} / w_{l}^{1}$ we consider two different cases suggested by Table 1. First, if $\hat{l}(\tilde{\beta})<l^{S}(\tilde{\beta})$ and $\hat{m}(\tilde{\gamma}) / H<h^{S}(\tilde{\gamma}) / H$ simultaneously hold (scenario 1), a marginal increase in $\phi$ raises the share of $L-$ labor assigned to non-routinized jobs (as argued above). This stimulates demand for non-production workers, thereby raising $m / H$. Both the implied reallocation of $H$-labor away from production and the increase in efficiency units of $L$-labor reduce

[^13]education-intensity of production labor $\kappa=\tilde{h} / \tilde{l}$. This is a counteracting effect of an increase in $\phi$ on $\kappa .^{25}$ In sum, both $\kappa$ and thus between-group wage inequality $\omega$ remain unaffected. ${ }^{26}$ Second, if one of the two conditions $\hat{l}(\tilde{\beta}) \leq l^{S}(\tilde{\beta})$ and $\hat{m}(\tilde{\gamma}) \leq h^{S}(\tilde{\gamma})$ is binding, a marginal increase in $\phi$ reduces (scenario 2) or does not affect (scenarios 3 and 4) the share of non-production workers in the total supply of $H$-labor. This gives rise to a positive impact of an increase in $\phi$ on $\kappa$ and, therefore, reduces $\omega$, according to (15).

Taken together, Propositions 1 and 2 demonstrate that within-group wage inequality and between-group wage inequality can be adversely related, given the same factor supply shock. ${ }^{27}$ However, an increase in $\phi$ alone cannot explain the Anglo-American experience of rising wage inequality within and between groups in the 1980s and most of the 1990s. To address this fact, we now turn to the impact of technological change.

### 4.2.2 New Information and Communication Technologies (ICT)

Most of the literature on the relationship between wage inequality and technological change has focussed on biased changes in the production technology, in the sense of an (exogenous or endogenous) increase in marginal productivity of educated relative to less educated labor. ${ }^{28}$ In

[^14]contrast, we consider a kind of technological change which raises the productivity gain from allocating workers towards non-routine jobs, represented by shifts in $a$ and $b$ in our model.

An increase in $a$ raises the relative productivity of supported $H$-workers in production $\left(h^{2}\right)$, according to (17), and thus has a positive effect on the demand for non-production activities. This raises either the share of non-production workers in total supply of $H$-labor, $m / H$ (scenarios 1 and 2) or $w_{m}(\tilde{\gamma}) / w_{h}^{1}$ for all $\gamma \geq \tilde{\gamma}$ (scenarios 3 and 4). Thus, the impact of an increase in $a$ on within-group wage inequality of $H$-labor is completely analogous to the impact of an increase in $\phi$ on that of $L$-labor (which is depicted in Fig. 1). Regarding between-group wage inequality, $\omega$, however, the effect of a higher $a$ is less clear-cut (i.e., is positive in scenario 1 but negative otherwise). The reason is that, holding everything else constant, $a$ is positively related to the educationintensity of production labor, $\kappa=\tilde{h} / \tilde{l}$ (which in turn is negatively related to $\omega$ ), according to (3) and (4). ${ }^{29}$

To the contrary, an increase in $b$ raises $\omega$ in all scenarios, but the effects on within-group wage inequality are more ambiguous. First, a higher $b$ has a direct negative effect on $\kappa$ (analogous to the direct positive effect of $a$ on $\kappa$ ), which ultimately gives rise to an increase in between-group wage inequality $\omega$ in all four scenarios. Second, since an increase in $b$ raises the productivity of $L$-workers in non-routine relative to those in routine tasks, demand for $\beta$-abilities of $L$-labor is raised, all other things equal. However, an increase in $\omega$ is associated with an increase in the marginal cost of support activity, according to (16), and therefore reduces the demand for $\beta$-abilities. This counteracts the aforementioned effect, leaving the impact of an increase in $b$ on within-group wage inequality $w_{l}(\beta) / w_{l}^{1}, \beta \geq \tilde{\beta}$, generally ambiguous. The impact of an increase in $b$ on $w_{l}(\beta) / w_{l}^{1}, \quad \beta \geq \tilde{\beta}$, is only unambiguously positive if $\hat{l}(\tilde{\beta})<l^{S}(\tilde{\beta})$ (scenarios 1 and 4 ). We can thus conclude that the following robust relationships hold.

[^15]Proposition 3 (Wage inequality and advances in ICT): (i) Within-group wage inequality of $H$-workers is a non-decreasing (and continuous) function of $a$, and strictly increasing in $a$ over some ranges. (ii) Betweengroup wage inequality is strictly increasing in $b$.

In sum, we may conclude that technological changes which raise $a$ and $b$ simultaneously (and thus raise the incentive of firms to reassign workers to non-Tayloristic tasks) can explain both rising inequality within the group of educated workers and rising between-group wage inequality.

## 5 Concluding Remarks

This paper has argued that both rising relative supply of educated labor and technological change has led to restructuring processes within firms which raised the demand for typically (to empirical researchers) unobservable abilities like analytical and social skills. The contribution of our paper is to propose a theory on equilibrium responses of firms to changing technology and factor supply conditions in allocating labor to routine and non-routine tasks, and to analyze their implications for skill prices within and across education groups. Our model rests on the hypotheses that (i) a reallocation of labor towards non-routine tasks is productivity-enhancing but costly in terms of training provision, (ii) analytical and social ability is more relevant for performing non-routine tasks, and (iii) organizational (e.g., human resource management) activity within firms is skill-intensive. Our results are not only consistent with empirical evidence on a pervasive rise in within-group wage inequality and possibly stagnating or falling wage dispersion between education groups, but also contribute to an understanding of why these developments occurred at the same time as firms reorganized work towards non-Tayloristic jobs throughout the developed world. In fact, the empirical literature on "skill-biased technological change" (e.g., Berman et al., 1994; Bresnahan, 1999; Bresnahan et al., 2002) has always pointed out that understanding changes in the demand for skills requires to take into account restructuring processes within firms. However, surprisingly little theoretical work has been done in this area so far.

An important and still ongoing debate in the context of wage inequality is why the education premium in Continental Europe evolved so differently as opposed to the U.S. and UK. Although relative supply of educated labor has increased considerably during the last few decades in
basically all advanced countries, one center of this debate has been whether differences in skill supply growth rates across countries or institutional differences is the main reason for rather stable education premia in Europe and sharply rising ones (at least in the 1980s and most of the 1990s) in the U.S. (e.g., Gottschalk and Smeeding, 1997). In a well-recognized paper, Blau and Kahn (1996) find no evidence in favor of market forces to reconcile these different wage patterns (under the hypothesis of similar technology-induced changes in the labor demand composition across countries), based on the years of schooling as a measure of skill. However, as pointed out by Acemoglu (2002), the institutional view cannot explain why unemployment rates have risen almost proportionally for educated and less educated workers (Nickell and Bell, 1996, 1997). A recent paper by Leuven et al. (2004) is capable to reconcile this conflicting evidence. By constructing internationally comparable measures of skill, they find that up to " 60 percent of the variation in skill wage differential is explained by relative net supply" (Leuven et al., 2004, p. 482), diametrical to the findings by Blau and Kahn. ${ }^{30}$ Insofar as skill supply shifts can be viewed as exogenous events (e.g., significantly affected by public policy), this is consistent with our analysis. ${ }^{31}$ In addition, our model can explain why, at the same time, both within-group wage inequality and the organizational set up in firms has changed also in Continental Europe.

Our final remark is a tentative policy conclusion. Our model has emphasized the role of unobserved heterogeneity for both observed wage patterns and the incentive of firms to restructure towards organizational forms which require analytical and interactive abilities. In other words, shortage of such skills may be an impediment for firms to enhance productivity and may be the major source of low earnings individually, partly irrespective of formal education. In fact, as argued by Heckman (2000), labor market programmes aiming at raising qualifications of workers

[^16]often turn out to be almost ineffective to boost earning prospects due to the lack of social skills. Our model suggests that this problem may require an increased emphasis on social abilities in high-school education or even earlier in order to reverse inequality trends.

## Appendix

## Proof of Lemma 1

Perfect competition in the labor market implies that $0<\hat{l}(\beta)<l^{S}(\beta)$ $\left(0<\hat{m}(\gamma)<h^{S}(\gamma)\right)$ is only consistent with an equilibrium if $w_{l}(\beta)=w_{l}^{1}$, $\beta \in B\left(w_{m}(\gamma)=w_{h}, \gamma \in \Gamma\right)$. Hence, it is an immediate consequence of profit maximization that $0<\hat{l}(\tilde{\beta}) \leq l^{S}(\tilde{\beta}) \quad\left(0<\hat{m}(\tilde{\gamma}) \leq h^{S}(\tilde{\gamma})\right)$ requires $\hat{l}(\beta)=l^{S}(\beta)$ for all $\beta>\tilde{\beta}\left(\hat{m}(\gamma)=h^{S}(\gamma)\right.$ for all $\left.\gamma>\tilde{\gamma}\right)$ and $\hat{l}(\beta)=0$ for all $\beta<\tilde{\beta}(\hat{m}(\gamma)=0$ for all $\gamma>\tilde{\gamma})$. Moreover, note from (13) that $\lambda=w_{m}(\gamma) / \gamma$ for all $\gamma \geq \tilde{\gamma}$. Thus, (11) and (12) imply for all $\beta \geq \tilde{\beta}$ and $\gamma \geq \tilde{\gamma}$ that

$$
\begin{equation*}
b \beta w_{l}^{1}=w_{l}(\beta)+\frac{w_{m}(\gamma)}{\gamma}\left(g(\chi)-\chi g^{\prime}(\chi)\right) \tag{A.1}
\end{equation*}
$$

whereas (9) and (10) imply

$$
\begin{equation*}
a w_{h}=w_{h}+\frac{w_{m}(\gamma)}{\gamma} g^{\prime}(\chi) \tag{A.2}
\end{equation*}
$$

for all $\gamma \geq \tilde{\gamma}$. Equations (16) and (17) follow from (A.1), (A.2) and definition $\omega=w_{h} / w_{l}^{1}$. Moreover, using the facts that $w_{l}(\tilde{\beta})=w_{l}^{1}$ if $\hat{l}(\tilde{\beta})<l^{S}(\tilde{\beta})$ and $w_{m}(\tilde{\gamma})=w_{h}$ if $\hat{m}(\tilde{\gamma})_{\tilde{\beta}}<h^{S}(\tilde{\gamma})$, (A.1) and (A.2) confirm $w_{l}(\beta) / w_{l}^{1}=b(\beta-\tilde{\beta})+1$ for all $\beta \geq \tilde{\beta}$ and $w_{m}(\gamma) / w_{h}=\gamma / \tilde{\gamma}$ for all $\gamma \geq \tilde{\gamma}$, respectively. However, note that $w_{l}(\tilde{\beta})>w_{l}^{1}$ and $w_{m}(\tilde{\gamma})>w_{h}$ is possible if $\hat{l}(\tilde{\beta})=l^{S}(\tilde{\beta})$ and $\hat{m}(\tilde{\gamma})=h^{S}(\tilde{\gamma})$, respectively, since threshold ability types are a scarce resource in this case.

## Proof of Lemma 2

First, note that using $\tilde{h}^{2}=a h^{2}$, (3), (14) and $\chi=h^{2} / l^{2}, \kappa=\tilde{h}^{2} / \tilde{l}^{2}$ can be written as

$$
\begin{equation*}
\kappa=\frac{h^{1}+\tilde{h}^{2}}{l^{1}+\tilde{l}^{2}}=\frac{\phi\left(1-\frac{m}{H}\right)+(a-1) \chi \frac{l^{2}}{L}}{1-\frac{l^{2}}{L}+\frac{\tilde{l}^{2}}{L}} \tag{A.3}
\end{equation*}
$$

(recall $\phi=H / L)$. Also note that Lemma 1 implies

$$
\begin{align*}
l^{2} & =\sum_{\beta>\tilde{\beta}} l^{S}(\beta)+\hat{l}(\tilde{\beta}),  \tag{A.4}\\
\tilde{l}^{2} & =b\left(\sum_{\beta>\tilde{\beta}} \beta l^{S}(\beta)+\tilde{\beta} \hat{l}(\tilde{\beta})\right),  \tag{A.5}\\
m & =\sum_{\gamma>\tilde{\gamma}} h^{S}(\gamma)+\hat{m}(\tilde{\gamma})  \tag{A.6}\\
\tilde{m} & =\sum_{\gamma>\tilde{\gamma}} \gamma m^{S}(\gamma)+\tilde{\gamma} \hat{m}(\tilde{\gamma}), \tag{A.7}
\end{align*}
$$

according to (2), (4), (6), (7). We now explore comparative-static effects for scenarios $1-4$ separately.

Ad scenario 1: Recall from Lemma 1 that $\hat{l}(\tilde{\beta})<l^{S}(\tilde{\beta})$ and $\hat{m}(\tilde{\gamma})$ $<h^{S}(\tilde{\gamma})$ imply $w_{l}(\beta) / w_{l}^{1}=b(\beta-\tilde{\beta})+1$ for all $\beta \geq \tilde{\beta}$ and $w_{m}(\gamma) / w_{h}=$ $\gamma / \tilde{\gamma}$ for all $\gamma \geq \tilde{\gamma}$, respectively, thus confirming our results for within-group inequality. Moreover, recall $w_{l}(\tilde{\beta})=w_{l}^{1}$ and $w_{m}(\tilde{\gamma})=w_{h}$. Thus, (A.1) and (A.2) imply that $\left(g(\chi)-\chi g^{\prime}(\chi)\right) \omega=(b \tilde{\beta}-1) \tilde{\gamma}$ and

$$
\begin{equation*}
g^{\prime}(\chi)=(a-1) \tilde{\gamma} \Longleftrightarrow \chi=\left(g^{\prime}\right)^{-1}((a-1) \tilde{\gamma}) \equiv \tilde{\chi}(a), \tag{A.8}
\end{equation*}
$$

respectively, where $\tilde{\chi}(a)$ is increasing in $a$. Thus,

$$
\begin{equation*}
\omega=\frac{(b \tilde{\beta}-1) \tilde{\gamma}}{\left[g(\chi)-\chi g^{\prime}(\chi)\right]_{\chi=\tilde{\chi}(a)}} \equiv \tilde{\omega}(\underset{(+)}{a, b}(+), \tag{A.9}
\end{equation*}
$$

where $\partial \tilde{\omega} / \partial a>0,{ }^{32} \partial \tilde{\omega} / \partial b>0$ and $\partial \tilde{\omega} / \partial \phi=0$. Using $\omega=\Omega(\kappa)$, according to (15), we find that $\kappa=\Omega^{-1}(\tilde{\omega}(a, b)) \equiv \tilde{\kappa}(a, b)$, where $\partial \tilde{\kappa} / \partial a<0$ and $\partial \tilde{\kappa} / \partial b<0$, since $\Omega^{\prime}(\kappa)<0$, and $\partial \tilde{\kappa} / \partial \phi=0$. Also note that combining $l^{2}=\tilde{m} / g(\chi)$ from (5) with (A.4) and (A.7), and rearranging terms, yields

[^17]\[

$$
\begin{equation*}
\frac{\hat{l}(\tilde{\beta})}{L}=\frac{\phi\left(\sum_{\gamma>\tilde{\gamma}} \gamma \frac{h^{S}(\gamma)}{H}+\tilde{\gamma} \frac{\hat{m}(\tilde{\gamma})}{H}\right)}{g(\chi)}-\sum_{\beta>\bar{\beta}} \frac{l^{S}(\beta)}{L} \tag{A.10}
\end{equation*}
$$

\]

Substituting (A.4)-(A.6) into (A.3) and then using (A.8) and (A.10) yields

$$
\begin{equation*}
\frac{1-[1-\eta(\tilde{\chi}(a))] \frac{\hat{m}(\tilde{\gamma})}{H}+\sum_{\gamma>\tilde{\gamma}}\left(\eta(\tilde{\chi}(a)) \frac{\gamma}{\tilde{\gamma}}-1\right) \frac{h^{S}(\gamma)}{H}}{\frac{1}{\phi}+\frac{b \tilde{\beta}-1}{g(\tilde{\chi}(a))}\left(\sum_{\gamma>\tilde{\gamma}} \gamma \frac{h^{S}(\gamma)}{H}+\tilde{\gamma} \frac{\hat{m}(\tilde{\gamma})}{H}\right)+\frac{b}{\phi} \sum_{\beta>\tilde{\beta}}(\beta-\tilde{\beta}) \frac{l^{S}(\beta)}{L}}-\tilde{\kappa}(a, b)=0 \tag{A.11}
\end{equation*}
$$

where $\eta(\chi) \equiv \chi g^{\prime}(\chi) / g(\chi), \chi=\tilde{\chi}(a)$ and $\kappa=\tilde{\kappa}(a, b)$ have been used. (A.11) defines $\hat{m}(\tilde{\gamma}) / H$ implicitly as function of $(a, b, \phi)$. Comparativestatic results regarding $\hat{m}(\tilde{\gamma}) / H$ follow from applying the implicit function theorem to (A.11) and observing the properties of $\tilde{\chi}(a)$ and $\tilde{\kappa}(a, b) .{ }^{33}$

For the results regarding $\hat{l}(\tilde{\beta}) / L$, note that combining (5) with (A.7) yields

$$
\begin{equation*}
\frac{\hat{m}(\tilde{\gamma})}{H}=\frac{g(\tilde{\chi}(a))}{\tilde{\gamma} \phi} \frac{l^{2}}{L}-\sum_{\gamma>\tilde{\gamma}} \frac{\gamma}{\tilde{\gamma}} \frac{h^{S}(\gamma)}{H} \tag{A.12}
\end{equation*}
$$

Next, substitute (A.5), (A.6) and (A.8) into (A.3) and use both $\chi=\tilde{\chi}(a)$ and (A.12) to obtain

$$
\begin{equation*}
\kappa=\frac{\phi\left(1+\sum_{\gamma>\tilde{\gamma}}\left(\frac{\gamma}{\tilde{\gamma}}-1\right) \frac{h^{S}(\gamma)}{H}\right)-\frac{1}{\tilde{\gamma}}\left[g(\chi)-\chi g^{\prime}(\chi)\right]_{\chi=\tilde{\chi}(a) \frac{l^{2}}{L}}}{1-\frac{l^{2}}{L}+b \sum_{\beta>\tilde{\beta}} \beta \frac{l^{S}(\beta)}{L}+b \tilde{\beta} \frac{l^{S}(\tilde{\beta})}{L}} . \tag{A.13}
\end{equation*}
$$

Finally, substitute (A.4) into (A.13) and use $\kappa=\tilde{\kappa}(a, b)$ which leads to

$$
\begin{equation*}
\frac{\phi\left[1+\sum_{\gamma>\tilde{\gamma}} \frac{\gamma-\tilde{\gamma} \hat{\gamma}}{\tilde{m}(\tilde{\gamma})} \frac{\hat{m}}{H}\right] \frac{\hat{m}(\tilde{\gamma})}{H}-\frac{1}{\tilde{\gamma}} \frac{l^{2}}{L}\left[g(\chi)-\chi g^{\prime}(\chi)\right]_{\chi=\tilde{\chi}(a)}}{1+\sum_{\beta>\tilde{\beta}}(b \beta-1) \frac{l^{S}(\beta)}{L}+(b \tilde{\beta}-1) \frac{\hat{l}(\tilde{\beta})}{L}}-\tilde{\kappa}(a, b)=0 . \tag{A.14}
\end{equation*}
$$

[^18](A.14) defines $\hat{l}(\tilde{\beta}) / L$ implicitly as function of $(a, b, \phi)$. Comparativestatic results regarding $\hat{l}(\widetilde{\beta}) / L$ follow from applying the implicit function theorem to (A.14) and, again, observing the properties of $\tilde{\chi}(a)$ and $\tilde{\kappa}(a, b)$.

Ad scenario 2: First, note that $\hat{m}(\tilde{\gamma})<h^{S}(\tilde{\gamma})$ implies that $w_{m}(\gamma) / w_{h}=\gamma / \tilde{\gamma}$ for all $\gamma \geq \tilde{\gamma}$, thus confirming our results regarding inequality within the group of $H$-workers. Moreover, (A.8) still holds. Also note that $\hat{l}(\tilde{\beta})=l^{S}(\tilde{\beta})$ implies that $l^{2} / L$ is constant, according to (A.4). Thus, using (A.12) confirms the results regarding $\hat{m}(\tilde{\gamma}) / H$. Next, use $\hat{l}(\tilde{\beta})=l^{S}(\tilde{\beta})$ in (A.13) and recall $\tilde{\chi}^{\prime}(a)>0$ to confirm that $\kappa$ is strictly increasing in both $a$ and $\phi$, and strictly decreasing in $b$. Using $\omega=\Omega(\kappa)$ with $\Omega^{\prime}(\kappa)<0$, according to (15), confirms the results regarding $\omega$. Finally, use these results and substitute $\chi=\tilde{\chi}(a)$ from (A.8) into (16) to confirm the results regarding $w_{l}(\beta) / w_{l}^{1}, \beta>\tilde{\beta}$.

Ad scenario 3: First, note that $\hat{l}(\tilde{\beta})=l^{S}(\tilde{\beta})$ implies that $l^{2} / L$ is constant and $\tilde{l}^{2} / L=b \sum_{\beta \geq \tilde{\beta}} \beta l^{S}(\beta) / L$, according to (A.4) and (A.5), respectively. Similarly, $\hat{m}(\tilde{\gamma})=h^{S}(\tilde{\gamma})$ implies that $m / H$ is constant and $\tilde{m} / H=\sum_{\gamma \geq \tilde{\gamma}} \gamma m^{S}(\gamma)$, according to (A.6) and (A.7), respectively. Thus, $\tilde{m}=l^{2} g(\chi)$ implies that

$$
\begin{equation*}
\chi=g^{-1}\left(\frac{\phi \sum_{\gamma>\tilde{\gamma}} \gamma m^{S}(\gamma) / H}{l^{2} / L}\right) \equiv \underset{(+)}{\tilde{\chi}}(\phi), \tag{A.15}
\end{equation*}
$$

where $\tilde{\tilde{\chi}}(\phi)$ is increasing in $\phi$. Substituting both $\tilde{l}^{2} / L=b \sum_{\beta \geq \beta} \beta l^{S}(\beta) / L$ and (A.15) into (A.3) leads to

$$
\begin{equation*}
\left.\kappa=\frac{\phi\left(1-\frac{m}{H}\right)+(a-1) \tilde{\tilde{\chi}}(\phi) \frac{l^{2}}{L}}{1-\frac{l^{2}}{L}+b \sum_{\beta \geq \tilde{\beta}} \beta \frac{\beta^{S}(\beta)}{L}} \equiv \underset{\tilde{\kappa}}{(++)} \underset{(-)}{a}, \underset{(+)}{b}, \underset{(+)}{\phi}\right) . \tag{A.16}
\end{equation*}
$$

Thus, $\tilde{\tilde{\kappa}}(a, b, \phi)$ is increasing in $a$ and $\phi$, and decreasing in $b$. Noting that $\omega=\Omega(\tilde{\tilde{\kappa}}(a, b, \phi)) \equiv \tilde{\tilde{\omega}}(a, b, \phi)$ from (15) confirms the results regarding $\omega$. Moreover, substituting $\chi=\tilde{\tilde{\chi}}(\phi)$ and $\omega=\tilde{\tilde{\omega}}(a, b, \phi)$ into (16) and observing functional properties confirms our results for $w_{l}(\beta) / w_{l}^{1}, \beta \geq \tilde{\beta}$. Similarly, substituting $\chi=\tilde{\tilde{\chi}}(\phi)$ into (17) confirms the results regarding $w_{m}(\gamma) / w_{h}, \gamma \geq \tilde{\gamma}$.

Ad scenario 4: First, note that $\hat{l}(\tilde{\beta})<l^{S}(\tilde{\beta})$ implies $w_{l}(\beta) / w_{l}^{1}=$ $b(\beta-\tilde{\beta})+1$ for all $\beta \geq \tilde{\beta}$, thus confirming our results regarding inequality within the group of $L$-workers. Substituting (A.4) and (A.7) into $\tilde{m}=l^{2} g(\chi)$ from (5) and observing $\hat{m}(\tilde{\gamma})=h^{S}(\tilde{\gamma})$ implies that

$$
\chi=g^{-1}\left(\frac{\phi \sum_{\gamma \geq \tilde{\gamma}} \gamma \frac{h^{S}(\gamma)}{H}}{\sum_{\beta>\tilde{\beta}} \frac{l^{S}(\beta)}{L}+\frac{\hat{l}(\tilde{\beta})}{L}}\right) \equiv X\left(\begin{array}{cc}
\hat{l}(\tilde{\beta}) / L,  \tag{A.17}\\
(-) & (+)
\end{array}\right),
$$

where $X(\hat{l}(\tilde{\beta}) / L, \phi)$ is decreasing in $\hat{l}(\tilde{\beta}) / L$, and increasing in $\phi$. Next, recall that $\hat{l}(\tilde{\beta})<l^{S}(\tilde{\beta})$ implies $w_{l}(\tilde{\beta})=w_{l}^{1}$, i.e.,

$$
\begin{equation*}
\omega=\frac{b \tilde{\beta}-1}{a-1} \frac{g^{\prime}(\chi)}{g(\chi)-\chi g^{\prime}(\chi)}, \tag{A.18}
\end{equation*}
$$

according to (A.1). Combining (A.18) with $\omega=\Omega(\kappa)$ from (15) yields the relationship

$$
\begin{equation*}
\kappa=\Omega^{-1}\left(\frac{b \tilde{\beta}-1}{a-1} \frac{g^{\prime}(\chi)}{g(\chi)-\chi g^{\prime}(\chi)}\right) \equiv K(\underset{(-)}{\chi}, \underset{(+)}{a}, \underset{(-)}{b}), \tag{A.19}
\end{equation*}
$$

where $K(\chi, a, b)$ is decreasing in both $\chi$ and $b$, and increasing in $a$. Now, substituting $l^{2}=\left(\sum_{\gamma \geq \tilde{\gamma}} \gamma m^{S}(\gamma)\right) / g(\chi)$ into the numerator of (A.3) as well as substituting both (A.4) and (A.5) into the denominator of (A.3) yields

$$
\begin{equation*}
\kappa=\frac{\phi\left[1-\frac{m}{H}+\frac{(a-1) \chi}{g(\chi)} \sum_{\gamma \geq \tilde{\gamma}} \gamma \frac{h^{S}(\gamma)}{H}\right]}{1+\sum_{\beta>\tilde{\beta}}(b \beta-1) \frac{l^{S}(\beta)}{L}+(b \tilde{\beta}-1) \frac{\hat{l}(\tilde{\beta})}{L}} . \tag{A.20}
\end{equation*}
$$

Observing (A.17) and (A.19) then leads to

$$
\begin{align*}
0= & K\left(X\left(\frac{\hat{l}(\tilde{\beta})}{L}, \phi\right), a, b\right)\left[1+\sum_{\beta>\tilde{\beta}}(b \beta-1) \frac{l^{S}(\beta)}{L}+(b \tilde{\beta}-1) \frac{\hat{l}(\tilde{\beta})}{L}\right] \\
& -\phi\left[1-\frac{m}{H}+\frac{(a-1) X\left(\frac{\hat{l}(\tilde{\beta})}{L}, \phi\right)}{g\left(X\left(\frac{\hat{l}(\tilde{\beta})}{L}, \phi\right)\right)} \sum_{\gamma \geq \tilde{\gamma}} \gamma \frac{h^{S}(\gamma)}{H}\right] . \tag{A.21}
\end{align*}
$$

Note that $\hat{m}(\tilde{\gamma})=h^{S}(\tilde{\gamma})$ implies that $m / H$ is constant, according to (A.6). Thus, (A.21) gives us $\hat{l}(\tilde{\beta}) / L$ implicitly as function of $(a, b, \phi)$. Hence, observing the properties of functions $X(\hat{l}(\tilde{\beta}) / L, \phi)$ and $K(\chi, a, b)$ confirms the results regarding $\hat{l}(\tilde{\beta}) / L$. ${ }^{34}$

We now turn to wage inequality. First, note that combining $\hat{m}(\tilde{\gamma})=h^{S}(\tilde{\gamma})$ and (A.10) implies $\hat{l}(\tilde{\beta}) / L=\phi\left(\sum_{\gamma \geq \tilde{\gamma}} \gamma m^{S}(\gamma) / H\right) / g(\chi)-$ $\sum_{\beta>\tilde{\beta}} l^{S}(\beta) / L$. Substituting this expression into (A.20), and using $\kappa=K(\chi, a, b)$ from (A.19), leads to

$$
\begin{equation*}
\frac{1-\frac{m}{H}+\frac{(a-1) \chi}{g(\chi)} \sum_{\gamma \geq \tilde{\gamma}} \gamma \frac{h^{S}(\gamma)}{H}}{\frac{1}{\phi}+\frac{b \tilde{\beta}-1}{g(\chi)} \sum_{\gamma \geq \tilde{\gamma}} \gamma \frac{h^{S}(\gamma)}{H}+\frac{b}{\phi} \sum_{\beta>\tilde{\beta}}(\beta-\tilde{\beta}) \frac{l^{S}(\beta)}{L}}-K(\chi, a, b)=0 . \tag{A.22}
\end{equation*}
$$

Thus, (A.22) gives us $\chi$ implicitly as function of $(a, b, \phi)$. Hence, observing $\partial K(\chi, a, b) / \partial \chi<0$ reveals that $\chi$ is a decreasing function of $\phi \cdot{ }^{35}$ (Moreover, it is easy to check that changes in $a$ or $b$ affect $\chi$ in an ambiguous way.) According to (A.18), this implies that $\omega$ decreases with $\phi$, while within-group wage inequality $w_{m}(\gamma) / w_{h}$ for all $\gamma \geq \tilde{\gamma}$ increases in $\phi$, according to (17).

Next, we confirm that $\omega$ decreases with $a$. First, suppose $\chi$ is nonincreasing in $a$. In this case, $\kappa=K(\chi, a, b)$ is increasing in $a$ because of $\partial K(\chi, a, b) / \partial \chi<0$ and $\partial K(\chi, a, b) / \partial a>0$. Thus, since (15) implies $\omega=\Omega(\kappa)$ with $\Omega^{\prime}(\kappa)<0, \omega$ is decreasing in $a$ if $\chi$ is non-increasing in $a$. Now suppose to the contrary that $\chi$ is increasing in $a$. In this case, (A.22) imposes $\partial \kappa / \partial a>0$ and thus, $\partial \omega / \partial a<0$, according to (15). In sum, we have shown that whatever the sign of $\partial \chi / \partial a$ is, $\partial \omega / \partial a<0$.

In a similar fashion, we can show that $\partial \omega / \partial b>0$. First, suppose $\chi$ is non-decreasing in $b$. In this case, $\kappa=K(\chi, a, b)$ is decreasing in $b$ because of $\partial K(\chi, a, b) / \partial \chi<0$ and $\partial K(\chi, a, b) / \partial b<0$. Thus, since $\omega=\Omega(\kappa)$ with $\Omega^{\prime}(\kappa)<0, \omega$ is increasing in $b$ if $\chi$ is non-decreasing in $b$. Now suppose to the contrary that $\chi$ is decreasing in $b$. In this case, (A.22) imposes $\partial \kappa / \partial b<0$ and thus, $\partial \omega / \partial b>0$, according to (15). In sum, we have shown that whatever the sign of $\partial \chi / \partial b$ is, $\partial \omega / \partial b>0$.

34 Note that $\chi / g(\chi)$ is strictly increasing in $\chi$ since $g(\chi)-\chi g^{\prime}(\chi)>0$.
35 Note that this is no contradiction to (A.17) since $\hat{l}(\tilde{\beta}) / L$ increases with $\phi$ in scenario 4.

Finally, we show that $w_{m}(\gamma) / w_{h}$ is increasing in $a$ for all $\gamma \geq \tilde{\gamma}$. To see this, first, solve (A.18) for $(a-1) / g^{\prime}(\chi)$ and substitute the resulting expression into (17) to obtain

$$
\begin{equation*}
\frac{w_{m}(\gamma)}{w_{h}}=\frac{\gamma(b \tilde{\beta}-1)}{\omega\left[g(\chi)-\chi g^{\prime}(\chi)\right]}, \gamma \geq \tilde{\gamma} \tag{A.23}
\end{equation*}
$$

Suppose that $\chi$ is increasing in $a$. Then $\partial \omega / \partial a<0$ and $g^{\prime \prime}(\chi)>0$ unambiguously imply that $w_{m}(\gamma) / w_{h}$ is increasing in $a$ for all $\gamma \geq \tilde{\gamma}$, according to (A.23). Now suppose to the contrary that $\chi$ is non-increasing in $a$. According to (17), also in this case $w_{m}(\gamma) / w_{h}$ is increasing in $a$ for all $\gamma \geq \tilde{\gamma}$. (The impact of an increase in $b$ on inequality within the group of $H$-workers, however, is ambiguous, since its impact on $\chi$ is ambiguous.) This concludes the proof.

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[^0]:    1 For a comprehensive survey of the literature on the evolution of the distribution of wages and explanations related to technology changes, see Acemoglu (2002). Other explanations focus on trade-related factors (e.g., Das, 2001; 2003) and institutional factors (e.g., Fortin and Lemieux, 1997).

[^1]:    2 In Germany, wage dispersion within education groups has considerably increased among medium-educated and high-educated workers, whereas remaining fairly stable among the low-skilled (Fitzenberger, 1999; Fitzenberger et al., 2001).

[^2]:    3 We significantly extend our earlier framework (Egger and Grossmann, 2005), which does not allow for heterogeneity of workers within education groups.

[^3]:    4 As a consequence, the employment share of workers in routine tasks like administrative work and mere machine operating has dramatically declined over the last decades in favor of managing and professional tasks (e.g., Berman et al., 1994; Bresnahan, 1999; Falkinger and Grossmann, 2003).

[^4]:    5 It is interesting to note that the required nature of training seem to have changed along with this kind of restructuring. For instance, Barron et al. (1999) find for a random sample of 3600 US businesses from the Comprehensive Business Database in 1992 that the average time a worker is in "informal management training" is threefold the time she is in "formal training", and that off-site training programs are by far less important than on-site training in the firm.

[^5]:    6 For instance, note that formal education levels are highly related to public education policy (either by public scholling provision or by financial aid for university attendants), which, for simplicity, is treated as exogenous in our model. The human capital literature has identified various factors which are relevant for education decisions of individuals, e.g., credit constraints (Galor and Zeira, 1993), uncertainty (Levhari and Weiss, 1974; Gould et al., 2001) and social networks (Bénabou, 1996). Treating such factors as exogenous in an analysis of wage inequality, i.e., and abstracting from the educational attainment decision, follows, e.g., Acemoglu (1998; 1999) and Thesmar and Thoenig (2000), which focus on wage inequality across education groups.
    7 The few studies which control for measures of cognitive skills like scores from IQ and other mental ability tests argue that - because of the high correlation between cognitive skills and education levels - it is very difficult to separate the earning effects of cognitive ability from those of schooling (e.g., Cawley et al. 2001). This evidence suggests that differences in cognitive skills are of limited value to explain wage inequality within education groups (e.g., Heckman, 2000; Bowles et al., 2001). Consistent with this hypothesis, recent evidence from the U.S. Bureau of the Census (1998) suggests that personal attributes like "attitude" and "communication skills" are much more important for the hiring decisions of employers than "years of schooling" or "academic performance".

[^6]:    8 For instance, to solve some problem, a worker needs to gather information from coworkers, analyze the siutation in a reasonable amount of time, and make the right decision. The information received from non-production workers supports them in taking the right steps, but how a production worker uses this information depends on his/her personal traits.

    9 Alternatively, one may assume that unobservable abilities relevant for support tasks and production activities, respectively, are strongly positively correlated (as plausible), and that $H$-workers with a high level of such skills are more valuable for firms in support tasks. Such a modification would not yield much additional insight but implies significant cost, regarding the expositional simplicity of the paper.

[^7]:    10 The support activity may be time-consuming for employees. Implicitly, we assume that workers receive wages during that time, i.e., firms bear the entire cost of providing informal training to workers. Empirical evidence by Barron et al. (1999) indeed strongly supports this assumption.

    11 Thus, (5) may be viewed as joint production technology (e.g., Nadiri, 1987) with two outputs ( $h_{i}^{2}$ and $l_{i}^{2}$ ) and one input ( $\tilde{m}_{i}$ ), which has a strictly decreasing and strictly concave transformation curve.

[^8]:    12 As a consequence, "hold-up" problems, which are sometimes associated with firm-specific training, are not an issue in this context. For instance, as Batt (1999) points out, under new organizational forms, "...'learning' ... is a continuous process of using new ideas and information as sources of innovation" (p. 541f.). "[V]irtually all training and work related information (work procedures, system capabilities, product information, legal regulations) are on-line; employees receive eight to ten e-mail messages per day advising them of any updates in any of their systems" (p. 558).

[^9]:    13 For instance, in the U.S., the share of workers with less than high-school declined from 29 percent in 1970 to 19 percent in 1980 and 15 percent in 1990, whereas the share of workers with some college increased from 32 to 42 and 46 percent in these years, respectively (Topel, 1997, table 1). Similar patterns have been observed in Continental Europe (e.g., see Edin and Holmlund, 1995, for Sweden and Abraham and Houseman, 1995, for Germany).
    14 Insofar as analytical and social skills are related to non-cognitive abilities, this is in line with the empirical observation that education levels are highly correlated with cognitive but not with non-cognitive abilities (e.g., Cawley et al., 2001).

[^10]:    15 Implicitly, we assume that firms can perfectly screen workers with respect to abilities $\beta$ and $\gamma$, respectively, e.g., through job interviews and assessment centers.

    16 Note that (9) and (11) are equalities, according to the Inada conditions regarding $f$. Moreover, (12) is binding at least for one $\beta \in B$ under $l_{i}^{2}>0$.

[^11]:    20 Suppose that sufficient shares of workers in the economy are in Tayloristic and modern jobs within each education group, respectively, and the $\beta$-, $\gamma$-abilities are sufficiently dispersed. Then, these inequality measures correspond for some particular ability type to the $90-10$ wage differential within an education group. This measure is often used in empirical studies (e.g., Katz and Autor, 1999).

[^12]:    22 In terms of Corollary 1 and Fig. 1, respectively, given that $\tilde{\beta}=\beta^{k}$, scenarios 1 or 4 apply if $\phi$ rises in the interval $\left(\phi_{1}^{k}, \phi_{2}^{k}\right)$, whereas scenarios 2 or 3 apply if $\phi$ rises in the interval $\left[\phi_{2}^{k}, \phi_{3}^{k}\right)$.

[^13]:    24 In particular, this applies to Continental European countries, but also the U.S. in the 1970s.

[^14]:    25 Note that such a counteracting effect is absent in a conventional model which does not distinguish between production-related tasks on the one hand and non-production tasks on the other hand.

    26 The effects regarding $\omega$ and the allocation of labor in scenario 1 are similar to those discussed in Egger and Grossmann (2005), in which, however, we did not allow for within-group heterogeneity. Thus, scenarios 2-4 could not occur in this model.

    27 Typically, the (still rare) literature which allows for both between-group and within-group wage inequality does not acount for this possibility. A notable exception is the paper by Gould et. al. (2001), in which an increasing variance of the rate of technological progress across sectors raises residual wage inequality but, at the same time, depresses the education premium.

    28 In our model, this could be reflected by an increase in $F_{1} / F_{2}$ (for any given education-intensity $\kappa$ ). It has been established in a series of papers (focussing on different kinds of models and questions) that, somewhat surprisingly, such skillbiased technology change has an ambiguous effects on between-group wage inequality if one allows for some skill-intensive, productivity-enhancing technology like (5). Moreover, in such a model, biased technological change of this sort counterfactually leads to a decline in the non-production employment share. See also Grossmann (2002), Falkinger and Grossmann (2003), and Egger and Grossmann (2005) for these kinds of reasoning.

[^15]:    29 Only in scenario 1 in which a marginal increase in $a$ induces a reallocation of workers towards non-routine jobs in both education groups, $\omega=w_{h} / w_{l}^{1}$ rises. This is because, as is intuitive, an increase in $a$ raises the education-intensity of supported labor, $\chi=h^{2} / l^{2}$ (formally shown in the proof of Lemma 2). In turn, this lowers the marginal costs to support $L$-workers since $g^{\prime \prime}(\cdot)>0$. Hence, relative demand for non-production workers rises. Since $w_{m}(\tilde{\gamma})=w_{h}$ in scenario 1 , according to Lemma $1, \omega$ rises despite the fact that $a$ is positively related to $\kappa$ for a given allocation of labor.

[^16]:    30 Leuven et al. (2004) are capable of reproducing the findings of Blau and Kahn (1996) within their data set by employing years of schooling as skill measure, suggesting that results are very sensitive to the skill measure used.
    31 This is not to deny that also institutional factors (e.g., like strong unions or minimum wages) have contributed to stagnating education premia in Continental Europe. What it means, as pointed by Leuven et al. (2004, p. 484), is that the "relative contribution of institutions and market forces are, however, still unknown". Our model has abstracted from labor market institutions to focus on market forces.

[^17]:    32 Use the fact that $\left[g(\chi)-\chi g^{\prime}(\chi)\right]$ is strictly decreasing in $\chi$ together with the properties of $\tilde{\chi}(a)$.

[^18]:    33 Recall that $l^{S}(\beta) / L, \beta>\tilde{\beta}$, and $m^{S}(\gamma) / H, \gamma>\tilde{\gamma}$, are not affected by $\phi$ due to the assumption of constant within-group compositions of abilities $\beta$ and $\gamma$, respectively. Moreover, note that $\eta^{\prime}(\chi)>0$ since $g(\chi)-\chi g^{\prime}(\chi)>0$ and $g^{\prime \prime}(\chi)>0$.

