



# Entrepreneurial innovation and economic growth

Volker Grossmann \*

Department of Economics, University of Fribourg, Bd. de Pérolles 90, G 424, CH-1700 Fribourg, Switzerland  
 CESifo, Munich, Germany  
 Institute for the Study of Labor (IZA), Bonn, Germany

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## ABSTRACT

A fast growing empirical literature identifies an important role of entrepreneurs for productivity growth. This paper develops a simple overlapping-generations framework with endogenous occupational choice and productivity-enhancing entrepreneurial innovation. It shows that introducing these basic features into R&D-based growth theory has important implications. First, an equilibrium with price-taking firms can be supported despite a constant returns to scale production technology, once entrepreneurial human capital is accounted for. Second, in the proposed model, a larger size of the workforce capable to conduct R&D neither affects the long-run rate of economic growth (“strong scale effect”) nor per capita income or welfare (“weak scale effect”). Economic growth is sustained in the long-run and may be policy-dependent.

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## 1. Introduction

A recent empirical literature identifies an important role of entrepreneurs for productivity growth. For instance, [Baumol et al. \(2007\)](#) argue that the driving force behind the advanced countries' IT revolution and the associated productivity growth surge in the last 15 years is due to the development and growth of innovative entrepreneurial companies, like Microsoft, Intel, eBay, Amazon, Google, or Federal Express. In another recent paper, [Van Praag and Versloot \(2008\)](#) provide a meta-study of 57 recent high-quality studies on the contribution of entrepreneurs (young firms with less than 100 employees) for macroeconomic performance. They conclude that entrepreneurial firms “engender relatively much [...] productivity growth and produce and commercialize high quality innovations” (p. 91).

This paper argues that accounting for occupational choice and entrepreneurial innovation alters important implications of existing R&D-based growth theory. First, even when entrepreneurial innovators produce according to a constant-returns to scale technology (after entry costs and sunk costs for R&D investment are incurred), an equilibrium with price-taking can be supported. In contrast, as pointed out by [Romer \(1990\)](#), this is not possible in standard models with linearly homogenous, aggregate production functions. The fundamental difference to that kind of models is that, despite constant returns in the production technology, price-taking entrepreneurs may have an incentive to finance R&D and entry costs. The basic argument rests on the notion that entrepreneurial skill is crucial for organization and management of a firm and therefore

\* Address: Department of Economics, University of Fribourg, Bd. de Pérolles 90, G 424, CH-1700 Fribourg, Switzerland. Tel.: +41 26 300 93 83; fax: +41 26 300 96 78.

E-mail address: [volker.grossmann@unifr.ch](mailto:volker.grossmann@unifr.ch)

imperfectly substitutable to other kinds of labor. However, it is not a rented factor. Thus, the proposed framework allows for positive (rather than zero) operating profits. At the same time, it is consistent with the well-known replication argument (suggesting constant returns of rival inputs) and with non-rivalry of innovations in the manufacturing process. Second, the proposed set up with price-taking entrepreneurial firms which invest in productivity-enhancing R&D allows one to remove the typical scale effect properties of endogenous growth theory; namely, that a larger population size (of those capable to conduct R&D) is either positively associated with economic growth (“strong scale effect”) or with a higher level of per capita income (“weak scale effect”). Nevertheless, economic growth is sustained in the long run even in absence of population growth and may be policy-dependent.

In standard models of endogenous technical change, aggregate output depends on some composite commodity index of imperfectly substitutable intermediate products (Dixit and Stiglitz, 1977; Ethier, 1982). As well-known, the Dixit–Stiglitz–Ethier formulation implies that, by increasing the number of intermediate goods sectors while holding constant the amount of foregone consumption necessary for production in these sectors, total factor productivity rises. This property is interpreted as representing specialization gains. It is at the heart of horizontal innovation models (e.g. Romer, 1990; Jones, 1995a), by implying that economic growth can be driven by variety expansion over time. As pointed out by Jones (2005, p. 1089), since the number of intermediate goods sectors is increasing in the scale of the economy, “the weak form of scale effects is so inextricably tied to idea-based growth that rejecting one is largely equivalent to rejecting the other”.<sup>1</sup> In vertical innovation models, sometimes referred to as “Schumpeterian” models, this is not necessarily true, since variety expansion is not critical for growth. Like in horizontal innovation models, however, scale effects arise from the combination of two features: the aggregate production function is of the Dixit–Stiglitz–Ethier type and the number of sectors positively depends on population size (see e.g. Peretto, 1998; Young, 1998). The vertical innovation framework proposed in this paper, with entrepreneurial firms operating in perfect competition, shares the latter but avoids the former feature. It therefore removes weak scale effects, in addition to strong ones.

In sum, the analysis suggests that sustained R&D-driven growth is possible without weak or strong scale effects and when perfectly competitive producers operate under constant returns. The potential for scale-invariant endogenous growth is not new, but is achieved very differently to the previous literature. Dalgaard and Kreiner (2001) and Strulik (2005, 2007) employ infinite-horizon, monopolistic competition models with ever increasing average human capital levels. For instance, in Dalgaard and Kreiner (2001), the change in the aggregate human capital level over time is proportional to the aggregate final output level. This is different to Lucas (1988) who assumes that the change in the per capita level of human capital depends on per capita (time) resources invested in education. As a result, and unlike in Lucas (1988), higher population growth exerts a congestion effect on the level of human capital per worker. It has the same impact as an increase in the depreciation rate of human capital (see also Strulik (2005, 2007)). Consequently, long-run R&D-based growth is not necessarily related to population growth in a positive way. For simplicity, and to put the contrast to this previous literature in its sharpest relief, this paper leaves human capital as exogenous. This paper is also not the first one suggesting that innovation is possible under perfect competition. Two approaches exist in the literature. The first one assumes that (non-entrepreneurial) firms operate under decreasing returns to scale; it thus violates the replication argument. The second one, advanced in Boldrin and Levine (2005, 2008), assumes that “copies of ideas are rivalrous goods” (Boldrin and Levine, 2005, p. 1252). This creates a rent to the innovator for the first unit of a good, which can be copied and sold by others, even in a perfectly competitive environment. In contrast, this paper maintains the basic premises of seminal work in the endogenous growth literature (e.g. Romer, 1990) that there are constant returns in the production technology and knowledge is non-rival. Finally, and also related to this paper, a recent literature investigates occupational choice in the context of entrepreneurial risk-taking (e.g. Clemens, 2006; García-Peñalosa and Wen, 2008). However, unlike this paper, it employs the standard monopolistic competition framework which implies that scale effects prevail.

The paper is structured as follows. Section 2 presents the basic version of the model. For simplicity, it rests on the assumption of identical entrepreneurial skills. Section 3 analyzes the equilibrium. Section 4 discusses the framework in two respects. First, it shows that the main properties remain valid if the basic model is extended to allow for heterogeneity of entrepreneurs. Second, it compares important features of the model to the previous literature, in order to clarify the intuition of results and their contribution from a theoretical point of view and in light of empirical evidence. The last section concludes.

## 2. The basic model

Consider the following overlapping-generations, discrete-time framework with two-period lives and an endogenous mass (“number”) of entrepreneurs who can invest in R&D. Both goods and factor markets are perfect. Let  $t$  denote the time index, which is omitted whenever this does not lead to confusion. Each entrepreneur  $i = [0, N]$  produces a homogenous consumption good, the “numeraire”, according to

$$q_i = x_i^z (\ell_i^Y)^\beta (B_i)^{1-\alpha-\beta}, \quad \text{with} \quad \ell_{it}^Y = \bar{B}_{t-1} \ell_{it}^Y, \quad (1)$$

<sup>1</sup> Scale effect properties sometimes occur with respect to per capita utility rather than income. This happens if utility depends on a “love of variety” consumption index (Dixit and Stiglitz, 1977) and monopolistically competitive firms produce final rather than intermediate goods.

$\alpha, \beta \in (0, 1)$ , where  $q_i$  is final output of firm  $i$ ,  $x_i$  is its input of a homogenous producer good (“capital”),  $l_i^Y$  is labor input in manufacturing,  $B_i$  is an index of productivity in firm  $i$ , and  $\bar{B} \equiv (1/N) \int_0^N B_i di$  denotes average productivity of the  $N$  final goods producers;  $N_0 > 0$  is given. According to (1), an increase in the stock of knowledge in  $t - 1$ ,  $\bar{B}_{t-1}$ , raises efficiency units of manufacturing labor next period,  $l_{it}^Y$ . This captures an intertemporal knowledge spillover effect which is labor-saving. It implies that wages grow at the same rate as  $\bar{B}$  in equilibrium. This is required for existence of a balanced growth equilibrium (BGE).

Productivity  $B_i$  in firm  $i$  is determined by in-house R&D investments and the human capital (“ability”) of entrepreneur  $i$ , denoted by  $a_i$ . Let  $l_i^R$  denote R&D labor input of firm  $i$  and suppose  $B_i = A_i a_i$ , where  $A_i$  evolves according to  $A_{it} = \bar{B}_{t-1} h(l_{it}^R)$ . Thus

$$B_{it} = a_i \bar{B}_{t-1} h(l_{it}^R). \quad (2)$$

$\bar{B}_0 > 0$  is given.  $h$  is an increasing and strictly concave function. Moreover, assume  $\lim_{l \rightarrow 0} h'(l) \rightarrow \infty$  and  $\lim_{l \rightarrow \infty} h'(l) = 0$  in order to ensure interior solutions for R&D investment problems. Access to the previous stock of knowledge in the R&D process reflects the standard “standing on shoulders” effect.<sup>2</sup> It also means that R&D creates proprietary knowledge for one period. The overlapping-generations structure introduced below implies that this equals the duration an entrepreneurial firm is operated. Both patent rights or trade secrets are consistent with this assumption. R&D labor costs, despite being incurred contemporaneously, are sunk when the firm enters product market competition. This borrows from the IO literature on endogenous sunk costs for R&D (and marketing) outlays (e.g. Sutton, 1998).

Note that entrepreneurial skill  $a_i$  enters as rival human capital input, whereas technology indicator  $A_i$  is non-rival. Thus, in view of (2), production technology (1) has constant returns: doubling rented inputs ( $x_i, l_i^Y$ ) and entrepreneurial human capital ( $a_i$ ) doubles output, holding sunk investment for R&D labor ( $l_i^R$ ) constant. Since entrepreneurial skill is embodied in the founder of a firm and does not have to be rented, operating profits are positive, as will become apparent. This enables entrepreneurs to incur sunk costs for entering the market (if entry is costly) and for R&D, despite perfect competition. The critical role of entrepreneurs for the productivity of firms is reflected by the assumption that entrepreneurial skill  $a_i$  is imperfectly substitutable to labor input  $l_i^Y$ .

Each entrepreneur uses foregone consumption as input. One unit of foregone consumption can be transformed into one unit of any intermediate good. Thus, the aggregate capital stock in the economy is measured by  $K = \int_0^N x_i di$  and the marginal production cost is equal to the interest rate. For simplicity, capital can freely be rented at an internationally given interest rate, denoted by  $\bar{r} > 0$ .<sup>3</sup>

Denote by  $L_t$  the size of the population born in period  $t$ . It grows according to  $L_{t+1} = (1 + g_L)L_t$ , where  $L_0 > 0$ . In the first period of life, each individual inelastically supplies one unit of labor to the labor market and chooses savings for old age. In the second period, individuals decide whether to open a final good firm or to retire.<sup>4</sup> As entrepreneur, they may invest in R&D before competing in the product market. Opening up a firm may require a fixed labor input,  $f \geq 0$ . Each member  $i$  of generation  $t - 1$  maximizes the standard utility function

$$U_{it-1} = \ln c_{it-1,1} + \rho \ln c_{it-1,2}, \quad (3)$$

$0 < \rho < 1$ , where  $c_{i,1}$  and  $c_{i,2}$  denote the consumption level in the first and second period of life, respectively. In the basic model, all individuals are identical, i.e., entrepreneurial ability  $a_i = a$  for all  $i$ . (Heterogeneity is introduced in Section 4.1)

Finally, in order to examine whether economic growth potentially depends on public policy in the long-run, suppose the government may levy a time-invariant R&D subsidy, at rate  $\tau \in [0, 1)$ . It is financed by a proportional value-added consumption tax, with tax rate  $\lambda_t$  in  $t$ , such that the public budget is balanced each period. As will become apparent, such a tax neither affects entry nor R&D investment decisions. It thus fulfills the same role as a lump-sum tax in standard models (see Section 4.2).

### 3. Equilibrium analysis

This section derives the equilibrium properties of the basic model.

#### 3.1. Factor demand

Under perfect competition, the price of the intermediate good equals marginal production costs,  $\bar{r}$ . Thus, in the second period of life, entrepreneur  $i$  solves

<sup>2</sup> As is well known, constant returns to past knowledge is necessary for exponential long-run growth to be sustained in absence of population growth. Jones (2005) provides an excellent discussion of linearity assumptions in endogenous growth theory. Groth et al. (2008) show that under decreasing returns to knowledge there may still be unbounded growth in the long run, which however is less than exponential.

<sup>3</sup> Appendix A treats the case where the interest rate is endogenous. The analysis becomes significantly less tractable. However, the main insights from the basic model remain unchanged.

<sup>4</sup> This assumption follows Aghion et al. (2005) and is not critical. First, one may extend the model to allow for non-entrepreneurs to work in their second period of life. For instance, they could enter as experienced labor which may or may not be imperfectly substitutable to labor supplied by individuals in their first period of life. It is critical, however, that skill  $a_i$  is imperfectly substitutable to experienced labor as well. Otherwise, operating profits would be equal to wage income of experienced labor and nobody would be willing to become entrepreneur in the presence of sunk costs. Second, one may assume that individuals can become entrepreneur already in the first period of life without affecting the main results of the analysis.

$$\pi_i \equiv \max\{q_i - \bar{r}x_i - w l_i^Y\} \text{ s.t. (1),} \quad (4)$$

where  $w$  denotes the wage rate. Recalling  $l_{it}^Y = \ell_{it}^Y / \bar{B}_{t-1}$  we find that factor demand functions are given by

$$x_i = \left(\frac{\alpha}{\bar{r}}\right)^{\frac{1-\beta}{1-\alpha-\beta}} \left(\frac{\beta}{\omega}\right)^{\frac{\beta}{1-\alpha-\beta}} B_i, \quad (5)$$

$$\ell_i^Y = \left(\frac{\alpha}{\bar{r}}\right)^{\frac{\alpha}{1-\alpha-\beta}} \left(\frac{\beta}{\omega}\right)^{\frac{1-\alpha}{1-\alpha-\beta}} B_i, \quad (6)$$

where  $\omega_t \equiv w_t / \bar{B}_{t-1}$  is the wage rate in  $t$  adjusted for average productivity in the previous period  $t - 1$ . Consequently, operating profits are given by  $\pi_i = (1 - \alpha - \beta)q_i$ , where

$$q_i = \left(\frac{\alpha}{\bar{r}}\right)^{\frac{\alpha}{1-\alpha-\beta}} \left(\frac{\beta}{\omega}\right)^{\frac{\beta}{1-\alpha-\beta}} B_i. \quad (7)$$

As defined in national accounting, the gross domestic product,  $Y$ , is equal to the “sum” of the value of all entrepreneurs’ final output levels:

$$Y = \int_0^N q_i di. \quad (8)$$

By substituting (7) into (8), one finds that final output per worker,  $y = Y/L$ , is given by

$$y = \bar{B} \left(\frac{\alpha}{\bar{r}}\right)^{\frac{\alpha}{1-\alpha-\beta}} \left(\frac{\beta}{\omega}\right)^{\frac{\beta}{1-\alpha-\beta}} n, \quad (9)$$

where  $n \equiv N/L$  is the number of firms per worker.

### 3.2. Individual decisions

When saving  $s_{it-1}$  for old age, the budget constraint of a member  $i$  of generation  $t - 1$  (under consumption tax rate  $\lambda_{t-1}$ ) in the first period of life is given by

$$(1 + \lambda_{t-1})c_{it-1,1} + s_{it-1} = w_{t-1}. \quad (10)$$

Net profits of entrepreneur  $i$  are given by operating profits minus sunk costs for entry and R&D. Formally, with R&D employment  $l_i^R$ , fixed labor requirement  $f$  and R&D subsidy rate  $\tau$ , we have

$$\Pi_i = \pi_i - (1 - \tau)w l_i^R - wf. \quad (11)$$

In the second period of life, the consumption level is then given by

$$(1 + \lambda_t)c_{it,2} = \begin{cases} (1 + \bar{r})s_{it-1} + \Pi_{it} & \text{if } i \text{ is entrepreneur} \\ (1 + \bar{r})s_{it-1} & \text{otherwise.} \end{cases} \quad (12)$$

Combining (10) and (12), we find that

$$(1 + \lambda_{t-1})c_{it-1,1} + (1 + \lambda_t) \frac{c_{it,2}}{1 + \bar{r}} = I_{it-1}, \quad (13)$$

where  $I_{it-1}$  is the present discounted value (PDV) of income of an individual  $i$  from the perspective of period  $t - 1$ . We have

$$I_{it-1} = \begin{cases} w_{t-1} + \frac{\Pi_{it}}{1 + \bar{r}} & \text{if } i \text{ is entrepreneur,} \\ w_{t-1} & \text{otherwise.} \end{cases} \quad (14)$$

It follows that the optimal R&D labor input of entrepreneur  $i$ ,  $l_i^R$ , equals the level which maximizes net profits  $\Pi_i$ . Substituting  $\pi_i = (1 - \alpha - \beta)q_i$ , with  $q_i$  given by (7), into profit function (11), and using R&D technology (2), net profits of firm  $i$  in  $t$  can be expressed as

$$\Pi_{it} = (1 - \alpha - \beta) \left(\frac{\alpha}{\bar{r}}\right)^{\frac{\alpha}{1-\alpha-\beta}} \left(\frac{\beta}{\omega_t}\right)^{\frac{\beta}{1-\alpha-\beta}} a_i \bar{B}_{t-1} h(l_{it}^R) - (1 - \tau)w_t l_{it}^R - wf. \quad (15)$$

Recalling  $\omega_t \equiv w_t / \bar{B}_{t-1}$ , this implies first-order condition

$$(1 - \alpha - \beta) \left(\frac{\alpha}{\bar{r}}\right)^{\frac{\alpha}{1-\alpha-\beta}} \left(\frac{\beta}{\omega}\right)^{\frac{\beta}{1-\alpha-\beta}} a_i h'(l_i^R) = (1 - \tau)\omega. \quad (16)$$

Moreover, maximizing utility function  $u$  in (3) with respect to  $(c_{it-1,1}, c_{it,2})$ , subject to budget constraint (13), we obtain optimal consumption levels

$$c_{it-1,1} = \frac{I_{it-1}}{(1+\rho)(1+\lambda_{t-1})}, \quad (17)$$

$$c_{it,2} = \frac{\rho(1+\bar{r})I_{it-1}}{(1+\rho)(1+\lambda_t)}. \quad (18)$$

Substituting (17) and (18) into (3), we find that utility becomes

$$U_{it-1} = D(\lambda_{t-1}, \lambda_t) + (1+\rho) \ln I_{it-1} \quad \text{with} \quad (19)$$

$$D(\lambda_{t-1}, \lambda_t) \equiv \rho \ln \left[ \frac{\rho(1+\bar{r})}{1+\lambda_t} \right] - (1+\rho) \ln(1+\rho) - \ln(1+\lambda_{t-1}). \quad (20)$$

According to (14) and (19), an individual is indifferent whether or not to become entrepreneur if net profits are equal to zero. Consequently, with identical individuals ( $a_i = a$ ) and due to free entry, it holds that  $\Pi_i = 0$  for all  $i$  in equilibrium.

### 3.3. Steady state

Combining first-order condition (16) with  $\Pi_i = 0$  by using (15), we find that equilibrium R&D employment per firm,  $l^{R*}$ , is at all times uniquely defined by<sup>5</sup>

$$0 = \frac{h(l^{R*})}{h'(l^{R*})} - l^{R*} - \frac{f}{1-\tau} \equiv G(l^{R*}, \tau). \quad (21)$$

We thus find that, if (and only if)  $f > 0$ ,  $l^{R*}$  is increasing in the R&D subsidy rate,  $\tau$ . We write  $l^{R*} = \tilde{l}^R(\tau)$ . According to R&D technology (2) and  $a_i = a$  for all  $i$ , the productivity growth rate  $g_{\bar{B}} \equiv \bar{B}_t/\bar{B}_{t-1} - 1$  is for all  $t$  given by:

$$g_{\bar{B}} = ah(l^{R*}) - 1, \quad \text{with } l^{R*} = \tilde{l}^R(\tau). \quad (22)$$

Productivity growth is increasing in  $\tau$  if and only if  $f > 0$ .<sup>6</sup> It thus may be policy-dependent.  $g_{\bar{B}}$  is also increasing in entrepreneurial ability  $a$ . Most importantly, as  $l^{R*}$  is time-invariant and in particular remains unaffected when population size,  $L$ , changes, the same applies to  $g_{\bar{B}}$ . In view of (9), this means that there are no weak scale effects (i.e.,  $y$  is independent of  $L$ ) if the productivity-adjusted wage rate,  $\omega$ , and the number of entrepreneurs per worker,  $n \equiv N/L$ , are time- and scale-invariant.

To show this, first note that using  $l_i^R = \tilde{l}^R(\tau)$  and  $a_i = a$  in first-order condition (16) implies that the productivity-adjusted wage rate ( $\omega$ ) immediately jumps towards its steady state level, which is given by

$$\omega^* = (1 - \alpha - \beta)^{\frac{1-\alpha-\beta}{1-\alpha}} \left( \frac{\alpha}{\bar{r}} \right)^{\frac{\alpha}{1-\alpha}} \beta^{\frac{\beta}{1-\alpha}} \left( \frac{ah'(\tilde{l}^R(\tau))}{1-\tau} \right)^{\frac{1-\alpha-\beta}{1-\alpha}} \equiv \Omega(\tau, a, \bar{r}). \quad (23)$$

(Equilibrium values along a balanced growth path are denoted by superscript  $(*)$  throughout.) Using (21), which defines  $l^{R*} = \tilde{l}^R(\tau)$ , one can show that  $\omega^*$  is increasing in  $\tau$ . This is primarily due to the fact that a higher R&D subsidy raises the demand for R&D labor, for a given number of firms.  $\omega^*$  is also increasing in  $a$ . This reflects the complementary of entrepreneurial skill to R&D labor as captured by R&D technology (2). Higher capital costs, by contrast, reduce the demand for labor; thus,  $\omega^*$  is decreasing in the interest rate,  $\bar{r}$ . Most importantly, however, (23) shows that  $\omega^*$  is scale-invariant.

Finally, we can solve for the equilibrium number of firms per worker,  $n^*$ , by employing the labor market clearing condition. In labor market equilibrium, we have

$$\int_0^N l_i^Y di + \int_0^N (l_i^R + f) di = L \quad (24)$$

(recall that only young individuals are wage earners), where  $l_{it}^Y = \ell_{it}^Y/\bar{B}_{t-1}$ . Combining this with labor demand function (6) and observing that  $l_i^R = l^{R*}$  and  $B_{it} = a\bar{B}_{t-1}h(l^{R*})$  for all  $i$ , we obtain

$$n^* = \frac{1}{(\alpha/\bar{r})^{\frac{\alpha}{1-\alpha-\beta}} (\beta/\omega^*)^{\frac{1-\alpha}{1-\alpha-\beta}} ah(l^{R*}) + l^{R*} + f} \quad (25)$$

$$= \frac{1 - \alpha - \beta}{(1 - \alpha - \beta\tau)\tilde{l}^R(\tau) + (1 - \alpha)f} \equiv \tilde{n}(\tau, \bar{r}), \quad (26)$$

where the second equation follows after substituting the expression for  $\omega^*$  in (23) and using the definition of  $l^{R*} = \tilde{l}^R(\tau)$  in (21). This shows that in equilibrium the number of firms ( $N$ ) is proportional to scale  $L$  (recall  $n = N/L$ ) which is consistent

<sup>5</sup> To see that  $l^{R*}$  exists and is unique, confirm  $\lim_{l \rightarrow 0} G(l, \tau) < 0$ ,  $\lim_{l \rightarrow \infty} G(l, \tau) > 0$  from the properties of  $h'$ , and note that  $G(l, \tau)$  is increasing in  $l$ , since  $h'' < 0$ .

<sup>6</sup> The same result holds in the vertical innovation framework by Young (1998), as shown in Grossmann (2008). Young (1998) implicitly sets  $f = 0$ .

with empirical evidence (e.g. Laincz and Peretto, 2006). Moreover,  $n$  jumps from initial value  $n_0 = N_0/L_0$  to the steady state level  $n^*$ . We have seen that also R&D input per firm and the productivity-adjusted wage rate assume their steady state level immediately, i.e., there are no transitional dynamics in the economy.

### 3.4. The absence of scale effects

According to (9), (23) and (26), difference equation  $\bar{B}_t = \bar{B}_{t-1}ah(l^{R*})$  determines the evolution of equilibrium income per capita over time:

$$y_t^* = \bar{B}_t \left( \frac{\alpha}{\bar{r}} \right)^{\frac{\alpha}{1-\alpha-\beta}} \left( \frac{\beta}{\Omega(\tau, a, \bar{r})} \right)^{\frac{\beta}{1-\alpha-\beta}} \bar{n}(\tau, \bar{r}). \quad (27)$$

Thus, an increase in the scale of the economy (workforce size  $L$ ) does not have an effect on per capita income. The same is true for the capital stock per worker,  $k = K/L$ : the aggregate capital stock is equal to total output of the intermediate good,  $K = \int_0^N x_i di$ . Using capital input demand function (5) and recalling (9), we obtain  $k^* = (\alpha/\bar{r})y^*$ .

Also welfare is independent of scale at all times. This can be seen as follows: for all  $i$ , (indirect) utility of generation  $t-1$  is given by

$$U_{it-1} = D(\lambda_{t-1}, \lambda_t) + (1+\rho) \ln(\bar{B}_{t-2}\omega^*), \quad (28)$$

according to (19) and the fact that, in equilibrium, (the PDV of) income is  $I_{it-1} = (w_{t-1})\bar{B}_{t-2}\omega^*$  for all  $i$  (recall  $\omega_t = w_t/\bar{B}_{t-1}$ ). We already know that the productivity-adjusted wage rate,  $\omega^*$ , and average productivity level,  $\bar{B}$ , are independent of scale. It remains to be shown that consumption tax rates ( $\lambda$ ), used to finance R&D subsidies (at constant rate  $\tau$ ), are in equilibrium scale-invariant at all times as well. Total tax revenue in  $t$  is given by  $\lambda_t \int_0^{L_t} c_{it,1} di + \lambda_t \int_0^{L_{t-1}} c_{it,2} di$ . Using  $I_i = w$  in both (17) and (18), we find that a balanced government budget in period  $t$  implies

$$\tau N_t w_t l^{R*} = \lambda_t \left( L_t \frac{w_t}{(1+\rho)(1+\lambda_t)} + L_{t-1} \frac{\rho(1+\bar{r})w_{t-1}}{(1+\rho)(1+\lambda_t)} \right). \quad (29)$$

Using the facts that the population grows at constant rate  $g_L$  and, for all  $t$ ,  $w_t/w_{t-1} = \bar{B}_{t-1}/\bar{B}_{t-2} = ah(l^{R*})$ , we obtain  $\lambda_t = \lambda^*$  for all  $t$ , where  $\lambda^*$  is given by

$$\frac{\lambda^*}{1+\lambda^*} = \frac{\tau(1+\rho)n^*l^{R*}}{1 + \frac{\rho(1+\bar{r})}{(1+g_L)ah(l^{R*})}}. \quad (30)$$

Thus, at all times, the consumption tax rate  $\lambda$  is scale-invariant.

In sum, there are no weak scale effects. Neither are there strong ones. Growth rates of per capita income and the capital-labor ratio are independent of  $L$  at all times. In steady state, as in neoclassical growth theory and horizontal innovations models like Romer (1990) and Jones (1995a), both variables grow at the same rate as wages and productivity,  $g_{y^*} = g_{k^*} = g_{w^*} = g_{\bar{B}}$ . Finally, an increase in the population growth rate  $g_L$  neither affects the level of per capita income nor its growth rate in any period.

## 4. Discussion

This section discusses the model in three ways. First, it is shown how endogenous heterogeneity of firms can be introduced in a simple and natural fashion (Section 4.1). It is shown that the main results are robust to this extension. Second, in order to gain a deeper understanding for the results, the proposed model is compared to a standard vertical innovation framework (Section 4.2). Third, the main results derived in this paper are discussed in light of empirical evidence (Section 4.3).

### 4.1. Firm heterogeneity

To allow for heterogeneity in entrepreneurial skill suppose that, in each generation, entrepreneurial ability is distributed according to a time- and scale-invariant cumulative distribution function,  $\Phi(a)$ . The associated density function has support  $[0, \bar{a}]$ ,  $\bar{a} > 0$ . For simplicity, we focus on the case where there is no population growth,  $g_L = 0$ .

According to (14) and (19), an individual chooses to become entrepreneur if net profits are non-zero,  $\Pi_i \geq 0$ . Applying the envelope theorem to profit function (15), by using first-order condition for R&D investment (16), shows that  $\Pi_i$  is increasing in entrepreneurial skill,  $a_i$ . Thus, there exists a threshold value  $\underline{a}$  such that all individuals with  $a_i \geq \underline{a}$  choose to become entrepreneur and the others do not enter. For the marginal entrant, with ability level  $\underline{a}$ , we have  $\Pi_i = 0$ . Combining (15) and (16) shows that the marginal entrant hence chooses a time- and scale-invariant R&D labor input,  $\bar{l}^R(\tau)$ , which is implicitly defined by  $G(l^R, \tau) = 0$ , as in (21). Setting  $l_i^R = \bar{l}^R(\tau)$  and  $a_i = \underline{a}$  in (16) implies that, in BGE,

$$\underline{a}^* = \frac{1-\tau}{h'(\bar{l}^R(\tau))} \frac{1}{1-\alpha-\beta} \left( \frac{\bar{r}}{\alpha} \right)^{\frac{\alpha}{1-\alpha-\beta}} \left( \frac{(\omega^*)^{1-\alpha}}{\beta^\beta} \right)^{\frac{1}{1-\alpha-\beta}} \equiv \underline{a}(\omega^*, \tau, \bar{r}). \quad (31)$$

Along a balanced growth path, the number of final good firms per worker,  $n = N/L$ , is then given by

$$n^* = 1 - \Phi(\hat{a}(\omega^*, \tau, \bar{r})) \equiv \hat{n}(\omega^*, \tau, \bar{r}). \quad (32)$$

Function  $\hat{a}(\omega, \tau, \bar{r})$  is increasing in both productivity-adjusted wage rate  $\omega$  and interest rate  $\bar{r}$ , since higher factor costs reduce profits and thus impede the incentive to enter the market. Moreover, by making use of (21), one finds that  $\hat{a}(\omega, \tau, \bar{r})$  is decreasing in R&D subsidy rate  $\tau$  (i.e. raising  $\tau$  stimulates entry), holding  $\omega$  constant. The opposite effects hold with respect to function  $\hat{n}(\omega^*, \tau, \bar{r})$ .

From first-order condition (16), we also obtain that the optimal R&D labor input of individuals with  $a_i > \underline{a}^*$  in BGE reads

$$l_i^{R*} = (h')^{-1} \left( \frac{1-\tau}{a_i} \frac{1}{1-\alpha-\beta} \left( \frac{\bar{r}}{\alpha} \right)^{\frac{\alpha}{1-\alpha-\beta}} \left( \frac{(\omega^*)^{1-\alpha}}{\beta^\beta} \right)^{\frac{1}{1-\alpha-\beta}} \right) \equiv \hat{l}^R(\omega^*, \tau, a_i, \bar{r}). \quad (33)$$

As  $h$  is strictly concave, the complementarity between entrepreneurial skill and R&D labor input implies that function  $\hat{l}^R(\omega, \tau, a_i, \bar{r})$  is increasing in skill  $a_i$ ; that is, more highly skilled entrepreneurs invest more in R&D and therefore run more productive firms. Moreover,  $\hat{l}^R$  is decreasing in  $\omega$  and  $\bar{r}$ , and increasing in  $\tau$ .

We finally employ labor market clearing condition (24). Dividing it by  $L$  and using  $1/L = n/N$  yields

$$n \left( \frac{1}{N} \int_0^N l_i^Y di + \frac{1}{N} \int_0^N (l_i^R + f) di \right) = 1. \quad (34)$$

According to (2), (6) and  $l_{it}^Y = \ell_{it}^Y / \bar{B}_{t-1}$  we have

$$\frac{1}{N} \int_0^N l_i^Y di = \left( \frac{\alpha}{\bar{r}} \right)^{\frac{\alpha}{1-\alpha-\beta}} \left( \frac{\beta}{\omega} \right)^{\frac{1-\alpha}{1-\alpha-\beta}} \frac{1}{N} \int_0^N a_i h(l_i^R) di. \quad (35)$$

Using (31)–(33) and (35) implies that (34) can be rewritten as

$$1 = \left( \frac{\alpha}{\bar{r}} \right)^{\frac{\alpha}{1-\alpha-\beta}} \left( \frac{\beta}{\omega^*} \right)^{\frac{1-\alpha}{1-\alpha-\beta}} \int_{\hat{a}(\omega^*, \tau, \bar{r})}^{\bar{a}} a h(\hat{l}^R(\omega^*, \tau, a, \bar{r})) d\Phi(a) + \int_{\hat{a}(\omega^*, \tau, \bar{r})}^{\bar{a}} \hat{l}^R(\omega^*, \tau, a, \bar{r}) d\Phi(a) + f[1 - \Phi(\hat{a}(\omega^*, \tau, \bar{r}))]. \quad (36)$$

This equation implicitly defines the productivity-adjusted wage rate in BGE,  $\omega^*$ . Recalling the properties of functions  $\hat{a}$  and  $\hat{l}^R$ , we obtain that the right-hand side of (36) is decreasing in  $\omega^*$  and  $\bar{r}$ ; moreover, it is increasing in  $\tau$ . The following results are implied. First, we find that  $\omega^*$  is unique. Together with (31)–(33) we can thus conclude that the BGE is unique. Moreover, again, there are no transitional dynamics. Second, and also like in the basic model,  $\omega^*$  is increasing in R&D subsidy rate ( $\tau$ ) and decreasing in the interest rate ( $\bar{r}$ ).

Using (9), in equilibrium per capita income evolves according to

$$y_t^* = \bar{B}_t \left( \frac{\alpha}{\bar{r}} \right)^{\frac{\alpha}{1-\alpha-\beta}} \left( \frac{\beta}{\omega^*} \right)^{\frac{1-\alpha}{1-\alpha-\beta}} n^*. \quad (37)$$

It is thus scale-invariant and grows, like the capital-labor ratio,  $k = (\alpha/\bar{r})y$ , and the wage rate ( $w$ ), with the same rate as average productivity,  $\bar{B}$ . According to R&D technology (2) and (33), we obtain

$$B_{it} = a_i \bar{B}_{t-1} h(\hat{l}^R(\omega^*, \tau, a_i, \bar{r})). \quad (38)$$

Thus, average productivity  $\bar{B}$  grows according to

$$g_{\bar{B}} = \frac{1}{1 - \Phi(\underline{a}^*)} \int_{\underline{a}^*}^{\bar{a}} a h(\hat{l}^R(\omega^*, \tau, a, \bar{r})) d\Phi(a) - 1. \quad (39)$$

Hence,  $g_{\bar{B}}$  is independent of  $L$ , which confirms that not only weak scale effects but also strong ones are absent.

#### 4.2. Comparison to standard framework

We next work out the differences and similarities of the proposed model to a standard vertical innovation framework. Consider the following infinite-horizon growth model with a typical, aggregate constant-returns to scale production function of final output (the numeraire good):

$$Y = X^\gamma (L^Y)^{1-\gamma}, \text{ with } X = \left( \int_0^N (A_i)^{1-\gamma} (x_i)^\gamma di \right)^{1/\gamma}, \quad (40)$$

$0 < \gamma < 1$ .  $L^Y$  is labor input in the final goods sector,  $X$  is the quantity of a composite good consisting of (a mass of)  $N$  differentiated intermediate capital inputs, and  $x_i$  and  $A_i$  denote the quantity and the quality of intermediate good  $i$ , respectively. Intermediate goods are also produced by a constant-returns to scale technology: one unit of foregone consumption can be transformed into one unit of any intermediate good. Thus, the marginal production cost is equal to the interest rate,  $r$ . Each



intermediate goods firm produces one variety in a monopolistically competitive environment.<sup>7</sup> Like in the basic model with entrepreneurial innovation, labor force  $L$  is homogenous, is supplied inelastically to a perfect labor market, and grows at a constant rate,  $g_L \geq 0$ .

Quality-improvements occur according to

$$A_{it} = \bar{A}_{t-1} h(l_{it}^R), \quad (41)$$

where  $h$  has the same properties as before and  $\bar{A}_{t-1}$  is the average quality of goods in  $t-1$ ;  $\bar{A}_0 > 0$ . Thus, again, there are constant returns to past knowledge in the R&D technology. In addition to R&D labor input ( $l_i^R$ ), also a fixed labor input,  $f \geq 0$ , is required one period in advance of production.<sup>8</sup> Sunk investment  $f$  has to be made each period, i.e., firms have to re-establish each period.<sup>9</sup>

With production function (40) and perfect competition in the final goods sector, the inverse demand function of the representative final goods producer for capital input  $i$  is given by  $p_i = \gamma(A_i L^Y/x_i)^{1-\gamma}$ . Hence, an intermediate good monopolist  $i$ , with operating profits  $\pi_i = \max(p_i - r)x_i$ , chooses price  $p_i = r/\gamma$ ; thus, we obtain  $\pi_i = (1/\gamma - 1)rx_i$  with output

$$x_i = \left(\frac{\gamma^2}{r}\right)^{\frac{1}{1-\gamma}} A_i L^Y. \quad (42)$$

In period  $t-1$ , firm  $i$  chooses R&D labor input to maximize the present discounted value (PDV) of total profits,

$$\Pi_{it-1} \equiv \frac{\pi_{it}}{1+r_t} - w_{t-1}(1-\tau)l_{it-1}^R - w_{t-1}f, \quad (43)$$

where the R&D subsidy rate is financed by a lump-sum tax. In equilibrium under free entry,  $\Pi_i = 0$  for all  $i$  at all times.

Appendix B solves for a general equilibrium when an infinitely living representative dynasty behaves according to standard preferences. Like in the entrepreneurial innovation model, in a BGE an increase in population size ( $L$ ) induces a proportional increase in the number of goods/sectors,  $N$ . In turn, R&D input per firm is unaffected by larger scale. Formally, and identical to the basic entrepreneurial innovation model, the equilibrium R&D labor input per firm is uniquely given by  $l^{R*} = \bar{l}^R(\tau)$ , as defined by (21). Consequently, according to (41), the growth rate of average productivity is given by  $g_{\bar{A}} = h(l^{R*}) - 1$ . It again may be policy-dependent and is independent of scale. This illustrates the basic idea of “Schumpeterian” models without strong scale effects like Peretto (1998), Young (1998), Dinopoulos and Thompson (1998) and Howitt (1999). Because knowledge which is accessible for innovators equals the last period’s average productivity level (rather than, for instance, the “sum” of past productivity levels of firms),<sup>10</sup> the product proliferation effect of larger scale predicts that productivity growth is related to R&D investment per variety. According to (2), the same holds in the entrepreneurial innovation model developed in this paper.

But why do standard models imply existence of weak scale effects, in contrast to the one proposed in this paper? To see this, substitute the intermediate input levels given by (42) into (40) to find that per capita income,  $y = Y/L$ , can be written as

$$y = \bar{A}N \left(\frac{\gamma^2}{r}\right)^{\frac{1}{1-\gamma}} \frac{L^Y}{L}. \quad (44)$$

The interest rate  $r$  is independent of scale and time-invariant in BGE. Moreover, since in BGE the number of firms  $N$  is proportional to population size  $L$ , also the fraction of employment allocated to in the final goods sector,  $L^Y/L$  is time- and scale-invariant. According to (44), consequently, per capita income  $y$  is proportional to  $L$  and the growth rate of per capita income is increasing in the population growth rate,  $g_L$ . These results are implications of employing a Dixit–Stiglitz–Ethier type of aggregate production function, like (40). As well known, this class of production technology implies that output (or the composite commodity input  $X$ ) rises in the number of intermediate goods  $N$  when holding total capital input  $K = \int_0^N x_i di$  constant. This property, which technically is an immediate implication of declining marginal productivity of any variety  $i$ , is interpreted as capturing specialization gains. It is, due to the positive relationship between  $N$  and scale, the ultimate source of weak scale effects.<sup>11</sup>

The proposed entrepreneurial innovation framework abstains from employing an aggregate Dixit–Stiglitz–Ethier production function. Rather, there is an endogenous number of entrepreneurs, each operating a constant-returns production tech-

<sup>7</sup> As all rival inputs are rented, in contrast to the entrepreneurial innovation framework developed in this paper, price-taking would imply zero profits, leaving no incentives to enter and to invest in R&D.

<sup>8</sup> This is unlike in the proposed entrepreneurial innovation model analyzed above, where both kinds of sunk costs are assumed to be incurred in the same period. Most results in this paper would remain unchanged if, like in Young (1998), sunk costs had to be incurred one period in advance. It would induce, however, transitional dynamics and therefore would make the analysis less tractable. In an overlapping generations context, however, the alternative assumption of a one-period lag until R&D becomes effective seems less appropriate. (I am grateful for a referee for this point.)

<sup>9</sup> This feature is somewhat artificial in an infinite-horizon context and comes out more naturally in the proposed overlapping generations context with occupational choice. There entrepreneurs operate firms in their second and last period of life.

<sup>10</sup> For a discussion of alternative formulations, see e.g. Grossmann (2008).

<sup>11</sup> One could remove the “specialization gains property” of production technology (40), and thereby eliminate weak scale effects, in an ad hoc fashion. Modifying the technology for final goods production to  $Y = (1/N)X^\gamma(L^Y)^{1-\gamma}$  does the trick, as can easily be seen from (44). However, this is unsatisfactory as there is lack of a proper justification for such technology.



nology. Total output is then simply measured like in national accounting: it is equal to the aggregate value of final output levels of individual producers. This eliminates weak scale effects in addition to strong ones.

#### 4.3. Empirical evidence

But is it attractive, in light of empirical evidence, to remove scale effects? In view of international linkages and associated international technological spillovers it is beside the point to dismiss scale effects by arguing that small economies like Luxembourg, Switzerland or Hongkong are among the richest. However, in the post World War II era, *strong* scale effects, featured by first-generation models of endogenous growth (Romer, 1990; Grossman and Helpman, 1991; Aghion and Howitt, 1992), are also inconsistent with the fact that the number of researchers substantially increased over time while productivity growth rates remained relatively stable (see Jones, 1995a,b).<sup>12</sup>

Cross-country studies testing the hypothesis of *weak* scale effects account for international trade relations in order to separate the effects arising from a larger market size per se and those arising from more indirect effects of trade, like technology spillovers.<sup>13</sup> They provide mixed evidence in support of weak scale effects. For instance, Hall and Jones (1999) regress per capita income on population size while controlling for instrumented “social infrastructure” – an index which includes a measure of trade openness. They find that population size enters insignificantly. Frankel and Romer (1999) show that when trade volumes are instrumented for by geographical variables “there is a positive [...] relation between country size and income per person”, which however is “only marginally significant” (p.387). Rodrik et al. (2004) redo a similar analysis by instrumenting measures of institutional quality as well, in addition to instrumenting trade volumes. In contrast to Frankel and Romer (1999), they find insignificant and sometimes even negative effects of larger population size on per capita income. (Instrumented trade volumes enter insignificantly as well.) Similar evidence is provided by Bolaky and Freund (2006). Rose (2006) employs a large panel data set to examine the effect of population size of a country on many economic and social indicators (including GDP per capita). He concludes that small and large countries are not systematically different. Also consistent with the absence of scale effects, another strand of literature suggests that the impact of an increase in the population growth rate on the growth rate of per capita income is either insignificant or negative (Brander and Dowrick, 1994; Ahituv, 2001; Kelley and Schmidt, 2005).<sup>14</sup> The standard R&D-based growth literature, by contrast, predicts a positive relationship between income growth and population growth (see e.g. Section 4.2), whereas the proposed (basic) entrepreneurial innovation framework implies no relationship (see Section 3.4).

In sum, the evidence suggests that it may be useful to analyze endogenous growth in a framework where even the weak form of scale effects is absent. The proposed model also is, like “Schumpeterian” growth models, consistent with the evidence that there is a close relationship between productivity growth and R&D per variety. Ha and Howitt (2007) and Maddsen (2008) find strong support for this hypothesis by using cointegration tests for developed countries. The result is also consistent with the observation that both R&D intensity and productivity growth are fairly constant over time since World War II.

#### 5. Concluding remarks

This paper proposed a simple overlapping generations framework with occupational choice and endogenous vertical innovations of entrepreneurs. The fundamental premises of Romer (1990, p.572) still apply: technological change (“improvement in the instructions for mixing together raw materials”) is critical for economic growth and capital accumulation, arise due to intentional R&D investments, and can be applied without non-rivalry to the manufacturing process. But contrary to Romer’s conclusion from these premises made in the context of aggregate production functions, an equilibrium with price-taking agents *can* be supported despite constant returns in the context of entrepreneurial production, in line with the replication argument. This is possible since entrepreneurial skill is a rival and critical production factor which is imperfectly substitutable to other factors. Moreover, accounting for heterogeneity in entrepreneurial ability naturally introduces heterogeneity of firms with respect to their R&D investments and productivity.

Focussing on price-taking entrepreneurs shows, in addition, that R&D-based growth is possible without scale effects with respect to per capita income levels. This is fundamentally different to standard endogenous growth models which employ the notion of an aggregate production technology, using intermediate inputs supplied by monopolistically competitive firms. The property that also weak scale effects may be absent is important for a number of issues. First, from a theoretical point of view, it is potentially useful for the agenda of applying general equilibrium models with endogenous technical change to identify the determinants of long-run economic growth and their quantitative importance. In view of the mixed empirical evidence even for the weak form of scale effects, one may want to make sure that important results and mechanisms are

<sup>12</sup> Support in favor of strong scale effects is, among others, provided by Kremer (1993) for historical (pre-modern) times. This suggests that the importance of population size for economic growth decreased over time. Dalgaard and Jensen (2007) provide a possible explanation, based on a declining importance of the bequest motive to save relative to life-cycle motives.

<sup>13</sup> Doing so only accounts for scale effects associated with a higher domestic labor force, not those associated with trade liberalization. This is not problematic, however, provided that scale effects are not higher if market size increases due to an enlarged foreign market than due to a larger domestic market.

<sup>14</sup> Jones (2002, 2005) argues that semi-endogenous growth theory may, nevertheless, not be inconsistent with the latter finding. If one allows for dilution effects of larger population size, higher population growth depresses the capital-labor ratio in the transition to a steady state, similar to neoclassical growth theory. According to this reasoning, we do not yet observe steady state dynamics and a positive relationship between population growth and income growth eventually may materialize.

not driven by this property. Second, the analysis suggests that goods market integration, by increasing the size of the economy, does not necessarily raise living standards. In fact, the widely recognized instrumental variable approach to identify the determinants of per capita income by Rodrik et al. (2004), perhaps the most comprehensive study to date, shows that the impact of (endogenous) trade becomes insignificant once (endogenous) institutional quality is controlled for. The proposed framework may be useful to understand such evidence. Finally, the analysis showed that the widely discussed demographic change, which is projected for many developed countries, may not necessarily impede advancements of the world's technological frontier.

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## Appendix A. Closed economy version of basic model

This appendix adapts the basic model to a closed economy. That is, the interest rate is endogenized, rather than exogenously given by  $\bar{r}$ .

In equilibrium of a closed economy, aggregate savings of individuals for old age ( $K^S$ ) are equal to total capital demand ( $K^D$ ). Combining (10) with (17) and using the fact that, in equilibrium,  $I_i = w$  for all  $i$  yields optimal savings  $s_i = \frac{\rho w}{1+\rho}$  for all  $i$ . The aggregate capital supply in  $t$  is thus given by  $K_t^S = \frac{\rho L_{t-1} w_{t-1}}{1+\rho}$  or, using  $\omega_{t-1} = w_{t-1}/B_{t-2}$ ,

$$K_t^S = \frac{\rho \bar{B}_{t-2} L_{t-1} \omega_{t-1}}{1+\rho}. \quad (45)$$

Next we derive capital demand,  $K^D = \int_0^N x_i di$ . Using (5), we obtain

$$K_t^D = N_t \bar{B}_t \left( \frac{\alpha}{r_t} \right)^{\frac{1-\beta}{1-\alpha-\beta}} \left( \frac{\beta}{\omega_t} \right)^{\frac{\beta}{1-\alpha-\beta}}. \quad (46)$$

Now use that (for all  $t$ )  $\bar{B}_t/\bar{B}_{t-1} = ah(l^{R*})$  and  $N_t/L_{t-1} = (1+g_L)n_t$  when setting  $K^S = K^D$  to find

$$\frac{\rho \omega_{t-1}}{1+\rho} = (1+g_L)n_t \left( \frac{\alpha}{r_t} \right)^{\frac{1-\beta}{1-\alpha-\beta}} \left( \frac{\beta}{\omega_t} \right)^{\frac{\beta}{1-\alpha-\beta}} a^2 h(l^{R*})^2. \quad (47)$$

Consequently, using that  $\tilde{l}^{R*} = \tilde{l}^R(\tau)$  together with  $\omega = \Omega(\tau, a, r)$  and  $n = \tilde{n}(\tau, r)$ , according to (23) and (26), respectively, the evolution of  $r$  is governed by first-order difference equation

$$\frac{\rho \Omega(\tau, a, r_{t-1})}{1+\rho} = (1+g_L)\tilde{n}(\tau, r_t) \left( \frac{\alpha}{r_t} \right)^{\frac{1-\beta}{1-\alpha-\beta}} \left( \frac{\beta}{\Omega(\tau, a, r_t)} \right)^{\frac{\beta}{1-\alpha-\beta}} a^2 h(l^R(\tau))^2, \quad (48)$$

where  $r_0$  is given. Setting  $r_{t-1} = r_t$  gives us the long run interest rate,  $r^*$ ; in turn, we obtain  $\omega^* = \Omega(\tau, a, r^*)$  and  $n^* = \tilde{n}(\tau, a, r^*)$ . Note that  $r^*$ ,  $\omega^*$  and  $n^*$  are independent of scale in BGE. Thus, again, neither the level of per capita income in BGE,

$$y_t^* = \bar{B}_t \left( \frac{\alpha}{r^*} \right)^{\frac{1-\beta}{1-\alpha-\beta}} \left( \frac{\beta}{\omega^*} \right)^{\frac{\beta}{1-\alpha-\beta}} n^*, \quad (49)$$

nor its growth rate  $g_{\bar{B}} = ah(\tilde{l}^R(\tau)) - 1$  depend on scale.

## Appendix B. Solution of standard vertical innovation model

Suppose that in the vertical innovation model of Section 4.2 there is an infinitely living representative dynasty with standard intertemporal utility function

$$U = \sum_{t=0}^{\infty} L_t \rho^t \ln c_t, \quad (50)$$

$0 < \rho < 1$ , where  $c_t$  denotes per capita consumption in period  $t$ . This appendix shows that there exists a symmetric BGE of the model, where the number of goods per worker,  $n = N/L$ , the interest rate ( $r$ ), R&D labor per firm ( $l^R$ ), and the fraction of labor employed in the final goods sector ( $L^Y/L$ ), are time- and scale-invariant.

First, note that financial assets per capita, denoted by  $a$ , accumulate according to

$$(1+g_L)a_{t+1} = (1+r_t)a_t + w_t - c_t, \quad (51)$$

where  $w_t$  denotes the wage rate (recall that  $L_{t+1} = (1 + g_L)L_t$ ). Utility maximization thus leads to Euler equation  $c_{t+1} = \rho(1 + r_t)c_t$ .

The wage rate is equal to the marginal product of labor in the final goods sector; according to (40),  $w = (1 - \gamma)Y/L^Y$ . Combining this with expression (44) for  $y$ , we find that

$$\tilde{\omega} \equiv \frac{w}{AL} = (1 - \gamma) \left( \frac{\gamma^2}{r} \right)^{\frac{\gamma}{1-\gamma}} n. \quad (52)$$

Next, combine  $\pi_i = (1/\gamma - 1)rx_i$  with  $x_i = (\gamma^2/r)^{\frac{1}{1-\gamma}}A_iL^Y$  from (42) and substitute (41) to find that the PDV of profits of firm  $i$  from the perspective of period  $t - 1$  is given by

$$\Pi_{it-1} = \frac{1 - \gamma}{1 + r_t} \gamma^{\frac{1+\gamma}{1-\gamma}} (r_t)^{-\frac{\gamma}{1-\gamma}} \bar{A}_{t-1} h(l_{it-1}^R) L_t^Y - w_{t-1}(1 - \tau) l_{it-1}^R - w_{t-1}f, \quad (53)$$

according to (43). In  $t - 1$ , firm  $i$  chooses R&D labor input to maximize  $\Pi_{it-1}$ . In view of (53) and the definition of  $\tilde{\omega}$  in (52), the associated first-order condition implies

$$\frac{(1 - \gamma) \gamma^{\frac{1+\gamma}{1-\gamma}} (r_t)^{-\frac{\gamma}{1-\gamma}}}{1 + r_t} h'(l_{it-1}^R) \frac{L_t^Y}{L_{t-1}} = (1 - \tau) \tilde{\omega}_{t-1}. \quad (54)$$

Due to free entry, in equilibrium, the value of net profits becomes zero. In view of (53) and (54),  $\Pi_{it-1} = 0$  confirms that, for all  $i$  and  $t$ , the equilibrium R&D labor input is again given by  $l^{R*} = \bar{l}^R(\tau)$  as defined in (21). Thus,  $\bar{A}_{t+1}/\bar{A}_t = h(l^{R*})$ , according to (41). Moreover, the labor market clearing condition reads  $N_{t+1}(l^{R*} + f) + L_t^Y = L_t$ . (Note that firms founded in  $t$  produce in  $t + 1$ .) Thus, using  $n = N/L$  and  $L_{t+1} = (1 + g_L)L_t$ , we find that in BGE

$$\left( \frac{L^Y}{L} \right)^* = 1 - n^*(1 + g_L)(l^{R*} + f). \quad (55)$$

From asset accumulation equation (51), in BGE,  $c_{t+1}/c_t = w_{t+1}/w_t$ . Using  $w_t = \bar{A}_t L_t \tilde{\omega}_t$  together with (41) and the property that  $\tilde{\omega}$  is time-invariant in BGE (which will become apparent), we find  $c_{t+1}/c_t = (1 + g_L)h(l^{R*})$ . From the Euler equation, the equilibrium interest rate factor is thus given by

$$1 + r^* = \frac{(1 + g_L)h(l^{R*})}{\rho}. \quad (56)$$

Combining (52) with (54), by making use of  $L_t = (1 + g_L)L_{t-1}$ , we also find that in BGE

$$\gamma h'(l^{R*})(1 + g_L) \left( \frac{L^Y}{L} \right)^* = (1 - \tau)(1 + r^*)n^*. \quad (57)$$

Observing (55) and (56), from (57) we find

$$n^* = \frac{\rho \gamma \frac{h'(l^{R*})}{h(l^{R*})}}{1 - \tau + (1 + g_L) \rho \gamma (l^{R*} + f) \frac{h'(l^{R*})}{h(l^{R*})}}. \quad (58)$$

Combining (55) and (58), we also find that

$$\left( \frac{L^Y}{L} \right)^* = \frac{1 - \tau}{1 - \tau + (1 + g_L) \rho \gamma (l^{R*} + f) \frac{h'(l^{R*})}{h(l^{R*})}}. \quad (59)$$

Thus, there exists a BGE where  $n$ ,  $r$ ,  $l^R$ ,  $L^Y/L$  and  $\tilde{\omega}$  are time-invariant. Moreover, using (44), we obtain that in BGE the growth rate of per capita income is given by

$$g_y = \frac{y_{t+1} - y_t}{y_t} = \frac{\bar{A}_{t+1}}{\bar{A}_t} \frac{L_{t+1}}{L_t} - 1 = (1 + g_L)h(l^{R*}) - 1. \quad (60)$$

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