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May increased partisanship lead to convergence of parties' policy platforms?

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Abstract Parties face a trade-off between motivating partisans to participate in the election and appealing to issue-oriented middle-of-the-road voters. We show that, consequently, parties may diverge from the median voters' preferred policy by sending ambiguous messages to voters which include announcements of alternative platforms. Moreover, surprisingly, an increase in the size of a partisan constituency may lead to platform convergence towards the median voters' preferred policy. We identify two conditions for this outcome. First, the electorate is sufficiently divided such that full convergence does not occur and, second, the majority of the non-partisan voters is more inclined to the party with increased support of partisans.

Keywords Convergence · Ideology; Median voter · Two-party competition · Voting

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1 Introduction

Observed divergence across party platforms challenges the traditional median voter predictions (Downs 1957), that parties and political candidates will tend to cater to the middle-of-the-road voters. This alludes to the relevance of other factors, such as ideological preferences

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of candidates under probabilistic voting (see e.g., Wittman 1983; Calvert 1985; Alesina 1988; Roemer 1997a; Cukierman and Tommasi 1998a, 1998b), a strategic motive to cater to partisan constituencies (e.g., Glaeser et al. 2005) and/or the role of special interest groups (Olson 1982; Grossman and Helpman 1996, 2001; Ashworth 2006) in explaining parties' behavior.

This paper examines the role of partisanship in shaping policy platforms set by political parties. For the United States, empirical evidence suggests that political partisanship remained fairly stable in the 1980s and 1990s and has been increasing since 2000. Looking at the feeling of voters towards parties as captured by their responses to the "thermometer" questions included in the National Election Survey, Kimball and Gross (2007) measure the extent of partisanship by the correlation between thermometers for the Republican Party and the Democratic Party. They show that the correlation between party thermometers was never as strong as it was in 2004 (-0.47).[1] Moreover, it is more negative for strong partisans (-0.7) and campaign activists (-0.6) than for fairly independent voters and non-activists.[2] This suggests that party identifiers not only support their own party but also harbor strong negative feelings towards the other party.[3] In addition, the campaign contributions of economic elites increasingly are targeted to candidates with strong ideological predispositions (see McCarty et al. 2006 and the references therein).

Prima facie, one would expect that the existence of stronger (and more cohesive) partisan constituencies, or, larger support of special interest groups for candidates with extreme positions, would lead to greater divergence across party platforms, to the disadvantage of middle-of-the-road voters. However, this paper argues that stronger partisanship may well induce convergence of platforms towards the one preferred by the median voter and may thereby benefit middle-of-the-road voters.

We propose a simple two-party model where parties seek to maximize the (expected) number of votes. We assume that partisan voters never defect from their own party but may abstain from voting when the proposed policy platform is sufficiently far from their ideology.[4] We thus provide a microfoundation for the notion that parties/candidates seem, at least to some extent, to be ideology-driven and argue that this may be consistent with pure vote-maximization in the face of partisan constituencies. Parties face, hence, a fundamental trade-off between motivating partisans to participate in the election, on the one hand, and appealing to non-partisan, middle-of-the-road voters, on the other hand.

Our modeling strategy serves two goals. First, the existence of partisan voters together with the assumption that parties are motivated by vote share (rather than by the implemented policies) gives rise, in a simple way, to the possibility that parties diverge from the policy preferred by the majority of (middle-of-the-road) voters. Second, the proposed framework enables us to shed light on the question whether and under which circumstances more rabid partisanship (a larger fraction of party identifiers in the electorate) may lead to convergence of policy platforms.

[1] In the mid-1970s, by contrast, the correlation coefficient was around zero.

[2] Glaeser and Ward (2006: Fig. 4) measure partisanship by looking at the difference rather than the correlation between the feeling towards the Democratic party and the Republican party. They obtain similar results.

[3] The finding is consistent with social identification theory (Campbell et al. 1960; Greene 2004), according to which strong partisans suffer from perceptional biases in evaluating their preferred party relative to others.

[4] In fact, empirically, abstention in elections is strongly determined by alienation, i.e., is a function of the distance from a voter's ideal point to the nearest candidate. Such evidence has been found for both presidential elections (Zipp 1985; Adams and Merrill 2003) and midterm elections (Plane and Gershtenson 2004).

In our framework, a pure strategy equilibrium fails to exist if the middle-of-the-road electorate is sufficiently diverse. This raises a question about the interpretation of a mixed strategy political equilibrium. We follow the approach of Laslier (2000) and interpret a mixed strategy profile in a deterministic two-party game as reflecting the parties' proposed platforms, where the probability that a policy alternative is offered equals the fraction of voters identifying a party with this policy alternative. For instance, one may imagine that the mixed strategy profile reflects the fractions of time a party announces alternatives. In this sense, we show that in our setting "ambiguity is a rational behavior for the parties" (Laslier 2000).[5] In this context, where parties can send ambiguous messages to voters, we naturally define convergence (divergence) of party platforms as a higher (lower) fraction of voters who identify both parties with the policy alternative preferred by the median voter.

The structure of the remainder of the paper is as follows. The coming section discusses related literature. In Sect. 3, we present the basic structure of the model and discuss its assumptions. In Sect. 4 we characterize the political equilibrium. In Sect. 5 we examine the implications of increased partisanship for the parties' platform proposals. Section 6 concludes. All formal analysis and derivations are relegated to the appendix.

2 Related literature

As pointed out by Roemer (1997a: 479f.), "[t]he assumptions of ideological parties and uncertainty together enable us to overcome the tyranny of the median voter theorem" (see also Persson and Tabellini 2000: ch. 5).[6] Unlike many previous contributions, our framework does not derive divergence of party platforms by adopting the standard probabilistic voting model (where the election outcome is assumed to be uncertain for given policy alternatives proposed by parties). Rather, we offer a setting, which extends the approach of Laslier (2000), by allowing for partisan voters. It generates divergence in the sense, that in equilibrium, not all voters identify a party with the policy alternative preferred by the median voter. Notably, in our setting, the extent of partisanship is a simple and observable measure (number of partisans) which can be used to conduct comparative-static analysis.

Eyster and Kittsteiner (2007) propose a different argument why policy platforms diverge from the one preferred by the median voter. They show that divergence may occur when campaign costs are sufficiently high in order to mitigate political competition. Another interesting source of divergence of parties' platforms is non-policy related valence competition (Ashworth and Bueno de Mesquita 2009; Zakharov 2009). Ashworth and Bueno de Mesquita (2009) show in a model where candidates can invest in their reputations ('charisma'), that a higher valence advantage of a candidate over his/her opponent has a smaller impact on the fraction of voters the candidate attracts, the more distant platforms are from each other. Zakharov (2009) explains in a similar framework why party polarization and campaign spending have simultaneously risen in the United States. In both papers,

[5] Alesina and Cukierman (1990) study a different kind of policy ambiguity. In their model, there is a stochastic relationship between the policy instrument of a party and the policy outcome, where the variance of the stochastic element is treated as measure of ambiguity. In our model, ambiguity means that different alternatives are proposed by the same party.

[6] Roemer (1997a) endogenizes uncertainty by employing a probabilistic voting framework (e.g., Wittman 1983; Calvert 1985). In his model, the probability of winning an election given the policy platforms set by (two) parties becomes endogenous when parties are uncertain about the distribution of traits among voters who turn out in the election.

divergence softens costly valence competition and may thus occur even in the absence of policy preferences of parties/candidates.[7] Miller and Schofield (2003) and Schofield (2003) consider both elements, namely, valence competition and campaign contributions. They propose a model where candidates are pure vote-maximizers who cater to potential activists in order to elicit campaign contributions. Campaign contributions, in turn, affect valence. As a result, again, "candidates do not converge in a Downsian fashion to the center of the electoral distribution. Instead, a 'rational' candidate will choose a policy position so as to 'balance' off activist contributions and voter responses" (Miller and Schofield 2003: 253).

Most related to our paper, Glaeser et al. (2005) analyze a two-party game where divergence of policy platforms from that preferred by the median voter occurs if and only if "party affiliates" (holding on average more extreme bliss points than the average bliss point of the total population) are on average better informed about the platform proposed by a party than the average voter.[8] The extent of extremism (measured by the deviation from the bliss point preferred by the median voter) of vote-maximizing parties rises if the share of party affiliates rises. While this is a potential outcome also in our framework, we demonstrate that the result does not hold true in general and are able to come up with intuitive and testable conditions under which convergence (or divergence) occurs. Note that in our framework divergence does not stem from informational asymmetries.

3 Basic structure of the model

Consider an economy with two vote-maximizing political parties, called leftwing (L) and rightwing (R) party, and a one-dimensional, non-empty set of *policy alternatives*, $\mathcal{P}$. For technical reasons, to ensure existence of an equilibrium, suppose that $\mathcal{P}$ contains a finite (but possibly large) number of elements, where $\bar{P}_L$ and $\bar{P}_R$ denote the leftmost and rightmost policy of $\mathcal{P}$, respectively.

There is a unit mass of "middle-of-the-road" (i.e., non-partisan) voters. They differ in their preferences regarding the policy alternatives in $\mathcal{P}$. To capture this heterogeneity in a simple form, we assume that there are two types of individuals, indexed by $i = 1, 2$, where the fraction of type i voters is denoted by x_i. We assume that $x_1 \in [a, b]$, $0 < a < b < 1$. Thus, there is a strictly positive proportion of each type of voter. The preferences of voters of type i are represented by a single-peaked utility function $u_i(P)$. We let $P_i^* \equiv \arg\max_{P\in\mathcal{P}} u_i(P)$ denote type-i voters' preferred policy. We assume that $\bar{P}_L < P_2^* < P_1^* < \bar{P}_R$; that is, voters' preferred policy varies across types and is in the interior of the policy space. We further assume that when middle-of-the-road voters of a given type are indifferent between the policy proposals of the two parties, they split their votes evenly between the two parties. Finally, we assume that all non-partisan voters take part in the elections.

In addition to middle-of-the-road voters, there exists a mass n_L (n_R) of individuals, who identify themselves with the leftwing and rightwing party, respectively, and whose preferred policy (ideology) is the leftmost (rightmost) element of the policy space ($\bar{P}_L$ and $\bar{P}_R$). Diehard leftists never vote for party R and, similarly, diehard rightists never vote for party

[7] In general, valence differences affect platform choices (e.g. Groseclose 2001; Aragones and Palfrey 2002). We will further relate the insights of our paper to this literature, when we discuss our results in the concluding section.

[8] See also Carrillo and Castanheira (2008) for the role of imperfect information of voters on convergence of parties' platforms. In their model, candidates can invest in the quality of the platform (e.g., by selecting competent advisers), in addition to selecting policy.

L.[9] We define an increase in the partisan support for party j by an increase in the size of its diehard constituency (relative to that of middle-of-the-road voting population, which is normalized to one). That is, the parameters n_L and n_R measure the extent of partisanship. These two parameters play a key role in the comparative-static analysis in Sect. 5.[10]

Diehards of party $j = L, R$ derive intrinsic utility

$$\gamma - |P_j - \bar{P}_j| \tag{1}$$

from voting for party j. The (reservation) utility from not participating in the election is normalized to zero. Thus, partisans vote for the party they feel affiliated with if its proposed policy is not too distant from their preferred policy; that is, partisans of party j participate in the election if $\gamma \geq |P_j - \bar{P}_j|$. We assume that there is within-group heterogeneity among the partisans. Formally, we let $F_j(\gamma)$ denote the cumulative distribution function of γ for diehards of party $j = L, R$. We assume that F_j is continuously differentiable.[11]

As parties maximize the number of votes, the payoff of party $j = L, R$ is given by

$$[1 - F_j(|P_j - \bar{P}_j|)]n_j + N_j, \tag{2}$$

where N_j denotes the number of middle-of-the-road voters that cast their ballot for party $j = L, R$. The first term of (2) captures party j's loss of partisan votes associated with deviating from its ideal point $\bar{P}_j$.

Similar to Glaeser et al. (2005), our model captures the notion that parties must balance two conflicting goals associated with different types of voters. First, parties compete for the support of middle-of-the-road voters, who choose between the two parties. Second, parties try to gain support from their diehard constituencies, viewed as a zero-one decision of diehards, whether to vote or not. In other words, parties face a trade-off between attracting electoral support from non-partisan voters and catering to their core constituency in order to induce them to vote. We assume that the diehard constituencies are sufficiently similar in size or, alternatively, that n_L and n_R are sufficiently small, such that the median voter will be a member of the middle-of-the-road constituency. The preferences of both parties and voters are assumed to be common knowledge.

Two remarks are in order. First, the assumption of vote-maximizing parties implies that parties neither care about the implemented policy nor about winning the election per se (like in Roemer 1997b; Dixit and Londregan 1998; Glaeser et al. 2005, among others). It captures the notion of career-driven politicians in its sharpest relief and is particularly reasonable for parliamentary elections, i.e., when the seats in parliament depend on vote share. Accordingly, a larger number of parliament members affiliated with a party may be eligible for certain ‘perks’ or privileges. Also, some decisions in parliament may require a supermajority. Moreover, as pointed out by Glaeser et al. (2005, p. 1294), vote-maximization is equivalent to the assumption that parties are “maximizing the probability of victory if, for

[9] For instance, see Shachar (2003), who stresses that partisanship involves habit formation from voting. Empirical evidence also suggests a close relationship between the ideological dispositions of voters and party identification (Abramowitz and Saunders 1998; Schreckhise and Shields 2003).

[10] These simple measures have the advantage that there are readily observable and therefore could be used for hypotheses testing. They could, for instance, be derived from “thermometer” surveys like those conducted by the US National Election Survey.

[11] Although F_j is defined on a finite grid, assuming continuity ensures that there are no jumps in the number of supporting partisans when party j's platform changes, no matter how fine the policy grid is.

example, each party's vote totals were affected by exogenous shocks whose difference is uniformly distributed".

Second, parties' payoff (2) resembles a standard (reduced-form) objective function of parties which consists of an ideological component of policy-motivated parties/candidates and of an egorent associated with success in the election. Parameter n_j then measures the relative importance of these objectives for party j. What our microfoundation shows is that seemingly ideologically driven behavior of parties may not be inconsistent with parties/candidates which are *purely* power-driven, but reflects the importance for parties to mobilize partisans to participate in the election. Apart from our microfoundation, it is also conceivable to motivate the payoff function in (2) as reflecting the potential withdrawal of financial campaign contributions by partisans. That is, similar to the turnout decision captured in our framework, partisans could make their financial contributions dependent on platform announcements and these contributions would enter the parties' payoffs. Again, parties would have to resolve the trade-off between appealing to partisans, thereby raising more funds by setting the platform close to the partisans' bliss points, and gaining the support of middle-of-the-road voters (power hunger).[12]

As will become apparent, there does not always exist a pure-strategy equilibrium for our two-party game. For interpreting mixed-strategy profiles, we follow the notion of Laslier (2000) that parties may send ambiguous messages about the policy alternative they want to implement. To capture this ambiguity in policy choices, consider a mixed strategy profile in the political game denoted by Γ, which is characterized by the two parties L and R, the payoff functions in (2), and action sets being given by the set of policy alternatives, $\mathcal{P}$. Let $l(P_L)$ denote the probability assigned to policy $P_L \in \mathcal{P}$ by party L, in the mixed strategy profile of game Γ, whereas $r(P_R)$ denotes the probability assigned to policy $P_R \in \mathcal{P}$ by party R. A pair of platforms is a pair of probability distributions over $\mathcal{P}$, $(\mathbf{l}, \mathbf{r})$. For instance, $l(P_L)$ may be interpreted as the fraction of time that party L spends claiming to be in favor of an alternative P_L. For notational convenience, let us also define the sets

$$A_i = \{(P_L, P_R) : u_i(P_L) > u_i(P_R)\}, \qquad B_i = \{(P_L, P_R) : u_i(P_L) = u_i(P_R)\}, \tag{3}$$

$i = 1, 2$. From the preferences of voters, if all voters identified party L with P_L and party R with P_R, the payoff for party L would be given by

$$\pi_L(P_L, P_R) = [1 - F_L(|P_L - \bar{P}_L|)]n_L + \sum_{i=1}^{2}[x_i \mathbf{1}_{A_i}(P_L, P_R) + 0.5x_i \mathbf{1}_{B_i}(P_L, P_R)], \tag{4}$$

where $\mathbf{1}_{A_i}$ and $\mathbf{1}_{B_i}$ denote the indicator function for the sets A_i and B_i, respectively.[13] A payoff for party R in case of unambiguous voter identification of parties' proposals, $\pi_R(P_L, P_R)$, can be derived analogously.

Following Laslier (2000), suppose that each voter observes a single pair of proposals, (P_L, P_R), one of each party, which is randomly and independently drawn from the joint distribution of proposals (the pair of ambiguous platforms). For any voter, the probability

[12] Herrera et al. (2008) analyze a framework wherein campaign financing can be used to mobilize party sympathizers to vote. In Miller and Schofield (2003), voters are non-partisans but may become party activists who make campaign contributions after policy shifts in a second policy dimension.

[13] For instance, for a given pair of alternatives, (P_L, P_R), we have $\mathbf{1}_{A_i}(P_L, P_R) = 1$ if $u_i(P_L) > u_i(P_R)$ and zero otherwise. Also recall that if voters of a certain type derive the same utility from the policy alternatives offered by the two parties, half of them vote for party L and half for party R.

that he/she identifies party L with P_L is $l(P_L)$ and the probability that he/she identifies party R with P_R is $r(P_R)$. Therefore, due to the law of large numbers, the proportion of voters (including diehards), who identify party L with $P_L \in \mathcal{P}$ and party R with $P_R \in \mathcal{P}$ is $l(P_L)r(P_R)$. This implies that the payoff (number of votes) for party L is given by

$$\sum_{P_L \in \mathcal{P}, P_R \in \mathcal{P}} l(P_L)r(P_R)\pi_L(P_L, P_R). \tag{5}$$

Importantly, (5) equals the expected payoff, $E(\pi_L(P_L, P_R))$, for party L under the pair of platforms $(\mathbf{l}, \mathbf{r})$. In the following we derive the equilibrium of the (deterministic) two-party game Γ with payoff functions $\pi_L(P_L, P_R)$ and $\pi_R(P_L, P_R)$ for party L and R. Because the proportion of voters who identify a party with a certain alternative is reflected by the mixed strategy profile of this game, we can interpret the equilibrium mixed strategy profile of Γ as platforms reflecting ambiguity in the messages sent by parties to voters, when parties maximize their vote-share.

4 Equilibrium analysis

For the equilibrium analysis of game Γ, we will add another piece of notation and define

$$P_i^{\max}(P) \equiv \max(P' \in \mathcal{P} | u_i(P') \geq u_i(P)), \tag{6}$$

and

$$P_i^{\min}(P) \equiv \min(P' \in \mathcal{P} | u_i(P') \geq u_i(P)), \tag{7}$$

$i = 1, 2$, as the rightmost and leftmost policy alternatives such that utility of voters is at least as high as that associated with a given platform P.[14]

We will focus our analysis on the case where the sizes of the diehard constituencies, n_L and n_R for party L and R are not too large. Formally, we make the following assumption:

Assumption A1 $F_L(P_i^* - \bar{P}_L)n_L \leq 0.5$ and $F_R(\bar{P}_R - P_i^*)n_R \leq 0.5$, $i = 1, 2$, simultaneously hold.

The assumption rules out pure strategy equilibria in which both parties diverge to their partisans' respective ideal points. The following proposition characterizes the equilibrium for the political game.

Proposition 1 *Suppose that Assumption A1 holds. Then there exists a sufficiently fine policy grid, $\mathcal{P}$, which contains at least four policy elements (P_1^*, P_2^*, $\bar{P}_L$ and $\bar{P}_R$), such that:*

(i) *If $x_1 \geq 0.5 + [F_L(P_1^* - \bar{P}_L) - F_L(P_2^{\min}(P_1^*) - \bar{P}_L)]n_L \equiv \bar{x}(n_L, \bar{P}_L, P_1^*)$, then there is a unique Nash equilibrium in pure strategies of the game Γ such that $P_L = P_R = P_1^*$, i.e., parties unambiguously cater to the median voter which is of type* 1.
(ii) *If $x_1 \leq 0.5 - [F_R(\bar{P}_R - P_2^*) - F_R(\bar{P}_R - P_1^{\max}(P_2^*))]n_R \equiv \underline{x}(n_R, \bar{P}_R, P_2^*)$, then parties unambiguously cater to the median voter which is of type* 2.

[14] Note from the single-peakedness of u_i and (7) that $P_i^{\min}(P) = P$ for all $P \leq P_i^*$ and, similarly, $P_i^{\max}(P) = P$ for all $P \geq P_i^*$, according to (6), $i = 1, 2$.

(iii) *If* $x_1 \in (\underline{x}(n_R, \bar{P}_R, P_2^*), \bar{x}(n_L, \bar{P}_L, P_1^*))$, *then game* Γ *only possesses Nash equilibria in mixed strategies, i.e., both parties send ambiguous messages to voters.*

Proof See Appendix A. □

Assuming that the policy grid is sufficiently fine, Proposition 1 characterizes how the political equilibrium depends on the distribution of voter types. First, it implies that when type i voters form a sufficient majority of the middle-of-the-road constituency, then both parties unambiguously signal the same policy alternative P_i^*, which is the preferred policy from the point of view of the median voter. One way to interpret this result is that both parties will cater to the middle on the issue at stake when there is enough cohesion among voters (captured by a large x_i in the case where $x_i > 0.5$). Due to the existence of partisan voters, however, when one group of voter types forms only a small majority (that is, the middle-of-the-road electorate is sufficiently divided), full convergence of policy platforms to the median voter's ideal point fails. To gain insight, consider, for instance, a situation in which type 2 voters constitute slightly less than 50% of the electorate and both parties unambiguously propose P_1^* (thus catering to type 1 voters who form the majority). Thus, each party will attract one half of the middle-of-the-road electorate. Now consider, say, a leftward deviation of party L to some (unambiguously proposed) alternative $P_L > P_2^{\min}(P_1^*)$. In such a case, party L will lose all type 1 voters but at the same time will attract all type 2 voters. Since the number of type 2 voters is sufficiently close to 0.5, the loss of voters will be rather small as party L still attracts roughly half of the voters. On the other hand, party L gains utility through increased turnout of its diehard constituency by shifting its platform leftward. Hence, such a deviation will be profitable, rendering an equilibrium (P_1^*, P_1^*) impossible, although type 1 voters form the majority. Proposition 1 states that in such a case any Nash equilibrium of the political game necessarily involves mixed strategies, which according to our setup means that at least one party sends an ambiguous message. This occurs if there is insufficient consensus among middle-of-the-road voters.

The attempt of parties to balance catering to diehards and appealing to middle-of-the-road voters implies that within the interval $(\underline{x}, \bar{x})$, formally defined by Proposition 1, parties choose to diverge, in the sense that the policy alternative most preferred by the median voter is not unambiguously proposed by the two parties in the equilibrium of the political game, hence the median voter theorem does not apply. Notably, a rise in either n_L or n_R, capturing the relative importance of partisans, would (plausibly) enlarge the range in which equilibrium necessarily involves mixed strategies, by increasing (decreasing) the upper (lower) bound of the interval. The finite policy grid, $\mathcal{P}$, ensures the existence of an equilibrium. The proposition states that the equilibrium necessarily entails parties sending ambiguous messages, hence diverging, when $x_1 \in (\underline{x}, \bar{x})$. However, one may not conclude that there generically exists a unique (mixed strategy) equilibrium, which is crucial for conducting comparative static analysis. For this reason and in order to obtain some easily interpretable results, in the analysis that follows we will put some more structure on the model by imposing some additional restrictions.

5 Effects of increased partisanship

When the distribution of voter types is such that the political equilibrium initially implies full convergence of platforms to the preferred policy of the median voter (say $x_1 \geq \bar{x}$, which implies that both parties unambiguously propose P_1^*) stronger partisanship (say, in favor of party L) may imply that x_1 will shift inside the interval $(\underline{x}, \bar{x})$. This would unambiguously be harmful for the majority of middle-of-the-road voters (type 1) and may even lead to a decrease in the well-being of type 2 voters. However, the more interesting (and empirically relevant) scenario is where, to begin with, platforms do not converge (that is, $x_1 \in (\underline{x}, \bar{x})$). In this section, we examine the impact of greater partisanship, captured by an increase in n_L or n_R, on the parties' platforms. We will use the following intuitive notion of convergence of parties to (and divergence from) the median voter's preferred policy.

Definition 1 (Convergence to the median voter) Parties are said to converge to (diverge from) the median voter's ideal point if, in equilibrium, the fraction of voters who identify both parties with the policy alternative preferred by the median voter rises (falls).

We choose a simple environment which satisfies the equilibrium properties stated by Proposition 1 and leads to a unique equilibrium. Uniqueness of equilibrium does not hold true in general if $x_1 \in (\underline{x}, \bar{x})$ but is needed for a meaningful comparative-static analysis. Rather than coming up with a general characterization, the goal of this section is to demonstrate, by employing a simple example, that, somewhat surprisingly, increased partisanship may well lead to convergence to the median voter's ideal point (in the sense defined above), and hence, can benefit the median voter. The example will also allow us to identify readily interpretable (testable) conditions under which platforms tend to converge or diverge. A discussion of the strategic considerations involved will be taken up later. The example is specified by Assumption A2:

Assumption A2 The following structure applies:

(i) $a = 1/4, b = 3/4$,
(ii) $\mathcal{P} = \{P_1^*, P_2^*, \bar{P}_L, \bar{P}_R\}$,
(iii) $n_j \in (1/3, 2/3)$, $j = L, R$,
(iv) $F_L(P_2^* - \bar{P}_L) = F_R(\bar{P}_R - P_1^*) = 1/2$, $F_L(P_1^* - \bar{P}_L) = F_R(\bar{P}_R - P_2^*) = 3/4$, $F_L(\bar{P}_R - \bar{P}_L) = F_R(\bar{P}_R - \bar{P}_L) = 1$,
(v) $u_1(P_1^*) > u_1(P_2^*) = u_1(\bar{P}_R) > u_1(\bar{P}_L)$, $u_2(P_2^*) > u_2(P_1^*) = u_2(\bar{P}_L) > u_2(\bar{P}_R)$.

Assumption A2 imposes a symmetric structure (but abstains from imposing $n_L = n_R$), in which the policy grid, $\mathcal{P}$, comprises the minimal number of elements (part (ii) of A2). From part (v) of A2 (which specifies voters' preferences), it follows that $P_2^{\min}(P_1^*) = \bar{P}_L$ and $P_1^{\max}(P_2^*) = \bar{P}_R$. Together with part (iv) of A2 and the definitions given in Proposition 1 this implies that $\bar{x} = 1/2 + 3n_L/4$ and $\underline{x} = 1/2 - 3n_R/4$. It is straightforward to verify that under part (iii) of Assumption A2, $\bar{x} > 7/8$ and $\underline{x} < 1/8$, so that combined with part (i) of Assumption A2 we have $[a, b] \subset (\underline{x}, \bar{x})$. Moreover, one can verify from parts (iii) and (iv) of Assumption A2 on the ideological motives of parties that Assumption A1 holds.

Table 1 gives us, for a given x_1, the payoff matrix for the political game Γ under action sets $\mathcal{P}$ for the two parties and payoff π_L for party L as given by (4) and π_R for party R which can be derived by symmetric considerations. As a guide for interpreting the table, consider the case where both parties propose policy P_1^*. In such a case voters split evenly between the two parties. As P_1^* differs from the partisans' ideal points, the losses of diehard voters experienced by party L and party R are given by $F_L(P_1^* - \bar{P}_L)n_L = 3n_L/4$ and $F_R(\bar{P}_R - P_1^*)n_R = n_R/2$, respectively. Thus, the payoff for party L (R) is given by $n_L/4 + 1/2$ ($n_R/2 + 1/2$, respectively). As can be straightforwardly verified from Table 1, there exists no Nash equilibrium in pure strategies for the game Γ. This is consistent with part (iii) of Proposition 1, as Assumption A2 implies that $[a, b] \subset (\underline{x}, \bar{x})$.

Table 1 Payoff matrix for the example

L \ R	P_1^*	P_2^*	$\bar{P}_R$	$\bar{P}_L$
P_1^*	$\frac{n_L}{4}+\frac{1}{2}, \frac{n_R}{2}+\frac{1}{2}$	$\frac{n_L}{4}+x_1, \frac{n_R}{4}+1-x_1$	$\frac{n_L}{4}+1, n_R$	$\frac{n_L}{4}+\frac{1+x_1}{2}, \frac{1-x_1}{2}$
P_2^*	$\frac{n_L}{2}+1-x_1, \frac{n_R}{2}+x_1$	$\frac{n_L}{2}+\frac{1}{2}, \frac{n_R}{4}+\frac{1}{2}$	$\frac{n_L}{2}+1-\frac{x_1}{2}, n_R+\frac{x_1}{2}$	$\frac{n_L}{2}+1, 0$
$\bar{P}_R$	$0, \frac{n_R}{2}+1$	$\frac{x_1}{2}, \frac{n_R}{4}+1-\frac{x_1}{2}$	$\frac{1}{2}, n_R+\frac{1}{2}$	$x_1, 1-x_1$
$\bar{P}_L$	$n_L+\frac{1-x_1}{2}, \frac{n_R}{2}+\frac{1+x_1}{2}$	$n_L, \frac{n_R}{4}+1$	n_L+1-x_1, n_R+x_1	$\frac{n_L}{2}+\frac{1}{2}, \frac{1}{2}$

5.1 Positive analysis

Equilibrium in mixed strategies of the deterministic two-party game Γ means that parties' platforms reflect ambiguity in the sense that parties choose to propose more than one policy alternative to voters and the proportions of voters which identify a party with a certain alternative corresponds to the respective probability in the mixed strategy profile of that party in the game Γ. We let $l_1 = l(P_1^*)$, $l_2 = l(P_2^*)$ and $l_3 = l(\bar{P}_L)$. Correspondingly, r_1, r_2 and r_3 denote the proportion of voters which associate party R with alternative P_1^*, P_2^* and $\bar{P}_R$, respectively. It is easy to check that for party L, $\bar{P}_R$ is a dominated strategy and, for party R, $\bar{P}_L$ is a dominated strategy (dominated by both P_1^* and P_2^*). Thus, in equilibrium (indicated by superscript (*) on probabilities), $l_1^* + l_2^* + l_3^* = 1$ and $r_1^* + r_2^* + r_3^* = 1$. One can further show the following.

Lemma 1 *Suppose Assumption A2 holds. Then there exists a unique equilibrium which can be characterized as follows*:

(i) *If* $x_1 \geq 1 - 3n_R/4 \equiv \hat{x}$, *then* $r_1^* = \frac{4x_1 - 3n_L}{2x_1} \equiv g(x_1, n_L) > 0$, $l_1^* = \frac{x_1 + n_R - 1}{x_1} \equiv h(x_1, n_R) > 0$, $r_2^* = l_2^* = 0$, $r_3^* > 0$, $l_3^* > 0$.

(ii) *If* $\tilde{x} \equiv 3n_L/4 < x_1 < \hat{x}$, *then* $r_1^* = g(x_1, n_L) > 0$, $r_2^* = \frac{2x_1 - n_L}{2(1-x_1)} - 2 + \frac{3n_L}{2x_1} \equiv z(x_1, n_L) > 0$, $l_1^* = z(1 - x_1, n_R)$, $l_2^* = g(1 - x_1, n_R) > 0$, $r_3^* = 1 - r_1^* - r_2^* > 0$, $l_3^* = 1 - l_1^* - l_2^* > 0$.

(iii) *If* $x_1 \leq \tilde{x}$, *then* $r_1^* = l_1^* = 0$, $r_2^* = h(1 - x_1, n_L) > 0$, $l_2^* = g(1 - x_1, n_R)$, $r_3^* > 0$, $l_3^* > 0$.

Proof See Appendix B. □

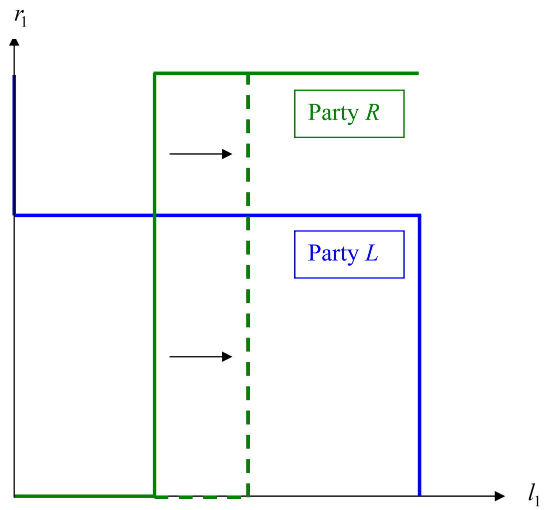
We will now discuss the impact of an increase in the number of partisan voters on the political equilibrium.[15]

Proposition 2 *Suppose that Assumption A2 holds. If* $x_1 > 1/2$ (*i.e., the majority of middle-of-the-road voters prefers* $\bar{P}_R$ *to* $\bar{P}_L$), *parties converge to the median voter's ideal point if* n_R *rises and diverge from the median voter if* n_L *rises. If* $x_1 < 1/2$, *the opposite holds.*

Proof First, note that $\hat{x} > 1/2$ and $\tilde{x} < 1/2$, according to part (iii) of Assumption A2. If $x_1 > 1/2$, voters of type 1 are the median voter. Hence, applying Definition 1, parties converge to the median voter if the proportion of voters that identify both parties with P_1^*, given by $r_1^* l_1^*$, rises. According to part (i) of Lemma 1, if $x_1 \geq \hat{x}$, then $r_1^* l_1^* = g(x_1, n_L) h(x_1, n_R)$.

[15] We do not consider the knife-edge case $x_1 = 1/2$.

Fig. 1 Reaction functions and effect of an increase in n_R for the case where$x_1 \geq \hat{x}$



Clearly, this proportion of voters is increasing in n_R and decreasing in n_L. If $1/2 < x_1 < \hat{x}$, then $r_1^* l_1^* = g(x_1, n_L) z(1 - x_1, n_R)$, according to part (ii) of Lemma 1. Hence, $r_1^* l_1^*$ is again increasing in n_R (note that $\partial z(x_2, n_R)/\partial n_R > 0$ if $x_1 = 1 - x_2 > a = 1/4$) and decreasing in n_L. For the case where $x_1 < 1/2$, calculate $r_2^* l_2^*$ by distinguishing the ranges $\tilde{x} < x_1 < 1/2$ and $x_1 \leq \tilde{x}$, using parts (ii) and (iii) of Lemma 1, respectively. The result thus can be confirmed in an analogous way. This concludes the proof. □

To gain some intuition for the result stated by Proposition 2, suppose first that a large majority of the middle-of-the-road electorate is of type 1, i.e., $x_1 \geq \hat{x}$. In this case, parties will never propose the preferred policy alternative of the minority of type 2 voters, P_2^*, in equilibrium, i.e., $l_2^* = 0$ (part (i) of Lemma 1). Now consider an upward shift in n_R. Starting from equilibrium, other things equal, when n_R increases, party R would gain from proposing policy alternative $\bar{P}_R$ unambiguously instead of proposing both P_1^* and $\bar{P}_R$. As can be observed from Fig. 1, the range of values of l_1 (the proportion of voters who identify party L with alternative P_1^*) in which party R finds it optimal to exclusively to play $\bar{P}_R$ (that is, to set $r_1 = 0$) increases. This means that the reaction curve of party R shifts rightward (dashed line in Fig. 1) as a response to an increase in n_R. So as to maintain the equilibrium platform, it is necessary that party L will increase l_1^*, thus balancing against the upward shift in n_R.

To grasp the intuition for this result, one can decompose the effect of a rise in the number of party R's diehards into a direct effect and a strategic one. The direct effect derives from the fact that an increase in n_R, other things equal, would induce party R to shift its policy to the right (that is, to diverge from the center). To see the strategic effect, recall that type 1 voters are more inclined to the ideal point of rightwing partisans, $\bar{P}_R$, compared with $\bar{P}_L$ (as $u_1(\bar{P}_R) > u_1(\bar{P}_L)$) and form the majority of the electorate (as $x_1 > 1/2$). Thus, party L, in response to the rightward shift of party R, would gain from shifting its policy towards the center, thus converging towards the preferred policy alternative of the majority. In turn, this behavioral response of party L has a restraining effect on party R which, as a result of party L's shift to the center, finds it less profitable to diverge from the median voter. In equilibrium, this restraining effect turns out to be extreme when $x_1 \geq \hat{x}$, as party R does not change its equilibrium strategy.

Now, consider the case $x_1 \in (1/2, \hat{x})$. This means that the median voter is still of type 1, but the majority of type 1 voters is small enough such that some voters identify both parties with alternative P_2^* in equilibrium (i.e., $l_2^* > 0$, $r_2^* > 0$). Again, an increase in n_R implies that party L will propose P_1^* more often (i.e., l_1^* increases) balancing against the desire of party R to move to the right in response to an increase in n_R. In this sense, there is still convergence. The increase in l_1^* is accompanied, however, by an increase in the equilibrium fraction of time party L announces its partisans' preferred policy, $\bar{P}_L$ (i.e., l_3^* goes up). That is, party L also caters more to partisan voters, but less to the minority of middle-of-the-road voters (i.e., l_2^* decreases).

Consider next an increase in n_L, when $x_1 > 1/2$. Note that in analogy to an upward shift in n_R, the direct effect of a shift in n_L would be, other things equal, a leftward shift in party L's platform. However, as the majority of middle-of-the-road voters is more inclined to the ideal point of rightwing partisans, the latter will respond to the leftward shift in party L's policy by shifting its policy to the right (reflected by a decrease in r_1^* and an increase in r_3^*). This in turn will have a restraining effect on party L, similar to the former case, but now party R has diverged from the median voter.

To sum up, when the party that would win the support of the majority of middle-of-the-road voters in the case where both parties propose their respective partisans' ideal points, $\bar{P}_L$ and $\bar{P}_R$, as their platforms (the favored party in a "partisan battle", which in a simple sense captures the political bias of the middle-of-the-road constituency) experiences greater partisanship, the other (non-favored) party tends to moderate its position (i.e., it tends to announce the median voter's ideal point with a higher probability) in order to remain politically competitive. This in turn moderates the incentive for the favored party to announce the ideal point of its partisans, leading all-in-all to convergence. However, when the party which would lose a partisan battle (the non-favored party) experiences an increase in the number of partisans, the other party would tend to move more to the extreme, resulting in divergence.

5.2 Normative analysis

We now turn to the normative implications of our previous comparative-static (positive) analysis. We aim to show that the median voter actually may benefit from increased partisanship. For this purpose, one generally has to determine what the implemented policies are. As parties choose to send ambiguous messages (playing mixed strategies) there is no clear-cut answer to this question. However, dwelling on Laslier's (2000) interpretation of mixed strategies, one can invoke a simple ex-ante welfare measure for the well-being of the median voter. Recall that by assumption each voter observes a single pair of proposals (P_L, P_R), one from each party, which is randomly and independently drawn from the joint distribution of proposals (the pair of ambiguous platforms). Each such voter, assuming naively that all voters observe the same pair of policies, conjectures that the implemented policy will be the one (of the two observed) supported by the majority of the voters (assuming a majority voting rule). Aggregating over voters according to the probabilities assigned to different pairs of proposals by the mixed strategy profiles, we can calculate the (ex-ante) average utility gained by the members of the median voter constituency.

Now consider the specific environment defined by Assumption A2 for the range $x_1 \geq \hat{x}$. Recall from part (i) of Lemma 1 that both parties mix between P_1^* and their partisans' ideal points only when $x_1 \geq \hat{x}$ (i.e., $r_2^* = l_2^* = 0$). Since $x_1 \geq \hat{x}$ implies $x_1 > 1/2$, as the political outcome is determined by majority rule, it is easy to see from Table 1 that voters will believe that the implemented policy will be P_1^* unless they observe the pair of platforms that coincide with the two parties respective partisans' bliss points. The fraction of voters that observe the latter pair of platforms is given by $r_3^*l_3^* = (1 - r_1^*)(1 - l_1^*)$. These voters believe that the implemented policy will be either $\bar{P}_R$ or $\bar{P}_L$, depending on the parametric assumptions. As the argument we make applies to both cases, let us assume for concreteness that $n_R \geq n_L$, thus as $x_1 > 1/2$, the implemented policy will be $\bar{P}_R$. Hence, according to part (i) of Lemma 1, the (ex-ante) average utility of the type-1 voter (who is the median voter, in this case) is given by

$$\begin{aligned} E(u_1) &= r_3^* l_3^* u_1(\bar{P}_R) + (1 - r_3^* l_3^*) u_1(P_1^*) \\ &= u_1(P_1^*) - [1 - g(x_1, n_L)][1 - h(x_1, n_R)][u_1(P_1^*) - u_1(\bar{P}_R)]. \end{aligned} \tag{8}$$

Recalling that $\partial h(x_1, n_R)/\partial n_R > 0$ and observing $u_1(P_1^*) > u_1(\bar{P}_R)$ shows that the average utility of the median voter increases if the number of rightwing partisans, n_R, increases.[16] By the same token, it is easy to see that when Assumption A2 and $x_1 \leq \tilde{x}$ hold (i.e., the median voter is of type 2), then an increase in n_L raises average utility of type 2 voters, $E(u_2)$.

6 Concluding remarks

In this paper, we introduced a simple two-party political setting and examined the impact of stronger partisanship on the degree of divergence of policy platforms from the policy preferred by the median voter. Parties are assumed to be pure vote-maximizers, and as shown, need to balance between catering to the median voter on the one hand and obtaining the support of partisans on the other hand. Voters decide according to the policy alternatives they observe. Parties may choose to send ambiguous messages to voters, in the case of which different voters observe different platforms and, therefore, form different perceptions about the implemented policy in the aftermath of the elections. We show that when the middle-of-the-road electorate is sufficiently divided with respect to its preferred policy, in equilibrium, parties indeed choose to send ambiguous messages and the median voter theorem fails to hold. Our analysis suggests that in this case, somewhat surprisingly, an increase in the number of partisans may lead to platform convergence in the sense of an increase in the fraction of voters who identify both parties with the policy alternative preferred by the median voter. In turn, this behavioral response may benefit the median voter. This holds true under two conditions. First, to begin with, the electorate is sufficiently divided such that full convergence does not occur (as plausible) and, second, the majority of the non-partisan voters is more inclined to the party that benefits from stronger partisanship. In such a case, our model predicts that the other party will become more moderate. It would be interesting for future research to examine whether this theoretical possibility is supported empirically.

Two final remarks are in order. First, the result that the favored party may not become more extreme is somewhat reminiscent of the intuitions underlying equilibria in spatial models with fixed valence advantages for one candidate, e.g., due to incumbency (Groseclose 2001; Aragones and Palfrey 2002). Considering a probalistic voting model in which candidates are politically motivated, Groseclose (2001) shows that there are two conflicting forces when the valence advantages for one candidate increases. First, the favored candidate wants to go in the direction of his/her preferred policy; ceteris paribus, this would lead to divergence across candidates' platforms. Second, moving towards the rival increases the chances

[16] In fact, all non-partisan voters may gain from greater partisanship. For instance, this is the case when $x_1 \geq \hat{x}$ and n_R rises.

of winning. For instance, if both candidates offer the same policy, the candidate with the valence advantage wins with certainty. The second incentive, associated with the egorent derived from winning the election, may potentially dominate the first one, which is associated with the candidate's political agenda. In contrast to our result, however, this combined effect typically leads to divergence. The reason is that the non-favored candidate has an incentive to move away from the median voter, to de-emphasize his/her valence disadvantage. What our paper shares with the relevant literature is the general lesson that strategic considerations may bear surprising implications for the behavior of parties and candidates in response to some advantage they may possess.

Second, our analysis also relates to the debate on campaign finance policy (see, e.g., Prat 2002; Coate 2004; Ashworth 2006). The literature emphasizes a trade-off between the beneficial role of campaign contributions in providing voters with information regarding candidates' pertinent attributes (such as positions and/or competence) and its distortion of policy away from the median voter's ideal. Banning contributions is warranted when the costs of policy distortion exceed the losses in terms of information, and vice versa.[17] Interpreting the seemingly ideological component in the parties' objective as driven by contributions coming from special interest groups, our analysis suggests that even when information gains are absent (or small), banning contributions may be undesirable for the median voter due to strategic considerations.

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Appendix A: Proof of Proposition 1

The result is proven in five steps. Steps 1–4 characterize the equilibrium of game Γ when the policy space $\mathcal{P}$ would be given by the interval $[\bar{P}_L, \bar{P}_R] \subset \mathbb{R}$ rather than by a grid. Step 5 then shows that there exists a sufficiently fine grid which preserves the claims in step 1–4 and ensures that there always exists an equilibrium.

Step 1. In the first step, we show that, when the policy space is continuous, strategy pairs other than $P_j = \bar{P}_j$ for $j = L, R$ or $P_L = P_R \in \{P_1^*, P_2^*\}$ cannot be Nash equilibria in pure strategies of game Γ.

To confirm this claim, first consider the behavior of party L in response to $P_R \in (P_1^*, \bar{P}_R]$. Note that it may be optimal to set $P_L = \bar{P}_L$ (e.g., when n_L is high). Also note that, if $x_i > 0$, setting P_L slightly above $P_i^{\min}(P_R)$ is always preferred to $P_L = P_i^{\min}(P_R)$, $i = 1, 2$. To see the latter, note that choosing P_L slightly above $P_i^{\min}(P_R)$ attracts at least a mass x_i of middle-of-the-road voters, whereas setting $P_L = P_i^{\min}(P_R)$ attracts only $0.5x_i$ of type i voters, according to (4). The utility loss from the ideology motive when deviating slightly from $P_i^{\min}(P_R)$, however, is marginal (by continuity of $F_L(\cdot)$). But since policy space $\mathcal{P}$ is continuous, if choosing P_L slightly above $P_i^{\min}(P_R)$ yields a higher payoff for party L than when choosing $\bar{P}_L$, then there does not exist a best response to $P_R \in (P_1^*, \bar{P}_R]$. By an analogous argument, if $P_R \in [\bar{P}_L, P_2^*)$, then $P_R = \bar{P}_R$ is the only candidate for a best response of party R. This implies that $P_j = \bar{P}_j$, $j = L, R$, may be an equilibrium, but no

[17] For recent empirical evidence on the role of campaign finance and contribution limits in shaping political outcomes, see e.g., Stratmann (2005, 2006), Stratmann and Aparicio-Castillo (2006), Houser and Stratmann (2008).

strategy pair such that $P_R \in (P_1^*, \bar{P}_R)$ or $P_R \in (\bar{P}_L, P_2^*)$. Second, if $P_R = P_1^*$, the optimal response of party L may be P_1^* or $\bar{P}_L$. Given $P_L = \bar{P}_L$, we have already seen that $P_R = \bar{P}_R$ is the only candidate for a best response of party R. Also note that by a similar argument as used above, setting P_L slightly above $P_2^{\min}(P_1^*)$ is always preferred to $P_L = P_2^{\min}(P_1^*)$ if $x_2 > 0$. (If $x_2 = 0$, party L cannot gain from deviating from P_1^* or $\bar{P}_L$, respectively, in response to $P_R = P_1^*$.) Thus, if $P_R = P_1^*$, then no other strategy than $P_L = P_1^*$ can be part of an equilibrium. The same holds vice versa. Similarly, if $P_R = P_2^*$, then no other strategy than $P_L = P_2^*$ can be part of an equilibrium, and vice versa. This concludes step 1.

Step 2. In the second step we show that in a pure strategy Nash equilibrium of game Γ such that $P_L = P_R = P_i^*$, Assumption A1 and $x_i > 0.5$, $i = 1, 2$, must hold.

To see this, we first show that in a pure strategy Nash equilibrium with $P_L = P_R = P_i^*$, we have $x_i > 0.5$, $i = 1, 2$. For instance, suppose to the contrary that $P_L = P_R = P_1^*$ is an equilibrium and $x_1 \leq 0.5$. Now consider a deviation of party L to $P_L \in (P_2^*, P_1^*)$. In this case, party L would gain a mass $x_2 - 0.5 = 0.5 - x_1$ of middle-of-the-road voters, in addition to a utility gain from the ideology motive. Thus, if $x_1 \leq 0.5$, both parties setting P_1^* cannot form an equilibrium. Analogously, $P_L = P_R = P_2^*$ cannot be an equilibrium if $x_2 = 1 - x_1 \leq 0.5$. Next, suppose again that $P_L = P_R = P_1^*$. If party R deviates by moving to $\bar{P}_R$, it gains $n_R - ([1 - F_R(\bar{P}_R - P_1^*)]n_R + 0.5)$. Thus, if $F_R(\bar{P}_R - P_1^*)n_R > 0.5$ it would be profitable to do so. Analogously, if party L moves to $\bar{P}_L$, it at least gains $n_L - ([1 - F_L(P_1^* - \bar{P}_L)]n_L + 0.5)$. Thus, conditions $F_R(\bar{P}_R - P_1^*)n_R \leq 0.5$ and $F_L(P_1^* - \bar{P}_L)n_L \leq 0.5$ are both necessary for $P_L = P_R = P_1^*$ to be an equilibrium. In an analogous way, it is easy to see that both $F_R(\bar{P}_R - P_2^*)n_R \leq 0.5$ and $F_L(P_2^* - \bar{P}_L)n_L \leq 0.5$ are necessary for $P_L = P_R = P_2^*$ to be an equilibrium. In sum, Assumption A1 must hold. This concludes step 2.

Step 3. We next show that, given x_1, any Nash equilibrium in pure strategies is unique.

To confirm this claim, first, recall from Step 1 that $P_j = \bar{P}_j$ for $j = L, R$ and $P_L = P_R \in \{P_1^*, P_2^*\}$ are the only candidates for Nash equilibria in pure strategies. The claim in step 3 is proven by distinguishing all possible scenarios regarding the relationship of voters' utility at different policies $P \in \{\bar{P}_L, \bar{P}_R\}$.

Scenario 1: First, suppose that type 1 individuals are strictly better off under platform $\bar{P}_R$ than under $\bar{P}_L$ and type 2 voters are strictly better off under platform $\bar{P}_L$ than under $\bar{P}_R$; that is,

$$u_1(\bar{P}_L) < u_1(\bar{P}_R) \quad \text{and} \quad u_2(\bar{P}_L) > u_2(\bar{P}_R). \tag{A.1}$$

Then both $P_1^{\min}(\bar{P}_R) > \bar{P}_L = P_2^{\min}(\bar{P}_R)$ and $P_2^{\max}(\bar{P}_L) < \bar{P}_R = P_1^{\max}(\bar{P}_L)$ hold. Now suppose $P_L = \bar{P}_L$ and $P_R = \bar{P}_R$. Then party L attracts a fraction $x_2 = 1 - x_1$ of the electorate, whereas party R attracts the remaining fraction x_1. Now, for instance, if party L deviates by proposing a platform slightly above $P_1^{\min}(\bar{P}_R)$ (given $P_R = \bar{P}_R$) it attracts all middle-of-the-road voters. Thus, $\bar{P}_L$ is the (unique) optimal response to $\bar{P}_R$ if and only if $n_L + x_2 \geq [1 - F_L(P_1^{\min}(\bar{P}_R) - \bar{P}_L)]n_L + 1$, which is equivalent to $F_L(P_1^{\min}(\bar{P}_R) - \bar{P}_L)n_L \geq 1 - x_2 = x_1$. Similarly, for party R, $\bar{P}_R$ is the (unique) optimal response to $\bar{P}_L$ if and only if $F_R(\bar{P}_R - P_2^{\max}(\bar{P}_L))n_R \geq 1 - x_1 = x_2$. Now note that $P_1^{\min}(\bar{P}_R) < P_1^*$ and $P_2^{\max}(\bar{P}_L) > P_2^*$. Thus, if $P_L = P_R = P_1^*$ is a Nash equilibrium, i.e., $x_1 > 0.5$ and $F_L(P_1^* - \bar{P}_L)n_L \leq 0.5$, according to step 2, it is impossible that $F_L(P_1^{\min}(\bar{P}_R) - \bar{P}_L)n_L \geq x_1 > 0.5$ holds at the same time. Similarly, if $P_L = P_R = P_2^*$ in Nash equilibrium (such that $x_2 > 0.5$ and $F_R(\bar{P}_R - P_2^*)n_R \leq 0.5$, according to step 2) it is impossible that $F_R(\bar{P}_R - P_2^{\max}(\bar{P}_L))n_R \geq x_2 > 0.5$ holds at the same time. However, if $P_j = \bar{P}_j$ for $j = L, R$ is a Nash equilibrium for some x_1, then either $F_L(P_1^{\min}(\bar{P}_R) - \bar{P}_L)n_L \geq 0.5$ or $F_R(\bar{P}_R - P_2^{\max}(\bar{P}_L))n_R \geq 0.5$ must hold, with strict inequality if $x_1 \neq 0.5$. Thus, for any x_1 such that $P_j = \bar{P}_j$ for $j = L, R$ is a Nash equilibrium, at least one necessary condition for $P_L = P_R = P_i^*$ to be an equilibrium is violated, $i = 1, 2$. One can show that analogous arguments apply if $u_1(\bar{P}_L) > u_1(\bar{P}_R)$ and $u_2(\bar{P}_L) < u_2(\bar{P}_R)$.

Scenario 2: Next suppose $u_i(\bar{P}_L) < u_i(\bar{P}_R)$ for $i = 1, 2$. That is, if $P_L = \bar{P}_L$ and $P_R = \bar{P}_R$, then party L gets payoff n_L, whereas party R gets $n_R + 1$. (Again, the following arguments apply in a similar way to the opposite case in which $u_i(\bar{P}_L) > u_i(\bar{P}_R)$ for $i = 1, 2$.) Now, given $P_R = \bar{P}_R$, party L can attract at least a mass x_i of middle-of-the-road voters by choosing P_L slightly above $P_i^{\min}(\bar{P}_R)$. Thus, for $P_L = \bar{P}_L$ being the optimal response to $P_R = \bar{P}_R$, it is necessary that $n_L \geq [1 - F_L(P_i^{\min}(\bar{P}_R) - \bar{P}_L)]n_L + x_i$ for $i = 1, 2$. That is, $F_L(P_1^{\min}(\bar{P}_R) - \bar{P}_L)n_L \geq x_1$ and $F_L(P_2^{\min}(\bar{P}_R) - \bar{P}_L)n_L \geq 1 - x_1$ must hold *simultaneously*. Now note that $P_1^{\min}(\bar{P}_R) < P_1^*$ and $P_2^{\min}(\bar{P}_R) < P_2^*$. Thus, if Assumption A1 holds and $P_L = P_R = P_i^*$ is a Nash equilibrium for some $x_i (> 0.5)$, $i = 1, 2$, it is impossible that $P_j = \bar{P}_j$ for $j = L, R$ is a Nash equilibrium at the same time. Vice versa, if $P_j = \bar{P}_j$ for $j = L, R$ is a Nash equilibrium for some x_1, then either $F_L(P_1^{\min}(\bar{P}_R) - \bar{P}_L)n_L \geq 0.5$ or $F_L(P_2^{\min}(\bar{P}_L) - \bar{P}_L)n_L \geq 0.5$ must hold, with strict inequality if $x_1 \neq 0.5$. Thus, for any x_1 such that $P_j = \bar{P}_j$ for $j = L, R$ is a Nash equilibrium, at least one necessary condition for $P_L = P_R = P_i^*$ to be an equilibrium is violated, $i = 1, 2$, according to step 2.

Scenario 3: Finally, consider the case in which $u_i(\bar{P}_L) = u_i(\bar{P}_R)$ for at least one $i = 1, 2$. In this case, given $P_R = \bar{P}_R$, a slight deviation of party L from $\bar{P}_L$ yields a gain of at least a mass $0.5x_i$ of voters, whereas the loss of partisan votes is marginal by continuity of F_L. Thus, if $u(\bar{P}_L, S_i) = u(\bar{P}_R, S_i)$ for all $i = 1, 2$, the strategy pair $P_j = \bar{P}_j$, $j = L, R$, cannot be an equilibrium. If, say, $u_1(\bar{P}_L) = u_1(\bar{P}_R)$ and $u_2(\bar{P}_L) < u_2(\bar{P}_R)$, for $\bar{P}_L$ being the optimal response to $P_R = \bar{P}_R$, it is necessary that both $x_1 = 0$ and $F_L(P_2^{\min}(\bar{P}_R) - \bar{P}_L)n_L \geq 1 - 0.5x_1$ simultaneously hold. Thus, $x_1 = 0$ and $F_L(P_2^{\min}(\bar{P}_R) - \bar{P}_L)n_L \geq 1$ must hold. In this case, however, neither $P_L = P_R = P_1^*$ nor $P_L = P_R = P_2^*$ can be a Nash equilibrium, since necessary condition $x_1 > 0.5$ is violated for the former and $F_L(P_2^* - \bar{P}_L)n_L \leq 0.5$ is violated for the latter (recall $P_2^* > P_2^{\min}(\bar{P}_R)$). Similar arguments hold whenever $u_i(\bar{P}_L) = u_i(\bar{P}_R)$ for one $i = 1, 2$. This concludes step 3.

Step 4. Next, we show that when the policy space is continuous and Assumption A1 holds, then $P_L = P_R = P_1^*$ is a unique Nash equilibrium in pure strategies of game Γ if $x_1 \geq \bar{x}(n_L, \bar{P}_L, P_1^*)$, $P_L = P_R = P_2^*$ is a unique Nash equilibrium in pure strategies if $x_1 \leq \underline{x}(n_R, \bar{P}_R, P_2^*)$, and no pure-strategy equilibrium exists if $x_1 \in (\underline{x}(n_R, \bar{P}_R, P_2^*), \bar{x}(n_L, \bar{P}_L, P_1^*))$.

To confirm this claim, first, suppose $P_L = P_R = P_1^*$ and $x_1 > 0.5$. (Recall from step 2 that $x_1 > 0.5$ is necessary for $P_L = P_R = P_1^*$ to be an equilibrium.) For party R, any deviation to the left of P_1^* is not beneficial because it deviates further from its ideal point and loses (at least) a mass $0.5 - x_2 = x_1 - 0.5 > 0$ of voters. Similarly, any deviation of party L to the right of P_1^* is not beneficial. Now let us consider three other possible scenarios for deviating behavior from $P_L = P_R = P_1^*$, starting with party R. If party R moves to the right of P_1^*, it loses all voters, i.e., the best is to go to $\bar{P}_R$. We already know from the proof of step 2 that this does not pay if $F_R(\bar{P}_R - P_1^*)n_R < 0.5$, which is implied by presumption $F_R(\bar{P}_R - P_2^*)n_R \leq 0.5$ since $P_2^* < P_1^*$. Now, we turn to party L. Consider first a deviation of party L to the left of $P_2^{\min}(P_1^*)$. (Note that $P_2^{\min}(P_1^*) < P_1^*$.) Since this implies a loss of all middle-of-the-road voters, the best is to go to $\bar{P}_L$. We already know from the proof of step 2 that this does not raise the payoff of party L if $F_L(P_1^* - \bar{P}_L)n_L \leq 0.5$, as presumed. Finally, consider the case in which party L deviates to a point $P_L \in (P_2^{\min}(P_1^*), P_1^*)$. In this case party L will get support from exactly a mass $x_2 = 1 - x_1$ of the middle-of-the-road electorate. Since the best is to go as far to the left as possible while retaining these voters, P_L is set slightly above $P_2^{\min}(P_1^*)$. This will *not* raise the payoff of party L if and only if $0.5 + [1 - F_L(P_1^* - \bar{P}_L)]n_L \geq 1 - x_1 + [1 - F_L(P_2^{\min}(P_1^*) - \bar{P}_L)]n_L$, which is equivalent to

$x_1 \geq \bar{x}(n_L, \bar{P}_L, P_1^*) \in (0.5, 1)$. Observing the uniqueness result in step 3, this confirms that, when the policy space is continuous, $P_L = P_R = P_1^*$ is a unique Nash equilibrium in pure strategies if $x_1 \geq \bar{x}$ holds.

Next, suppose $P_L = P_R = P_2^*$ and $x_2 > 0.5$, i.e., $x_1 < 0.5$. For similar reasons as above, any deviation of party L to the right of P_2^* and any deviation of party R to the left of P_2^* is not profitable. Moreover, analogously to the previous case, it is easy to show that, by presumption, it does not pay for party L to deviate in any other way. For party R, any deviation to the right of $P_1^{\max}(P_2^*)$ is equally unprofitable. (Note that $P_1^{\max}(P_2^*) > P_2^*$.) Finally, consider the remaining deviation for party R, i.e., $P_R \in (P_2^*, P_1^{\max}(P_2^*))$.) Setting P_R slightly below $P_1^{\max}(P_2^*)$ does not raise the payoff for party R if and only if $0.5 + [1 - F_R(\bar{P}_R - P_2^*)]n_R \geq x_1 + [1 - F_R(\bar{P}_R - P_1^{\max}(P_2^*))]n_R$, which is equivalent to $x_1 \leq \underline{x}(n_R, \bar{P}_R, P_2^*) \in (0, 0.5)$. Using step 3, this confirms that, when the policy space is continuous, $P_L = P_R = P_2^*$ is a unique Nash equilibrium in pure strategies if $x_1 \leq \underline{x}(n_R, \bar{P}_R, P_2^*)$ holds.

To show that no equilibrium in pure strategies of game Γ exists when $x_1 \in (\underline{x}, \bar{x})$, first, note that Assumption A1 implies

$$F_L(P_1^{\min}(\bar{P}_R) - \bar{P}_L)n_L < 0.5 \quad \text{and} \quad F_R(\bar{P}_R - P_2^{\max}(\bar{P}_L))n_R < 0.5 \tag{A.2}$$

since $P_1^{\min}(\bar{P}_R) < P_1^*$ and $P_2^{\max}(\bar{P}_L) > P_2^*$, respectively; moreover,

$$F_L(P_2^{\min}(\bar{P}_R) - \bar{P}_L)n_L < 0.5 \quad \text{and} \quad F_R(\bar{P}_R - P_1^{\max}(\bar{P}_L))n_R < 0.5 \tag{A.3}$$

since $P_2^{\min}(\bar{P}_R) < P_2^* < P_1^*$ and $P_1^{\max}(\bar{P}_L) > P_1^* > P_2^*$, respectively. Now recall from step 1 together with the previous line of reasoning to confirm step 4 that, if $x_1 \in (\underline{x}, \bar{x})$, the only candidate for a Nash equilibrium in pure strategies is $P_j = \bar{P}_j$, $j = L, R$. Moreover, recall from the line of reasoning to confirm step 3 that for $P_j = \bar{P}_j$, $j = L, R$, to be a Nash equilibrium, $F_L(P_i^{\min}(\bar{P}_R) - \bar{P}_L)n_L \geq 0.5$ or $F_R(\bar{P}_R - P_i^{\max}(\bar{P}_L))n_R \geq 0.5$ must hold for at least one $i = 1, 2$. However, it is impossible that these conditions hold if both (A.2) and (A.3) are fulfilled. This concludes step 4.

Step 5. Note that, according to standard existence theorems of Nash equilibrium, there exists a Nash equilibrium in mixed strategies of game Γ when the strategy space is finite. Hence, the final step is to show that the result in step 4 also holds when the policy space is a sufficiently fine grid rather than continuous.

Consider first the scenario described by part (i) of Proposition 1; that is, when $x_1 \geq \bar{x}$. Obviously, any Nash equilibrium in the continuum case is also a Nash equilibrium with a grid. However, we need to ensure that no other Nash equilibrium than (P_1^*, P_1^*) exists. For any pair (P_L, P_R), we define the following two sets, for party L and R, respectively, that describe the payoff derived by each party for any possible strategy:

$$\Pi_L(P_R) \equiv \{\pi_L(P, P_R) \mid P \in [\bar{P}_L, \bar{P}_R]\}, \tag{A.4a}$$

$$\Pi_R(P_L) \equiv \{\pi_R(P_L, P) \mid P \in [\bar{P}_L, \bar{P}_R]\}. \tag{A.4b}$$

(Recall that $\pi_j(P_L, P_R)$ is the payoff for party $j = L, R$ for a given pair of strategies (P_L, P_R).)

Let $\bar{\Pi}_L(P_R) \equiv \sup[\Pi_L(P_R)]$ and $\bar{\Pi}_R(P_L) \equiv \sup[\Pi_R(P_L)]$ denote, correspondingly, the least upper-bounds associated with the two sets. It is easy to verify using our earlier notation that the following holds:

$$\bar{\Pi}_L(P_R) \in \left\{\pi_L(\bar{P}_L, P_R), \pi_L(P_2^{\min}(P_R), P_R) + \frac{x_2}{2}, \pi_L(P_1^{\min}(P_R), P_R) + \frac{x_1}{2}\right\}, \quad \text{(A.5a)}$$

$$\bar{\Pi}_R(P_L) \in \left\{\pi_R(P_L, \bar{P}_R), \pi_R(P_L, P_2^{\max}(P_L)) + \frac{x_2}{2}, \pi_R(P_L, P_1^{\max}(P_L)) + \frac{x_1}{2}\right\}. \quad \text{(A.5b)}$$

Let $H_L(P_L, P_R) \equiv \bar{\Pi}_L(P_R) - \pi_L(P_L, P_R)$ and $H_R(P_L, P_R) \equiv \bar{\Pi}_R(P_L) - \pi_R(P_L, P_R)$ denote the upper-bound gains of deviating from P_L and P_R for party L and R, respectively, and let $\bar{H}(P_L, P_R) \equiv \max[H_L(P_L, P_R), H_R(P_L, P_R)]$. We need to show that any $(P_L, P_R) \neq (P_1^*, P_1^*)$ does not form a Nash equilibrium for a sufficiently fine grid. By construction, $\bar{H}(P_L, P_R) > 0$ for $(P_L, P_R) \neq (P_1^*, P_1^*)$. We will separate now between two cases.

Case 1: Consider first the case in which either $\bar{H}(P_L, P_R) = H_L(P_L, P_R)$ and $\bar{\Pi}_L(P_R) = \pi_L(\bar{P}_L, P_R)$, or $\bar{H}(P_L, P_R) = H_R(P_L, P_R)$ and $\bar{\Pi}_R(P_L) = \pi_R(P_L, \bar{P}_R)$. In such a case, it is easy to verify, as $\bar{P}_L$ and $\bar{P}_R$ are part of the grid, that this does not form equilibrium. We turn next to the other, more complicated case.

Case 2: The other possible scenarios can be described as a union of two sets (defined for party L and R, respectively). Let $\Theta = \Theta_L \cup \Theta_R$, where

$$\Theta_L(P_L, P_R) \equiv \left\{\begin{array}{c} H_L(P_L, P_R) | (P_L, P_R) \neq (P_1^*, P_1^*) \\ H_L(P_L, P_R) = \bar{H}(P_L, P_R) \ \wedge \ \bar{\Pi}_L(P_R) \neq \pi_L(\bar{P}_L, P_R) \end{array}\right\}, \quad \text{(A.6a)}$$

$$\Theta_R(P_L, P_R) \equiv \left\{\begin{array}{c} H_R(P_L, P_R) | (P_L, P_R) \neq (P_1^*, P_1^*). \\ H_R(P_L, P_R) = \bar{H}(P_L, P_R) \ \wedge \ \bar{\Pi}_R(P_L) \neq \pi_R(P_L, \bar{P}_R) \end{array}\right\}. \quad \text{(A.6b)}$$

Denote by $\underline{\Theta} \equiv \inf[\Theta]$ the largest lower bound of the set Θ. By construction $\Theta \geq 0$. We turn next to prove that $\underline{\Theta} > 0$. Assume by negation that $\underline{\Theta} = 0$. This implies that, for any $\varepsilon > 0$, there exists a pair (P_L, P_R) such that $\bar{H}(P_L, P_R) \in \Theta$ and $\bar{H}(P_L, P_R) \in (0, \varepsilon)$. Consider an arbitrary small $\varepsilon > 0$, and without loss in generality, let $\bar{H}(P_L, P_R) = H_L(P_L, P_R) = \varepsilon' < \varepsilon$. If ε is small, the gain from deviation necessarily derives from increased turnout of partisans, thus P_L necessarily lies in a small neighborhood to the right of either $P_2^{\min}(P_R)$ or $P_1^{\min}(P_R)$. First, assume the latter, namely, that party L sets its policy slightly above the point at which type 1 voters are just indifferent between the two parties. Thus, $\varepsilon' = [F_L(P_L - \bar{P}_L) - F_L(P_1^{\min}(P_R) - \bar{P}_L)]n_L$. Denoting by $f_j(\cdot)$ the derivative of $F_j(\cdot)$, $j = L, R$, this can be rewritten as $\varepsilon' = n_L \int_{P_1^{\min}(P_R)}^{P_L} f_L(P - \bar{P}_L)dP$. Hence, $\varepsilon' \geq n_L[P_L - P_1^{\min}(P_R)]f_L^{\min}$, where $f_L^{\min} \equiv \min_{P \in [\bar{P}_L, P_R]} f_L(P)$,[18] implying $\chi \equiv P_L - P_1^{\min}(P_R) \leq \varepsilon'/[n_L f_L^{\min}]$. The infimum distance (from P_R) that party R has to shift its policy in order to attract the type 1 voters (of measure x_1) is given by $P_R - P_1^{\max}(P_L)$. This could be either positive or negative. We first look at the case when $P_R > P_1^{\max}(P_L)$. By substitution $P_R - P_1^{\max}(P_L)$ may be rewritten as $Q(\chi) \equiv P_R - P_1^{\max}(\chi + P_1^{\min}(P_R))$. It is easy to verify that $Q(0) = 0$. Furthermore, $Q(\cdot)$ is increasing and continuous. Thus, $Q(P_L - P_1^{\min}(P_R)) \leq Q(\varepsilon'/[n_L f_L^{\min}])$. By construction, it follows that $H_R(P_L, P_R) \geq x_1 - n_R \int_{P_R - Q(P_L - P_1^{\min}(P_R))}^{P_R} f_R(\bar{P}_R - P_R)dP$. However, using the fact that $Q(P_L - P_1^{\min}(P_R)) \leq Q(\varepsilon'/[n_L f_L^{\min}])$ and defining $f_j^{\max} \equiv \max_{P \in [\bar{P}_L, P_R]} f_j(P)$, $j = L, R$, we obtain

[18] Note that $f_L^{\min}$ is well defined by the continuity of f_L (recall that F_L is assumed to be continuously differentiable).

$$\varepsilon' = H_L(P_L, P_R) \geq H_R(P_L, P_R) \geq x_1 - Q\left(\frac{\varepsilon'}{n_L f_L^{\min}}\right) n_R f_R^{\max}. \tag{A.7}$$

For $\varepsilon' = 0$, as $Q(0) = 0$ and $x_1 \geq \bar{x}$, this inequality is violated, i.e., $H_L(P_L, P_R) < H_R(P_L, P_R)$. By virtue of continuity, this holds for sufficiently small $\varepsilon' > 0$. This establishes that $\underline{\Theta} > 0$ by contradiction. For the case where $P_R < P_1^{\max}(P_L)$ it immediately follows that by attracting the type 1 voters party R gains from increased turnout of partisans as well. Thus, $H_R(P_L, P_R) > x_1 \geq \bar{x}$. The same line of reasoning applies to the case where party L deviates to a point slightly to the right of $P_2^{\min}(P_R)$ as x_2 is bounded away from zero by assumption.

Now define $d_i \equiv \underline{\Theta}/\max(f_L^{\max} n_L, f_R^{\max} n_R)$ and consider a grid where the distance between any two adjacent points is lower than d_i. It follows by construction that for any point in a grid other than (P_1^*, P_1^*) at least one of the parties can deviate and profit. Thus, the only equilibrium is (P_1^*, P_1^*). We can repeat the same argument for the other two scenarios (described by parts (ii) and (iii) of Proposition 1) and define correspondingly d_{ii} and d_{iii}. We further let $d = \min(d_i, d_{ii}, d_{iii})$. Note that $d > 0$. We conclude that any grid including the bliss points (P_i^*, $i = 1, 2$) and the end points ($\bar{P}_L$ and $\bar{P}_R$), such that the distance between two adjacent points is lower than d, maintains the results claimed in step 4. This completes the proof of Proposition 1.

Appendix B: Proof of Lemma 1

First, note that restriction $n_j \in (1/2, 2/3)$, $j = L, R$ (part (iii) of Assumption A2) implies $\tilde{x} = 3n_L/4 \in (3/8, 1/2)$ and $\hat{x} = 1 - 3n_R/4 \in (1/2, 5/8)$, i.e., $\tilde{x} < 1/2 < \tilde{x}$. We now start by proving that the mixed strategy profile of game Γ in part (ii) forms an equilibrium when $x_1 \in (\tilde{x}, \hat{x})$ and then establish uniqueness for this range. We will then follow a similar procedure for parameter ranges $x_1 \geq \hat{x}$ (part (i)) and $x_1 \leq \tilde{x}$ (part (iii)).

Part (ii): Suppose there exists an equilibrium of game Γ such that both parties mix over P_1^*, P_2^* and their partisans' respective ideal point with positive probability. According to Table 1, in this case (l_1^*, l_2^*) solve

$$\pi_{R,1}(l_1, l_2, n_R, x_1) = \pi_{R,2}(l_1, l_2, n_R, x_1) = \pi_{R,3}(l_1, l_2, n_R, x_1), \tag{B.1}$$

where

$$\pi_{R,1}(l_1, l_2, n_R, x_1) \equiv l_1\left(\frac{n_R}{2} + \frac{1}{2}\right) + l_2\left(\frac{n_R}{2} + x_1\right) + (1 - l_1 - l_2)\left(\frac{n_R}{2} + \frac{1 + x_1}{2}\right), \tag{B.2}$$

$$\pi_{R,2}(l_1, l_2, n_R, x_1) \equiv l_1\left(\frac{n_R}{4} + 1 - x_1\right) + l_2\left(\frac{n_R}{4} + \frac{1}{2}\right) + (1 - l_1 - l_2)\left(\frac{n_R}{4} + 1\right), \tag{B.3}$$

$$\pi_{R,3}(l_1, l_2, n_R, x_1) \equiv l_1 n_R + l_2\left(n_R + \frac{x_1}{2}\right) + (1 - l_1 - l_2)(n_R + x_1) \tag{B.4}$$

are expected payoffs of party R when playing P_1^*, P_2^*, $\bar{P}_R$, respectively, given that party L mixes with $l_1, l_2, l_3 = 1 - l_1 - l_2$. Similarly, (r_1^*, r_2^*) solve

$$\pi_{L,1}(r_1, r_2, n_L, x_1) = \pi_{L,2}(r_1, r_2, n_L, x_1) = \pi_{L,3}(r_1, r_2, n_L, x_1), \tag{B.5}$$

where

$$\pi_{L,1}(r_1, r_2, n_L, x_1) \equiv r_1\left(\frac{n_L}{4} + \frac{1}{2}\right) + r_2\left(\frac{n_L}{4} + x_1\right) + (1 - r_1 - r_2)\left(\frac{n_L}{4} + 1\right), \tag{B.6}$$

$$\begin{aligned}\pi_{L,2}(r_1, r_2, n_L, x_1) \equiv{}& r_1\left(\frac{n_L}{2} + 1 - x_1\right) + r_2\left(\frac{n_L}{2} + \frac{1}{2}\right) \\ &+ (1 - r_1 - r_2)\left(\frac{n_L}{2} + 1 - \frac{x_1}{2}\right),\end{aligned} \tag{B.7}$$

$$\pi_{L,3}(r_1, r_2, n_L, x_1) \equiv r_1\left(n_L + \frac{1 - x_1}{2}\right) + r_2 n_L + (1 - r_1 - r_2)(n_L + 1 - x_1) \tag{B.8}$$

are expected payoffs of party L when playing P_1^*, P_2^*, $\bar{P}_L$, respectively, given that party R mixes with r_1, r_2, $r_3 = 1 - r_1 - r_2$. It is straightforward to show that $r_1 = g(x_1, n_L)$, $r_2 = z(x_1, n_L)$ solve (B.5) and $l_1 = z(1 - x_1, n_R)$, $l_2 = g(1 - x_1, n_R)$ solve (B.1). To see that this indeed forms an equilibrium if $x_1 \in (\tilde{x}, \hat{x})$, note first that $g(x_1, n_L) > 0$ iff $x_1 > \tilde{x}$ and $g(1 - x_1, n_R) > 0$ iff $x_1 < \hat{x}$. Thus, $r_1 > 0$ and $l_2 > 0$. Moreover, note that $z(x_1, n_L) > 0$ for $n_L \in (1/2, 2/3)$ if

$$\tilde{z}(x, n) \equiv 6x^2 - 4x - 4xn + 3n > 0 \tag{B.9}$$

for $n \in (1/2, 2/3)$. Since $x = (1 + n)/3$ minimizes $\tilde{z}(x, n)$ and $\tilde{z}((1 + n)/3, n) = (2n - 1)(2 - n) > 0$, we indeed have $\tilde{z}(x, n) > 0$ for $n \in (1/2, 2/3)$. Analogously, $z(1 - x_1, n_R) = z(x_2, n_R) > 0$ for all $x_2 = 1 - x_1$ when $n_R \in (1/2, 2/3)$, as presumed by part (iii) of Assumption A2. Thus, also $r_2 > 0$ and $l_1 > 0$. Finally, note that

$$r_1 + r_2 = g(x_1, n_L) + z(x_1, n_L) = \frac{2x_1 - n_L}{2(1 - x_1)} < 1 \text{ if } x_1 < \frac{1}{2} + \frac{n_L}{4} \equiv \overline{\overline{x}}, \tag{B.10}$$

$$\begin{aligned}l_1 + l_2 = z(1 - x_1, n_R) + g(1 - x_1, n_R) = \frac{2(1 - x_1) - n_R}{2x_1} < 1 \\ \text{if } x_1 > \frac{1}{2} - \frac{n_R}{4} \equiv \underline{\underline{x}}.\end{aligned} \tag{B.11}$$

Since $\overline{\overline{x}} \in (5/8, 2/3)$ and $\underline{\underline{x}} \in (1/3, 3/8)$, according to part (iii) of Assumption A2, we have $\hat{x} < \overline{\overline{x}}$ and $\tilde{x} > \underline{\underline{x}}$. Hence, also $r_1 + r_2 < 1$ and $l_1 + l_2 < 1$. This confirms that part (ii) of Lemma 1 characterizes an equilibrium.

To confirm that this equilibrium is unique, it remains to be shown that no equilibrium exists in which party L mixes between two policies in the set $\{P_1^*, P_2^*, \bar{P}_L\}$ with positive probability and assigns zero probability to the remaining policy and party R assigns positive probability to two policies within the set $\{P_1^*, P_2^*, \bar{P}_R\}$ and zero probability to the remaining policy. In such a situation, we could have the scenarios: (I) $l_1 = r_1 = 0$, (II) $l_1 = r_2 = 0$, (III) $l_1 = r_3 = 0$, (IV) $l_2 = r_1 = 0$, (V) $l_2 = r_2 = 0$, (VI) $l_2 = r_3 = 0$, (VII) $l_3 = r_1 = 0$, (VIII) $l_3 = r_2 = 0$ or (IX) $l_3 = r_3 = 0$. In the following, we will show that none of these nine scenarios can hold in equilibrium when $x_1 \in (\tilde{x}, \hat{x})$.

- First, suppose $l_2 = 0$. Thus, $\pi_{R,2} = -l_1 x_1 + n_R/4 + 1$ and $\pi_{R,3} = n_R + x_1 - l_1 x_1$, according to (B.3) and (B.4), respectively. This implies that $\pi_{R,2} > \pi_{R,3}$ if $x_1 < \hat{x}$, which holds by presumption. Hence, if $l_2 = 0$ in equilibrium, then also $r_2 > 0$, $r_3 = 0$ and thus $r_1 > 0$ in equilibrium. This rules out scenarios (IV) or (V) to hold in equilibrium.
- Next suppose $r_1 = 0$. Thus, $\pi_{L,1} = r_2 x_1 + n_L/4 + 1 - r_2$ and $\pi_{L,3} = n_L + 1 - x_1 + r_2 x_1 - r_2$, according to (B.6) and (B.8), respectively. This implies that $\pi_{L,1} > \pi_{L,3}$ if $x_1 > \tilde{x}$, as presumed. Hence, if $r_1 = 0$ in equilibrium, then also $l_1 > 0$, $l_3 = 0$ and thus $l_2 > 0$ in equilibrium. This rules out scenario (I) to hold in equilibrium (and, once again, scenario (IV)).
- Suppose now $l_3 = 1 - l_1 - l_2 = 0$. Thus, $\pi_{R,1} = l_1/2 + n_R/2 + x_1 - l_1 x_1$ and $\pi_{R,2} = l_1/2 - l_1 x_1 + n_R/4 + 1/2$, according to (B.2) and (B.3), respectively. This implies that $\pi_{R,1} > \pi_{R,2}$ if $x_1 > 1/2 - n_R/4 = \underline{\underline{x}}$, which holds by presumption since $\tilde{x} > \underline{\underline{x}}$. Thus, if $l_3 = 0$ in equilibrium, then also $r_1 > 0$, $r_2 = 0$ and thus $r_3 > 0$ in equilibrium. This rules out scenarios (VII) or (IX) to hold in equilibrium.
- Suppose $r_3 = 1 - r_1 - r_2 = 0$ next. Thus, $\pi_{L,1} = r_1/2 + n_L/4 + x_1 - r_1 x_1$ and $\pi_{L,2} = r_1/2 + n_L/2 + 1/2 - r_1 x_1$, according to (B.6) and (B.7), respectively. This implies that $\pi_{L,1} > \pi_{L,2}$ if $x_1 > \underline{x}$, which as we know holds by presumption $x_1 > \tilde{x}$. Hence, if $r_3 = 0$ in equilibrium, then also $l_1 > 0$, $l_2 = 0$ and thus $l_3 > 0$ in equilibrium. This rules out scenarios (III) and (IX) to hold in equilibrium.
- Now suppose $r_2 = 0$. Thus, $\pi_{L,1} = -r_1/2 + n_L/4 + 1$, $\pi_{L,2} = n_L/2 + 1 - x_1/2 - r_1 x_1/2$ and $\pi_{L,3} = n_L + 1 - x_1 - r_1/2 + r_1 x_1/2$.
  - First suppose $l_1 = r_2 = 0$ in equilibrium (scenario (II)). That is, party L must be indifferent between P_2^* and $\bar{P}_L$ in equilibrium, i.e., $\pi_{L,2} = \pi_{L,3}$, which implies $r_1 = (x_1 - n_L)/(2x_1 - 1)$. By supposing that $l_1 = 0$ in equilibrium, we must have $\pi_{L,1} \le \pi_{L,2}$ if $r_1 = (x_1 - n_L)/(2x_1 - 1)$. Since $\pi_{L,1} \le \pi_{L,2}$ implies $x_1 - n_L/2 \le r_1(1 - x_1)$, when $r_1 = (x_1 - n_L)/(2x_1 - 1)$, this condition is equivalent to $\tilde{z}(x_1, n_L) \le 0$. However, we have already confirmed that $\tilde{z}(x_1, n_L) > 0$. Hence, scenario (II) cannot hold in equilibrium.
  - Next suppose $l_3 = r_2 = 0$ in equilibrium (scenario (VIII)). That is, $\pi_{L,1} = \pi_{L,2}$, which implies
    $r_1 = (x_1 - n_L/2)/(1 - x_1)$. By supposing that $l_3 = 0$ in equilibrium, we must have $\pi_{L,3} \le \pi_{L,2}$ if $r_1 = (x_1 - n_L/2)/(1 - x_1)$. Since $\pi_{L,3} \le \pi_{L,2}$ implies $r_1(2x_1 - 1) \le x_1 - n_L$, when $r_1 = (x_1 - n_L)/(2x_1 - 1)$, this condition is again equivalent to $\tilde{z}(x_1, n_L) \le 0$. But since $\tilde{z}(x_1, n_L) > 0$, scenario (VIII) cannot hold in equilibrium.
- It remains to be shown that $l_2 = r_3 = 0$ cannot hold in equilibrium (scenario (VI)). Suppose the contrary. Thus, we must have $\pi_{L,2} \le \pi_{L,1} = \pi_{L,3}$ when $r_3 = 0$. From (B.6) and (B.8), it is easy to see that, when $r_3 = 0$, we have $\pi_{L,1} = \pi_{L,3}$ if $r_1 x_1 = 2(x_1 - 3n_L/4)$ and $\pi_{L,2} \le \pi_{L,3}$ if $r_1 x_1 \le 1 - n_L$. Thus, if $\pi_{L,2} \le \pi_{L,1} = \pi_{L,3}$ when $r_3 = 0$, then $x_1 \ge 1/2 + n_L/4 = \bar{\bar{x}}$. But since $x_1 < \hat{x}$ by presumption and we know $\hat{x} < \bar{\bar{x}}$, this is impossible. This also rules out scenario (VI) to hold in equilibrium.

Hence, the equilibrium in part (ii) is unique.

Part (i): Now consider the case $x_1 \ge 1 - 3n_R/4 = \hat{x}$. To prove existence of the equilibrium in part (i), first note that $\pi_{R,1}(l_1, 0, n_R, x_1) = \pi_{R,3}(l_1, 0, n_R, x_1)$ implies $l_1 = h(x_1, n_R)$ and $\pi_{L,1}(r_1, 0, n_L, x_1) = \pi_{L,3}(r_1, 0, n_L, x_1)$ implies $r_1 = g(x_1, n_L)$. Also note that $h(x_1, n_R) > 0$ if $x_1 > 1 - n_R$ which holds according to presumption $x_1 \ge \hat{x}$. Moreover, $h(x_1, n_R) < 1$ if $n_R < 1$ which holds in view of part (iii) of Assumption A2. That $g(x_1, n_L) > 0$ follows immediately from the fact that $x_1 \ge \hat{x}$ implies $x_1 > 3n_L/4 = \tilde{x}$ (recall $\hat{x} > \tilde{x}$). Furthermore, note that $g(x_1, n_L) < 1$ if $x_1 < 3n_L/2$ which holds since $x_1 \le b = 3/4$ and $3n_L/2 > 3/4$ (recall $n_L > 1/2$). For existence of equilibrium in which both $r_1 = g(x_1, n_L)$ and $l_1 = h(x_1, n_R)$ hold, it remains to be shown that $r_2 = l_2 = 0$ are consistent with an equilibrium under this mixture. To see this, note from Table 1 that for party R, P_2^* is dominated by $\bar{P}_R$ when $x_1 \geq \hat{x}$. Hence, $r_2^* = 0$ if $x_1 \geq \hat{x}$. Moreover, note that when $r_2 = 0$, $\pi_{L,1} > \pi_{L,2}$ if $x_1 - n_L/2 > r_1(1 - x_1)$. If $r_1 = g(x_1, n_L)$, this condition becomes $\tilde{z}(x_1, n_L) > 0$, which we know is true. Hence, indeed $l_2 = 0$ is indeed best response $r_1 = g(x_1, n_L)$ and $r_2 = 0$.

We now turn to show uniqueness of equilibrium when $x_1 \geq \hat{x}$. Since we already know that $r_1^* = g(x_1, n_L)$ and $r_2^* = 0$ in any equilibrium, it remains to be shown that no equilibrium with $l_2 > 0$ exists. Suppose the contrary, which means that an equilibrium with either $l_1 = 0$ or $l_3 = 0$ exists. However, $l_1 = 0$ cannot hold in equilibrium since we already know from the uniqueness proof for part (ii) that when $r_2 = 0$, we have $\pi_{L,1} > \pi_{L,2}$ if $\pi_{L,2} = \pi_{L,3}$. Similarly, $l_3 = 0$ cannot hold since we already know that when $r_2 = 0$, $\pi_{L,3} > \pi_{L,2}$ if $\pi_{L,1} = \pi_{L,2}$. This confirms uniqueness of the equilibrium in part (i).

Part (iii): Finally, consider the case $x_1 \leq 3n_L/4 = \tilde{x}$. Using $x_1 = 1 - x_2$, this implies $x_2 \geq 1 - 3n_L/4$. The situation is thus symmetric to part (i), as implied by Assumption A2. The result follows analogously.

This completes the proof of Lemma 1.

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