Socially Optimal Redistribution and Growth: A Further Warning on the Welfare Significance of Representative Consumers' Preferences

Socially Optimal Redistribution and Growth: A Further Warning on the Welfare Significance of Representative Consumers' Preferences

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In this paper, it is illustrated in a simple balanced growth model with redistributive capital income taxation that it is generally misleading to attribute welfare significance to the preferences of a representative consumer, if lump sum redistribution is unfeasible. This result holds even if a representative agent exists for all endowment distributions, i.e. even if there always exists an endowment distribution such that utility of the representative consumer has welfare significance for any social welfare function. Moreover, it can be concluded from our example that, in general, the net return to capital should be lower than the (social) marginal productivity of capital. (JEL: D11, D30, H20)

1. Introduction

In the (politico-economic) literature on the relationship between inequality and growth (e.g. Bertola, 1993; Alesina and Rodrik, 1994; Persson and Tabellini, 1994), the concept of a representative agent is often argued to play a particular role for evaluating "intertemporal efficiency" of redistribution policies[1]. The commonly assumed homothetic preferences in endogenous growth models imply the existence of a *representative consumer*. The preferences of this (fictional) agent are viewed as to "coincide with those of a social planner concerned only with *intertemporal efficiency*, in the sense of being indifferent to the distribution of consumption across individuals" (Bénabou, 1996, p. 22; italics original).

In this paper, in a world populated by agents differing in their wealth endowments, a representative consumer is said to exist if the following holds. Consider a fictional individual who is endowed with the aggregate wealth of

* I am grateful to Hartmut Egger, Josef Falkinger, Hannes Schellhorn, Martin Summer and two anonymous referees for helpful comments and suggestions.

1 In this literature, higher inequality (of capital endowments) leads to higher demands for redistribution through the political process, in turn depressing the economy's rate of growth.

FinanzArchiv 57(2000), 182–196. © 2000 Mohr Siebeck Verlag – ISSN 0015-2218

the economy. If his/her preference relation (or utility function, respectively) is such that his/her Walrasian demand function equals the aggregate demand function of the economy, then this agent is called representative. (See also Mas-Colell et al. (1995, ch. 4).) It has long been argued that generally such a preference relation may not exist and, even if it does, a (fictional) representative consumer may not represent the preferences of anybody in the economy. That is, a representative consumer may choose alternative 'A' instead of 'B' but every agent would prefer 'B' to 'A'. (See e.g. Kirman (1992) for further discussion.)

Rather than going into this debate, this paper argues that even if a representative consumer exists for *all* endowment distributions, such that there always exists an endowment distribution so that his/her preferences have welfare significance for *any* social welfare function one wishes to employ, this representative agent may nevertheless be meaningless for social welfare. This conclusion is drawn from a simple balanced growth model with a linear capital income tax and heterogeneous agents, who differ in their capital endowments. The tax revenue is redistributed to the individuals by uniform transfers. Due to the assumptions of infinite horizons and perfect capital markets, taxation of capital income depresses investment-driven growth (e.g. Rebelo, 1991). It is shown that although the economy's representative agent prefers a tax rate which implies that the net return to capital equals the (social) marginal productivity of capital, a higher tax rate (and thus slower growth) may be socially optimal[2]. In addition, capital income taxation cannot generally be viewed as inefficient. The reason for these results is simple but frequently neglected. In order to achieve a distribution of endowments such that utility of a representative consumer has welfare significance, one has to be engaged in *lump sum redistribution*. Concerning redistribution policies, however, if such a "first-best" policy were available, there would be no need to analyze the effects of any other redistribution policy in the first place. It may thus be very misleading to evaluate redistribution policies by looking at the preferences of the representative agent when informational or legislative constraints prevent the possibility of lump sum redistribution[3].

It is also argued that the critique in this paper of analyzing welfare effects of tax policies by looking at the preferences of a representative consumer

2 In our model, the social marginal productivity of capital equals the gross private marginal return to capital since there are no external effects of capital accumulation on the production technology. Thus, the capital tax rate that maximizes utility of a representative consumer equals zero. In contrast, Bertola (1993) analyzes a model with positive spill-over effects of investment. This implies that a capital subsidy is necessary to maximize utility of a representative consumer.

3 For illuminating discussions of such constraints to lump sum redistribution, see e.g. Stiglitz (1987) and Putterman et al. (1998).

(given that such an individual exists) is likely to carry over to models in which there is just one "representative" agent[4]. This is because the only justification to make the representative consumer assumption (i.e. to disregard distributional effects of taxation) in a normative analysis of taxation is that, even in a model with heterogeneous agents, the preferences of this consumer would have welfare significance in some conceptually meaningful sense.

Moreover, it should be noted that the arguments in this paper are not confined to growth models but apply to any standard Arrow-Debreu model (e.g. the Heckscher-Ohlin trade theory). The choice of illustrating the points made in this paper in a growth model is due to the fact that in (endogenous) growth theory homothetic preferences are a common assumption (in order to deal with steady states). Thus, a representative consumer in the above sense always exists in these models.

The paper is organized as follows. Section 2 sets up the model. In section 3, the equilibrium growth paths of all variables as well as the individually preferred tax rates are derived. Section 4 reviews the argument of the welfare significance of a representative consumer under the assumption of lump sum redistribution. Moreover, the socially optimal redistribution policy and its growth effects are discussed, given that lump sum redistribution is unfeasible. The last section concludes.

2. A Simple Growth Model

Consider a closed economy with perfect competition, populated by a large number n of infinitely living individuals. Time is continuous. Each individual i privately owns a capital endowment $k_0^i > 0$ at date zero. The total capital stock at this date is denoted by K_0. Capital is the only factor of production. The production technology for a homogenous consumption good exhibits constant returns to scale. Output Y at date t is produced by a representative firm according to

$$Y(t) = a K(t), \tag{1}$$

where $a > 0$ is a productivity parameter and $K(t)$ is the total capital stock at date t.

It is assumed that the income distribution cannot be changed in a lump-sum fashion due to informational or legislative constraints. Informational constraints in our context mean that policy makers only have very limited in-

4 Prominent normative studies of taxation under the representative consumer assumption are e.g. Chamley (1986) and Lucas (1990).

formation about individual asset holdings. For instance, there is no way of taxing innate wealth levels in a lump-sum manner by imposing differentiated poll taxes (and transfers)[5]. Legislative constraints are very severe in most developed countries as wealth is protected even in constitutions, i.e. governments cannot redistribute capital endowments directly[6].

Let there be a proportional capital income tax with a time-invariant tax rate $\tau \in [0, \tau^{\max}]$. ($\tau^{\max} < 1$ is defined as the maximal tax rate such that growth is non-negative and is derived in the next section.) Using (1), the after-tax return to capital r is time-invariant as well and given by

$$r = (1-\tau)a. \tag{2}$$

Note that, because there are no external (spill-over) effects of the capital stock on productivity, r equals the marginal productivity of capital a if and only if $\tau = 0$. The tax revenue

$$T(t) = \tau a K(t) \tag{3}$$

is distributed uniformly to the individuals at each date (i.e. all individuals receive transfer income $T(t)/n$). Thus, the government budget is balanced in any point of time and the capital stock accumulates according to

$$\dot{K}(t) = Y(t) - C(t), \tag{4}$$

where $C(t)$ denotes the aggregate level of consumption at date t. (For simplicity, there is no depreciation of capital.) According to (2) and (3), the individual budget constraint is given by

$$\dot{k}^i(t) \leq r k^i(t) + \frac{T(t)}{n} - c^i(t) = (1-\tau)ak^i(t) + \frac{\tau a K(t)}{n} - c^i(t), \tag{5}$$

where $k^i(t)$ and $c^i(t)$ denote the capital stock and the consumption level of individual i at date t, respectively. As usual, individuals also have to regard the "No-Ponzi-Game" condition

$$\lim_{t\to\infty} k^i(t)e^{-rt} \geq 0. \tag{6}$$

The redistribution scheme under consideration implies the following. First, the net transfer to an individual at date t is positive (negative) whenever the individual capital stock $k^i(t)$ is below (above) the average capital stock $K(t)/n$. Second, the ranking of net incomes at each date is preserved for any $\tau < 1$.

5 For further discussion and an analysis of optimal taxation under informational constraints, see e.g. Stiglitz (1987).

6 Note that also a proportional consumption tax with a constant tax rate would not cause any substitution effects in our model. For illustrative reasons we exclude such a policy.

And third, income inequality unambiguously decreases with the tax rate τ for any $\tau > 0$.

Individuals are assumed to have perfect foresight about aggregate variables and thus about their transfer income at each date. Life-time utility U^i is additive separable and identical for all individuals. We assume

$$U^i = \int_0^\infty u\big(c^i(t)\big)e^{-\varrho t}\,dt, \tag{7}$$

where $\varrho \in (0, a)$ is the subjective discount rate. As usual in the endogenous growth literature, instantaneous utility is of the CRRA-type (implying steady state growth), i.e.

$$u(c^i) = \begin{cases} \dfrac{(c^i)^{1-\sigma} - 1}{1-\sigma} & \text{for } \sigma \neq 1, \\ \ln c^i & \text{for } \sigma = 1, \end{cases} \tag{8}$$

where $\sigma > 0$ is the elasticity of the marginal utility of consumption. Note that with (8), life-time utility (7) is a monotonically positive transformation of a homogenous function, i.e. preferences are homothetic. This property is known to imply the most favorable conditions for the preferences of a representative consumer (who always exists in this case) to have welfare significance. As the purpose of this paper is to illustrate the bottlenecks which occur if one uses representative consumers' preferences for welfare analysis (in the context of growth), the assumption of homothetic preferences will make our points as clear as possible.

3. Equilibrium Growth and Policy Preferences

Each individual chooses his/her consumption path $\{c^i(t)\}_{t=0}^{\infty}$ such that life-time utility (7) is maximized subject to constraints (5) and (6), taking transfer income $T(t)/n$ as given in any point of time. Using (8), it is straightforward to show that the optimal growth rate of individual consumption $\hat{c}^i(t)$ at any date t is constant and given by

$$\hat{c}^i(t) = \frac{(1-\tau)a - \varrho}{\sigma} \equiv \vartheta \tag{9}$$

for all i and t[7]. Note that there are no transitional dynamics towards the steady state. This allows for closed-form solutions of intertemporal utility which,

7 The utility maximization problem is a simple optimal-control problem with one control variable $c^i(t)$, one state variable $k^i(t)$, the initial condition $k^i(0) = k_0^i > 0$ and the "terminal condition" $\lim_{t\to\infty} k^i(t)e^{-rt} \geq 0$. The current value Hamiltonian function is given by $\Im(c^i, k^i, \lambda^i) = ((c^i)^{1-\sigma} - 1)/(1-\sigma) + \lambda^i((1-\tau)a\,k^i + \tau a\,K/n - c^i)$, where the multiplier λ^i equals the shadow value of transfer income.

technically, is crucial for welfare analysis. (9) implies that all aggregate variables grow at the same rate ϑ, such that the initial level of aggregate consumption equals

$$C(0) = (a - \vartheta) K_0 , \tag{10}$$

according to (4). According to (5), which holds with equality, the individual capital stock also accumulates at this rate. Moreover, as usual in infinite horizon growth models with perfect capital markets, capital income taxation reduces the rate of growth since it depresses the return to capital[8]. As there is no depreciation of capital and since it is plausible to assume that investments are irreversible, the tax rate has to be restricted to $\tau \leq 1 - \varrho/a \equiv \tau^{\max}$, according to (9). (Note that $\tau^{\max} < 1$.)

The optimal initial consumption level of individual i is derived as follows. First, note that $r = (1 - \tau) a > \vartheta$ has to be assumed in order to obtain both bounded life-time consumption and bounded life-time transfer income. $r - \vartheta > 0$ implies $\lim_{t \to \infty} e^{-rt} k^i(t) = 0$ (transversality condition) since $k^i(t) = k_0^i e^{\vartheta t}$. Integrating (5), which holds with equality, leads to $\int_0^\infty c^i(t) e^{-rt} dt + \int_0^\infty \dot{k}^i(t) e^{-rt} dt = r \int_0^\infty k^i(t) e^{-rt} dt + \frac{\tau a}{n} \int_0^\infty K(t) e^{-rt} dt$. Using the transversality condition, it is easy to show that $\int_0^\infty \dot{k}^i(t) e^{-rt} dt = r \int_0^\infty k^i(t) e^{-rt} dt - k_0^i$. Finally, substitute the latter expression, (2) as well as $c^i(t) = c^i(0) e^{\vartheta t}$ and $K(t) = k_0 e^{\vartheta t}$ to obtain

$$c^i(0) = \big((1 - \tau) a - \vartheta\big) k_0^i + \frac{\tau a K_0}{n} . \tag{11}$$

Note that $\sum_i c^i(0) = C(0)$ is a function of the aggregate capital endowment, but does not depend on the *distribution* of capital. That is, according to (11), individual consumption is linear in the individual capital endowment k_0^i, due to the assumption of homothetic preferences. This implies that a representative consumer exists for *any* distribution of capital endowments. In fact, since preferences are identical as well, any individual endowed with the total initial capital stock K_0 (and taking the total transfer income at each date as given) would choose the economy's aggregate consumption $C(t)$ at any date t.

In the following, the indirect (intertemporal) utility functions as well as the individually desired tax rates on capital income are derived. Substituting

8 This type of model is chosen for our illustration in order to get an unambiguously negative relationship between taxation and macroeconomic indicators such that the case for redistribution is as weak as possible. However, if, for instance, capital markets are imperfect, redistribution may even spur growth [e.g. Bénabou (1996)].

(9) into (11) yields

$$c^i(0)=\frac{(\sigma-1)(1-\tau)a+\varrho}{\sigma}k_0^i+\frac{\tau a K_0}{n}. \tag{12}$$

It is easy to show that $c^i(0)$ increases with the tax rate τ if $\sigma\leq 1$ or if $\sigma>1$ and $\kappa^i<\sigma/(\sigma-1)\equiv\bar{\kappa}$, where $\kappa^i\equiv k_0^i/(K_0/n)$ is the relative capital endowment of individual i. (Note that $\bar{\kappa}>1$ if $\sigma>1$). Thus if $\sigma\leq 1$ or if the relative capital endowment of an individual is not too large, he/she faces a trade-off between the (initial) consumption level and growth with respect to capital income taxation. Using (7), (8) and the optimal consumption path $c^i(t)=c^i(0)\,e^{\vartheta t}$, indirect life-time utility of individual i, denoted V^i, is given by

$$V^i=\begin{cases}\displaystyle\int_0^\infty\frac{\left(c^i(0)e^{\vartheta t}\right)^{1-\sigma}-1}{1-\sigma}e^{-\varrho t}\,dt & \text{for } \sigma\neq 1,\\ \displaystyle\int_0^\infty\left(\ln c^i(0)+\vartheta t\right)e^{-\varrho t}\,dt & \text{for } \sigma=1.\end{cases} \tag{13}$$

Note that the assumption $r>\vartheta$ implies that $\varrho-(1-\sigma)\,\vartheta>0$, according to (2) and (9)[9]. Using this fact (which implies that life-time utility is bounded), and substituting (9) and (12) into (13), we obtain

$$V^i(k_0^i,\tau,\cdot)\equiv\begin{cases}\dfrac{\left(\dfrac{(\sigma-1)(1-\tau)a+\varrho}{\sigma}k_0^i+\dfrac{\tau a K_0}{n}\right)^{1-\sigma}}{(1-\sigma)\left(\varrho-(1-\sigma)\dfrac{a(1-\tau)-\varrho}{\sigma}\right)}-\dfrac{1}{\varrho(1-\sigma)} & \text{for } \sigma\neq 1,\\ \dfrac{1}{\varrho}\left(\ln\left(\varrho k_0^i+\dfrac{\tau a K_0}{n}\right)+\dfrac{a(1-\tau)-\varrho}{\varrho}\right) & \text{for } \sigma=1.\end{cases} \tag{14}$$

The preferred tax rate of individual i is given by $\tau^i\equiv\arg\max_\tau V^i$ s.t. $0\leq\tau\leq\tau^{\max}$. Neglecting the restrictions for the tax rate for a moment, it is straightforward (but tedious) to show that

$$\tilde{\tau}^i\equiv\arg\max_\tau V^i=1-\frac{1-(1-\kappa^i)\varrho/a}{\sigma(1-\kappa^i)+\kappa^i}\ \text{if } \sigma\leq 1 \ \text{ or } \ \kappa^i<\bar{\kappa}. \tag{15}$$

Note that $\tilde{\tau}^i<1$ and $\tilde{\tau}^i<\tau^{\max}(=1-\varrho/a)$ if $\kappa^i>\underline{\kappa}\equiv 1-(a/\varrho-1)/\sigma$ (with $\underline{\kappa}<1$)[10]. Moreover, we have $\partial\tilde{\tau}^i/\partial\kappa^i<0$[11]. If $\sigma>1$, individuals with $\kappa^i\geq\bar{\kappa}$ (who do not

9 Also note that $r>\vartheta$ implies $a>\vartheta$ (since $a\geq r$). This ensures $C(0)>0$, according to (10).

10 The denominator of the latter term in (15) is positive if and only if $\kappa^i<\bar{\kappa}$ and the numerator is positive for all κ^i since $0<\varrho<a$.

11 Using (15), one finds that $\partial\tilde{\tau}^i/\partial\kappa^i<0$ if and only if $\varrho-(1-\sigma)a>0$. The latter inequality is fulfilled because the assumption $r>\vartheta$ implies $a>(a-\varrho)/\sigma$ by setting $\tau=0$, according to (2) and (9).

face a trade-off between the initial consumption level and growth with respect to capital income taxation) always prefer $\tau^i = 0$[12]. It is easy to check from (15) that also $\tilde{\tau}^i = \tau^i = 0$ if $\kappa^i = 1$ (i.e. if an individual has exactly average capital endowment). Thus, all individuals with $\kappa^i \geq 1$ prefer zero capital taxation and thus the maximal growth rate $\vartheta^{\max} \equiv (a - \varrho)/\sigma$. This is because these individuals do not receive a net transfer from the redistribution scheme for any $\tau > 0$, but suffer from a reduction in the economy's growth rate. In contrast, individuals with an initial capital endowment lower than the average K_0/n receive a net transfer in any point of time if capital income is taxed. However, this positive effect has to be weighted against the slowdown of their consumption growth since overall capital accumulation and thus the growth rate of transfer income is reduced by taxation. In sum, one can state:

The preferred capital income tax rate of individual i is given by $\tau^i = \max\{0, \min\{\tilde{\tau}^i, \tau^{\max}\}\}$. *If* $\kappa^i \geq 1$, *then* $\tau^i = 0$, *if* $\underline{\kappa} < \kappa^i < 1$, *then* $\tau^i \in (0, \tau^{\max})$, *and if* $0 < \kappa^i \leq \underline{\kappa}$ *then* $\tau^i = \tau^{\max}$. *Moreover, the preferred tax rate is strictly decreasing in the relative capital endowment of an individual, i.e.* $\partial\tau^i/\partial\kappa^i < 0$, *if (and only if)* $\underline{\kappa} \leq \kappa^i < 1$[13].

In the next section, we are concerned with *welfare effects* of redistributive taxation in this simple model. In particular, it is examined if it is appropriate to make welfare judgments with respect to the redistribution policy under consideration by looking at indirect utility of a representative consumer. This will lead to a discussion of socially optimal redistribution and growth policies.

4. Welfare Effects and "Second-best" Policy

As stated above, a representative consumer exists for any distribution of capital endowments since all individual consumption levels are linear in capital

12 Note that $\partial^2 V^i/\partial\tau^2 \geq 0$ if $\sigma > 1$ and $\kappa^i \geq \bar{\kappa}$, and $\partial^2 V^i/\partial\tau^2 < 0$ otherwise.

13 This kind of result has been exploited in the politico-economic literature on inequality and growth (e.g. Bertola, 1993, Alesina and Rodrik, 1994, Persson and Tabellini, 1994). Since policy preferences with respect to capital income taxation are single-peaked and monotonic in the relative factor endowment, the median voter theorem can be applied. It is certainly realistic to view the median income earner as being poorer than the average, given the stylized fact of a skewed income distribution. According to a well-known result, the poorer the median is relative to the mean, i.e. the larger is $1 - \kappa^m$, where $\kappa^m \in (\underline{\kappa}, 1)$ is the relative endowment of the median income earner, the higher will the level of taxation be in majority voting equilibrium (e.g. Meltzer and Richard, 1981). If the difference between median and mean pre-tax income is viewed as being associated with a more unequal income distribution, this would be consistent with the notion that inequality is harmful for growth through the political process. That is, more redistribution is demanded in a more unequal society, in turn depressing the rate of growth. However, these notions have been criticized on theoretical as well as on empirical grounds. (See Grossmann (2000) for an extensive discussion.).

endowments. Moreover, any individual endowed with the average capital stock K_0/n would choose the per capita consumption level at each date. Thus, an individual with $\kappa^i = 1$ can be viewed as "representative". (In the following, such an individual is referred to as individual R). Does that imply that we can evaluate the welfare effects of the redistribution policy by merely looking at the indirect utility function V^R of the representative consumer? According to the analysis of the preceding section, if this were the case, welfare would be maximized if $\tau = 0$ such that the economy would grow at the maximal rate $\vartheta^{\max}$. It can be shown that in our example, preferences of the representative consumer indeed have welfare significance for any social welfare function *in the following sense.* If the representative consumer prefers the tax rate $\bar{\tau}$ to τ, then there is a *lump-sum* redistribution scheme such that social welfare is higher for $\bar{\tau}$ than for τ, independent of the social welfare function employed. This is best illustrated by considering the special social welfare function

$$\tilde{W}(V^1, \ldots, V^n) = \frac{1}{n} \sum_{i=1}^{n} \tilde{V}^i, \tag{16}$$

where

$$\tilde{V}^i \equiv \begin{cases} \left((1-\sigma)V^i + \dfrac{1}{\varrho(1-\sigma)} \right)^{\frac{1}{1-\sigma}} & \text{for } \sigma \neq 1, \\ \exp(\varrho V^i) & \text{for } \sigma = 1. \end{cases} \tag{17}$$

Note that $\tilde{V}^i$ are monotonically positive transformations of V^i. Furthermore, it is easy to check that $\tilde{V}^i$ is of the form $\tilde{V}^i = A(\tau) K_0/n + B(\tau) k_0^i$ (i.e. $\tilde{V}^i$ is linear in the individual capital endowment with common partial derivatives $\partial \tilde{V}^i / \partial k_0^i = B(\tau)$ for all i), according to (14) and (16).[14] Thus, $\tilde{W}(\cdot) = \tilde{V}^R$, i.e. welfare equals utility of the representative individual. Now compare the two tax regimes τ and $\bar{\tau}$, assuming that $(A(\bar{\tau}) + B(\bar{\tau})) K_0/n - (A(\tau) + B(\tau)) K_0/n \equiv Q > 0$, i.e. utility of the representative consumer (and thus welfare as given in (16)) is higher for $\bar{\tau}$ than for τ. In this case, under the endowment distribution $(\bar{k}_0^1, \ldots, \bar{k}_0^n)$, where $\bar{k}_0^i$ are defined by $A(\bar{\tau}) K_0/n + B(\bar{\tau}) \bar{k}_0^i = A(\tau) K_0/n + B(\tau) k_0^i + Q$, $i = 1, \ldots, n$, *everybody* is better off if the tax rate changes from τ to $\bar{\tau}$. That is, $\bar{\tau}$ is ranked above τ in the "potential compensation test" since there is an endowment distribution such that $\tilde{V}^i$ (and thus V^i) is raised for all i if $\tilde{W}(\cdot) = \tilde{V}^R$ (and thus V^R) is raised[15]. Thus, welfare is raised unambiguous-

14 $A(\cdot)$ and $B(\cdot)$ are functions of τ and parameters of the model (the latter arguments are suppressed in these functions). For instance, in the case $\sigma = 1$ we have $A(\cdot) = \tau a \exp(a(1-\tau)/\varrho - 1)$ and $B(\cdot) = \varrho \exp(a(1-\tau)/\varrho - 1)$, according to (14) and (16).

15 See Mas-Colell et al. (1995, ch. 4) for a further discussion of the potential compensation test in the contexts of aggregate demand and the representative consumer.

ly for any social welfare function $W(V^1, \ldots, V^n)$ which is increasing in its arguments (i.e. the Pareto principle is adopted).

This kind of result is the implicit justification for a welfare analysis exclusively based on the representative consumer in many models (see introduction). However, remember that we explicitly excluded the possibility of a lump-sum redistribution of endowments. Thus, it does not help to know that there is a distribution of endowments which makes everybody better off since the necessary compensation cannot be conducted. Due to this fact, even in this very special example which is most favorable for the representative consumer to have welfare significance, it is in general misleading to look at the utility of a representative consumer in order to maximize social welfare (i.e. to set $\tau = 0$).

To put the argument forward, note that the utility possibility frontier if lump-sum redistribution were possible is a hypothetical one. This hypothetical utility possibility frontier in absence of capital income taxation (i.e. $\tau = 0$) is given by

$$UPF_{hyp} \equiv \left\{ \left(V^1(k_0^1, 0, \cdot), \ldots, V^n(k_0^n, 0, \cdot) \right) : \textstyle\sum_i k_0^i = K_0 \right\}. \tag{18}$$

With capital income taxation, the utility possibility frontier depends on the distribution of capital endowments, i.e.

$$UPF_{tax} \equiv \left\{ \left(V^1(k_0^1, \tau, \cdot), \ldots, V^n(k_0^n, \tau, \cdot) \right) : 0 \le \tau \le \tau^{\max} \right\}. \tag{19}$$

Note that UPF_{hyp} and UPF_{tax} coincide if and only if $\tau = 0$.

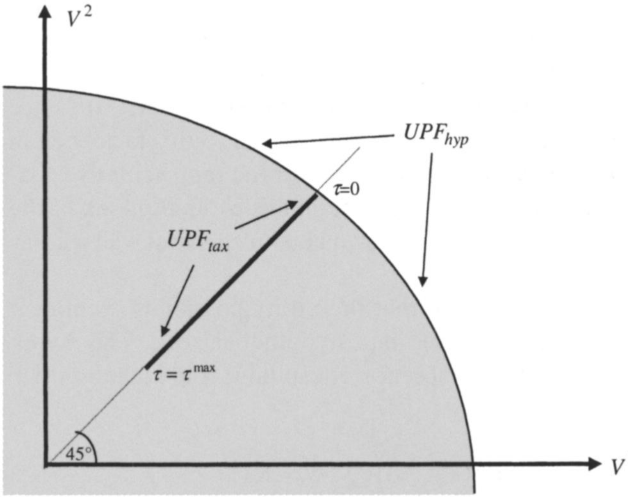
Let us start with a situation in which all individuals are identical as it is done in "representative agent models", i.e. let $k_0^i = K_0/n$ for all i. Figure 1 depicts this situation in the two-person case [16].

With identical individuals, the utility possibility frontier under capital income taxation lies on the 45°-line and is a subset of the hypothetical utility possibility set with lump-sum redistribution (the latter is indicated by the shaded area in figure 1). Not surprisingly, when all individuals are identical, any attempt to redistribute income in a non-lump-sum fashion makes the "representative" agent worse off due to excess burdens of capital income taxation. As a result, zero taxation (implying a growth rate $\vartheta^{\max}$) is socially optimal in this case. Generally, one has to distinguish *total effects* of taxation (on individual and aggregate variables) and excess burdens of taxation which arise on the individual level due to *substitution effects*. Concerning the total

16 Of course, in order to maintain the assumption that each individual takes transfer income T/n as given, the number of individuals n is supposed to be very large. We look at the case $n = 2$ merely for the graphical illustration. Alternatively, we could speak of two classes of individuals.

Figure 1

The Utility Possibility Frontier for a Perfectly Egalitarian Distribution of Capital Endowments and the Hypothetical ("First-best") Utility Possibility Set



(or macroeconomic) effects of taxation, the higher the tax rate τ, the higher the aggregate consumption level initially is, but the lower is its growth rate, according to (9) and (10). Thus, higher capital income taxation "flattens" the aggregate consumption path. Moreover, the higher τ, the lower are both total output and the (individual) capital stock(s) for any point in time $t > 0$ due to the growth slowdown. (Output at time zero is given by $Y(0) = a K_0$, where K_0 is exogenous). Concerning excess burdens, each individual would prefer to be taxed in a lump-sum manner if he/she could choose to pay the same amount of taxes either under lump-sum taxation or under capital income taxation. In our example, (given the same transfer under both lump-sum taxation and capital income taxation) an individual endowed with K_0/n would see his/her position unchanged with lump-sum taxation but unambiguously loses under capital income taxation[17].

17 Of course, nobody would on basis of this example conclude that one should abolish redistribution through capital income taxation since the normative justification of redistribution, i.e. to reach a more equal distribution of income, is defined away if one considers a perfectly egalitarian economy. However, a similar conclusion, namely that capital taxation should asymptotically tend to zero, has been drawn from standard Ramsey analysis with representative agents when markets are perfect and planning horizons infinite (e.g. Chamley, 1986, Lucas, 1990). This result was strongly criticized by Putterman et al. (1998), who argue that shifting taxation from capital to labor may improve aggregate economic performance but has drastic negative effects on the utility of workers, if heterogeneity of agents is taken into account.

To see how things change if one departs from the assumption of identical consumers (but there exists a representative consumer, whose preferences would have welfare significance for any endowment distribution if lump sum redistribution were feasible), now consider an unequal distribution of capital endowments. Figure 2 depicts UPF_{tax} in the two-person case for $k_0^1 > K_0/2 > k_0^2$.

As established in the preceding section, poor individuals do not necessarily prefer the maximal tax rate $\tau^{\max}$, which would yield zero growth. Thus, with $k_0^1 > k_0^2$, the utility possibility frontier in the $V^1 - V^2$ space has a maximum at the utility levels $\left(V^1(k_0^1, \tau^2, \cdot), V^2(k_0^2, \tau^2, \cdot)\right)$, if $\tau^2 \in (0, \tau^{\max})$ is assumed for the preferred tax rate of the poorer individual 2. Is it generally still welfare-maximizing to set the capital tax rate equal to zero, i.e. to maximize utility of a (fictional) representative consumer? The answer is: not necessarily, because we cannot compensate the poorer individual 2 to make him/her better off under $\tau = 0$ compared with any $\tau \in (0, \tau^2)$, due to the impossibility of lump-sum redistribution. This is why the "distortionary" capital income tax has been considered in the first place. We thus cannot regard $\tau = 0$ as generally leading to a *Pareto-improvement* to any $\tau > 0$ any longer since this term is meaningless if it refers to an unfeasible allocation on UPF_{hyp}. Clearly, any point in the hatched area in figure 2 (i.e. point B) is simply not feasible.

Since a Pareto-improvement is impossible, one may rather look at the socially optimal level of taxation which is given by

$$\tau^* \equiv \arg\max_{\tau} W\left(V^1(k_0^1, \tau, \cdot), \ldots, V^n(k_0^n, \tau, \cdot)\right) \text{ s.t. } (19). \tag{20}$$

Figure 2

The Utility Possibility Frontier for an Unequal Distribution of Capital Endowments and the Hypothetical ("First-best") Utility Possibility Frontier

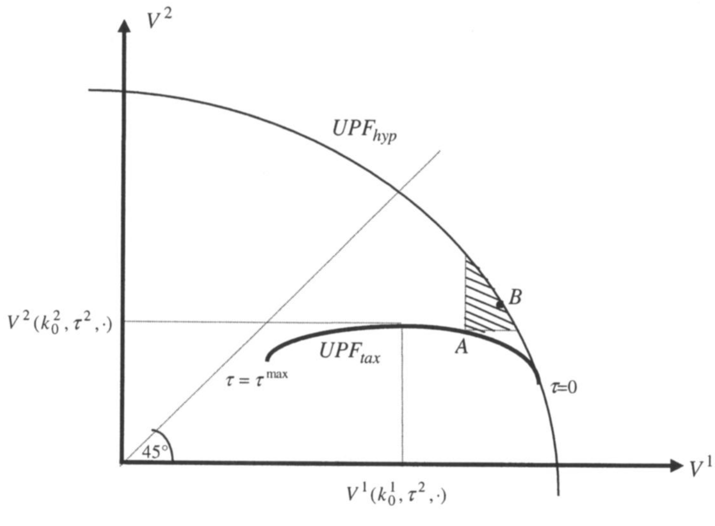
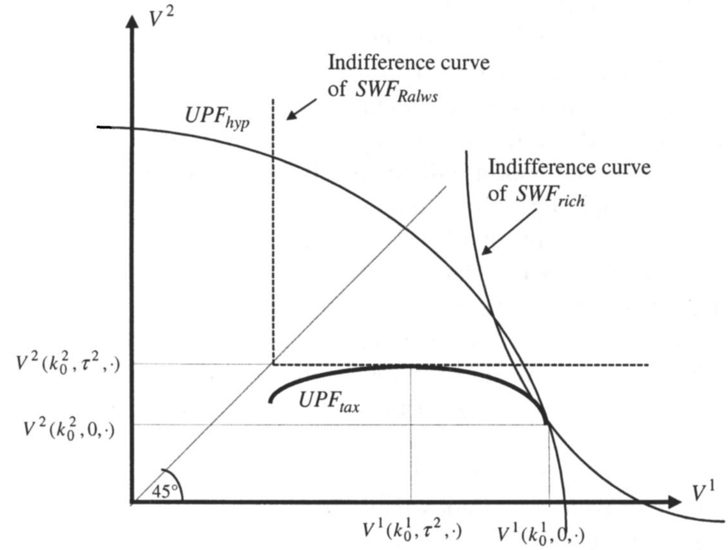


Figure 3
Socially Optimal "Second-best" Redistribution Policies under a Social Welfare Function Biased towards the Rich (SWF_{rich}) and a Ralwsian Social Welfare Function (SWF_{Ralws})



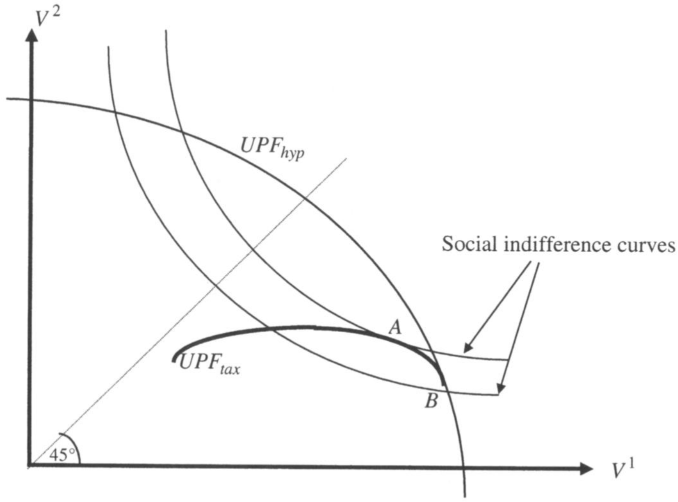
Unlike in figure 1, if individuals are not identical, $\tau^* = 0$ will only hold under welfare functions which, in the two-person case, put a higher weight on the rich compared with the poor individual. (There are probably not many people who would share this kind of social preferences). Figure 3 depicts the "second-best" solution to the social welfare maximization problem under both a Rawlsian social welfare function (the dashed indifference curve), yielding the preferred tax rate of the poorest individual as social optimum, and under a social welfare function strongly biased towards the rich (the solid indifference curve), yielding a corner solution with no redistribution[18].

Generally, the socially optimal redistribution lies between those two extremes. Moreover, in this example, the optimal growth rate of an economy also crucially depends on the social welfare function, and is generally lower than ϑ^{max}. That is, the optimal net return to capital is lower than the marginal productivity of capital.

To highlight how misleading welfare judgments on the basis of a representative consumer's preferences can be in evaluating tax reforms, consider the situation in figure 4.

18 The term "second-best" is in quotation marks since the "first-best" is supposed to be unfeasible.

Figure 4
Misleading Welfare Judgments from Preferences of a Representative Consumer



Suppose we start from a social optimum with positive redistribution at point A in figure 4. As argued by using figure 1, if all individuals were identical (like in "representative agent models") it would be both welfare-maximizing and Pareto-optimal to abandon the redistribution policy. However, if individuals are not identical, welfare is *reduced* with that policy change (as one reaches point B) when the heterogeneity is taken into account, contrary to what a representative agent model would tell us. This also raises doubts on the common practice of evaluating tax reforms from a normative point of view under the representative agent assumption.

5. Conclusion

In sum, our little exercise made clear that when lump-sum redistribution is unfeasible, attributing welfare significance to the preferences of a representative consumer may yield very misleading results in the normative analysis of taxation. This has been shown in a simple balanced growth model with heterogeneous individuals in which a representative agent exists for all endowment distributions, i.e. even if there always exists an endowment distribution such that utility of the representative consumer has welfare significance for any social welfare function.

Concerning economic growth, the example has shown that an *equality-growth trade-off* (as "second-best" redistribution policies may depress the growth rate) is very distinct from an *equality-efficiency trade-off.* In the absence of lump-sum redistribution, a "first-best" utility possibility frontier simply does not exist. Consequently, as is well known, we cannot regard this utility possibility frontier as being the locus of efficient outcomes. In fact, due to distributive concerns, socially optimal growth may be reached if the net return to capital is lower than the (social) marginal product of capital. However, this may not indicate an inefficient solution.

Finally, concerning the evaluation of tax reforms one should generally be aware of the fact that normative results are very difficult to reach. Given that *any* tax reform is likely to have distributional impacts when consumers are heterogeneous to some respect, looking at welfare changes under the representative agent assumption does not help to evaluate tax reforms from a normative point of view.

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