

Cross-Sectional Relations among Accruals, Cash Flows and Profitability: A Bayesian Model for Analyzing Earnings Management

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Abstract

It is an important stylized fact that cross-sectional accruals are positively related to earnings and negatively related to operating cash flows. In this paper, we reconsider the question of why these associations occur and how to deal with them in earnings management studies. We propose a simple Bayesian model in which unobservable true profitability is related to cash flows by normal accruals and to reported earnings by discretionary accruals. The overlay of uncertainty concerning true performance and normal and discretionary accruals gives rise to the observed cross-sectional relations, even if earnings are not managed and true performance does not affect accruals. In our model, earnings management appears in the Bayesian estimates of both discretionary and normal accruals. The most efficient way to detect earnings management depends on the sampling of suspect firms. For random sampling, the test should be based on estimated discretionary accruals; for sampling with respect to reported earnings, estimated normal accruals are more efficient; and for sampling with respect to the Bayesian estimate of true profitability, total accruals are most appropriate. We propose graphic diagnostics and a simple regression approach to identify the appropriate performance adjustment setting.

Keywords: Earnings management, Discretionary accruals, Normal accruals, Bayesian analysis, Performance adjustment.

JEL: M41, G17

1. Introduction

It is an important stylized fact that total accruals, defined as net income minus operating cash flow, contain a predictable component in the cross-section of firms: they are positively related to net income (Kothari et al., 2005) and negatively related to contemporaneous cash flows (Dechow, 1994; Dechow et al., 1995, 1998). The relation of total accruals to net income is particularly strong, in many years explaining more than 20% of the cross-sectional variation in total accruals.¹ In this paper, we reconsider the question of where these associations come from and how to deal with them in attempts to detect earnings management. Our model suggests that the relations—not only to cash flow but also to net income—arise almost automatically, even if earnings are not managed and performance does not affect accruals.

The intuition of this result is contained in the accrual quality model of Dechow and Dichev (2002). The model decomposes the accruals of a specific firm-year into three components: (1) past and future cash flows captured in current earnings (positive current accruals), (2) current cash flows captured in past or future earnings (negative current accruals), and (3) errors in accruals that are corrected when cash flow uncertainty is resolved (zero current accruals). For given current cash flows (i.e., fixed component (2)), accrual component (1) perfectly matches earnings, giving rise to a coefficient of one between current accruals and current earnings. Analogously, for fixed current earnings (i.e., fixed component (1)), accrual component (2) is equal to the negative value of current cash flows.

These considerations assume that the individual accrual components can be identified. The sum of the components may be zero, which calls into question that a natural correlation

¹Data are shown in Section 3.3 in this paper. Bushman et al. (2016) find that the negative correlation between accruals and cash flows has strongly declined over time. The adjusted R^2 from regressing accruals on cash flows was as high as 70% in the 1960s and, more recently, lower than 20%.

also exists for total accruals. But in fact, the initial intuition carries over to total accruals and cash flows as explained by Larson et al. (2018, p. 842): “To understand the intuition behind this approach [proposed by Dechow and Dichev (2002)], we consider a simple case where we assume that [...] a firm has constant predetermined earnings, such that all of the variation in cash flows relates to variation in accruals. In this case, [...] accruals are negatively related to contemporaneous cash flows with a coefficient of minus one.” Technically, the same argument can also be applied to earnings: If a firm has constant predetermined cash flows (e.g., a start-up firm with a fixed cash budget), all variation in earnings relates to accruals, and accruals are positively related to contemporaneous earnings with a coefficient of one. However, the first argument (based on constant earnings) appears to be more relevant because “accruals are used to smooth temporary fluctuations in cash flows” (Dechow, 1994, p. 19).

To transfer this intuition from the firm level to the cross-section of firms, we can rephrase the arguments analogously:² If all firms report the same earnings, accruals will show a cross-sectional correlation to cash flows of minus one, and if all firms have the same cash flows, accruals will show a cross-sectional correlation to earnings of one. Again, the first part of the statement seems to be more relevant because cash flows include temporary fluctuations that are reversed over time. However, there are two problems with this intuitive explanation. The first is that prior literature focuses only on the negative correlation of accruals to cash flows. The symmetrical argument for a positive relation of accruals to earnings is typically not considered further. The two arguments cannot easily be combined because earnings and cash flows cannot be fixed at the same time. The second problem is that the assumption

²Following a proposition by McNichols (2002), the accrual quality model is often integrated into the widely used Jones (1991) model or modified Jones model of Dechow et al. (1995) by including past, present and future cash flows in the Jones model regression (see the overview of the use of different accrual models in accounting studies in Breuer and Schütt, 2019, Fig. 1).

that all firms have the same earnings or cash flows is too extreme. A theoretical model is missing that reveals how the assumption can be weakened and what the consequences are in terms of more realistic associations of accruals with earnings and cash flows.

Our intended contribution is to propose such a model. It addresses the first problem by relating net income and cash flow to an unobservable variable, which we call true profitability, resulting from the most suitable translation of given business transactions into earnings according to the accounting standards in place. This concept corresponds to what Ball (2016, p. 551) describes as “the gold standard of complete comparability, the hypothetical property of financial reporting that two otherwise identical companies [...] would report identical financial statements”. The second problem is addressed by weakening the assumption that all firms have the same profitability. We assume instead that prior information on the *distribution* of true profitability is available.

In our model, cash flows deviate predictably or randomly from true profitability. A predictable (expected) deviation could be due to reversals from past accruals. Unpredictable (random) deviations can arise, for example, from the timing of customer payments. We refer to these deviations (expected and random) as “normal” accruals because they are a natural element of accrual accounting and arise from the most suitable application of the accounting rules.

Net income will also deviate from true profitability, even in a world without earnings management. On the one hand, accounting errors occur. On the other hand, accounting for transactions with uncertain cash flows typically implies some discretion. As Ball (2016, p. 553) notes, “[a]ll accounting accruals involve some degree of judgment about future cash flows. [...] Consequently, there is much leeway in practice in how all accounting standards are implemented.[...] The conclusion is that rules alone do not determine actual reporting

practice. Estimation and discretion are required.” Therefore, we refer to these deviations as “discretionary” accruals.

The combination of a prior distribution of true profitability with (partly) random normal and discretionary accruals implies that low cash flows will often go along with large normal accruals and that high net income will tend to be observed when discretionary accruals are large. Therefore, the cross-sectional relations among total accruals, cash flow and earnings arise, even though the accruals of each firm are random variables, independent of cash flow and earnings performance. This is not to say that accruals are not directly affected by performance. In fact, Kothari et al. (2005) show that firms with higher profitability tend to have higher performance-related abnormal accruals. This effect is complementary to the one on which we are focusing.

In our Bayesian model, there are three types of information on true profitability: prior information, net income information and cash flow information. Under normality, we can combine these types of information to obtain the posterior distribution and the maximum likelihood estimate of true profitability, which directly leads to estimates of normal and discretionary accruals.³ We find that these estimates are proportional to the residuals of regressing total accruals on net income and cash flow, respectively, so that the model is consistent with the methods used in earnings management studies. The model suggests that earnings management appears in the Bayesian estimates of both discretionary and normal accruals. The most efficient way to detect earnings management depends on the sampling of firms suspected of earnings management. For random sampling, the test should be based on discretionary accruals; for sampling with respect to reported earnings, using normal accruals

³The recent paper of Breuer and Schütt (2019) also explores Bayesian methods in earnings management studies, but with a different focus. Its aim is to “illustrate how Bayesian methods and tools can be applied to the estimation of the modified Jones model to better predict normal accruals” (p. 11).

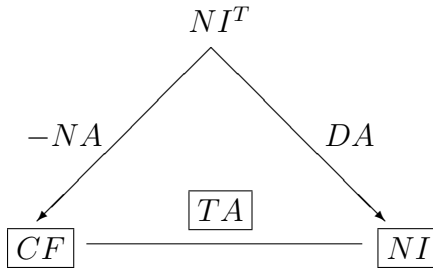


Figure 1: **Model variables.** NI^T : true profitability; CF : operating cash flow; NI : reported net income; NA : normal accruals; DA : discretionary accruals; TA : total accruals. The frame around TA , CF and NI indicates that the values of these variables are observable.

is more efficient; and for sampling with respect to the Bayesian estimate of true profitability, total accruals are most appropriate⁴. The model highlights the efficiency loss induced by performance adjustments, as shown by Keung and Shih (2014) (see also Ayers et al., 2006; Dechow et al., 2012). It also highlights how important it is to identify the most appropriate type of performance adjustment and how this can be achieved in practical applications. Overall, the results suggest that our model offers a useful theoretical framework for analyzing earnings management.

The remainder of the paper proceeds as follows. Section 2 derives the model and conducts tests for earnings management in simulated data. In Section 3, we apply the model to the empirical data. Section 4 briefly explores how the model is related to Jones-type models. Section 5 concludes the paper.

2. Bayesian Model

2.1. Assumptions and Basic Implications

Figure 1 shows the basic structure of the model. We assume that for a given set of a firm's business transactions, the accounting standards unambiguously define the "true" profit or loss, NI^T . If the accounting standards are not complete in this sense, we can think of NI^T

⁴Total accruals have been used in the early earnings management studies by Healy (1985) and DeAngelo (1986).

as the profit or loss that corresponds to the most suitable accounting treatment according to a panel of neutral experts. With a proper voting mechanism in place, the experts will always come to an unequivocal solution. While the specific level of NI^T is unobservable, we assume that prior information is available according to which NI^T is normally distributed with expected value μ_0 and variance σ_0^2 :

$$NI^T \sim N(\mu_0, \sigma_0^2). \quad (1)$$

The operating cash flow CF differs from NI^T by normal accruals, NA :

$$CF = NI^T - NA. \quad (2)$$

In this way, accruals are classified as normal if they arise from the most suitable application of the rules of accrual accounting to the cash-generating business transactions. By definition, a firm can only influence these normal accruals by changing cash flows and the corresponding operating activities. If the intention of such changes were to manage earnings, they would classify as “real” earnings management. We assume that the conditional probability of normal accruals given NI^T is normal with expected value μ_{NA} (predictable component) and variance σ_{NA}^2 :

$$NA|NI^T \sim N(\mu_{NA}, \sigma_{NA}^2). \quad (3)$$

The parameter μ_{NA} will typically be negative because depreciation is considered in NI^T but not in CF . Since NI^T and $NA|NI^T$ follow a normal distribution, the cash flow $CF =$

$NI^T - NA$ is also normal:

$$\begin{aligned} CF | NI^T &\sim N(NI^T - \mu_{NA}, \sigma_{NA}^2) \\ \Leftrightarrow CF + \mu_{NA} | NI^T &\sim N(NI^T, \sigma_{NA}^2). \end{aligned} \quad (4)$$

Its unconditional probability distribution is:

$$CF \sim N(\mu_0 - \mu_{NA}, \sigma_0^2 + \sigma_{NA}^2). \quad (5)$$

Besides cash flow CF , the second indicator of true profitability is the reported net income NI . It deviates from NI^T by discretionary accruals, DA , which we assume to be independent of normal accruals NA :

$$NI = NI^T + DA. \quad (6)$$

Discretionary accruals arise from accounting practices that deviate from what an expert panel would consider most suitable. This includes accounting errors and management discretion based on a different assessment of future implications of relevant material transactions. Discretionary accruals also arise from earnings management in which discretion is exercised with the intention to increase or reduce the reported profit. However, we do not explicitly consider earnings management at this stage. Our objective is to model accruals under the null hypothesis of no earnings management. Therefore, we assume that discretionary accruals conditional on NI^T are random variables with a symmetrical probability distribution, which is again the normal distribution:

$$DA | NI^T \sim N(\mu_{DA}, \sigma_{DA}^2). \quad (7)$$

This means that NI conditional on NI^T has variance σ_{DA}^2 :

$$\begin{aligned} NI | NI^T &\sim N(NI^T + \mu_{DA}, \sigma_{DA}^2) \\ \Leftrightarrow NI - \mu_{DA} | NI^T &\sim N(NI^T, \sigma_{DA}^2), \end{aligned} \quad (8)$$

while the unconditional variance of NI is $\sigma_{NI}^2 = \sigma_0^2 + \sigma_{DA}^2$:

$$NI \sim N(\mu_0 + \mu_{DA}, \sigma_0^2 + \sigma_{DA}^2). \quad (9)$$

A first implication of this framework is that CF and NI are positively related with a covariance of:

$$Cov(CF, NI) = Cov(NI^T - NA, NI^T + DA) = \sigma_0^2. \quad (10)$$

The reason is that both variables are based on NI^T . The larger the uncertainty of NI^T (measured by σ_0^2), the more important the common source of uncertainty in CF and NI . In the other direction, when σ_0^2 approaches zero, the covariance of CF and NI disappears because the remaining sources of uncertainty in CF and NI are NA and DA , respectively, which are independent.

The second immediate implication of the framework is that total accruals $TA = NI - CF = DA + NA$ are negatively correlated with CF and positively correlated with NI :

$$Cov(TA, CF) = Cov(NI - CF, CF) = \sigma_0^2 - \sigma_{CF}^2 = -\sigma_{NA}^2 \quad (11)$$

$$Cov(TA, NI) = Cov(NI - CF, NI) = \sigma_{NI}^2 - \sigma_0^2 = \sigma_{DA}^2. \quad (12)$$

These relationships are in line with the empirical observation that the cross-section of total

accruals is negatively related to cash flow and positively related to profitability. Our simple framework shows that these relationships will arise under general terms. They do not require earnings management, particular reversal patterns of accruals over time or causal effects of profitability (e.g., due to superior growth of profitable firms leading to growing accruals). It is sufficient that both CF and NI are linked to the same base, which is “true” profitability. With this anchor in place, NI and CF are endogenous variables because they *contain* DA and NA , respectively. If we have a sample of firms for which this framework holds (with identical parameters), a cross-sectional regression of TA on CF results in a slope coefficient of $-\sigma_{NA}^2/(\sigma_0^2 + \sigma_{NA}^2)$ and a regression of TA on NI provides a slope coefficient of $\sigma_{DA}^2/(\sigma_0^2 + \sigma_{DA}^2)$. Thus, for the special case of known NI^T ($\sigma_0^2 = 0$), the slopes are equal to minus one and one, respectively. The less precise the prior information (higher σ_0^2), the more the slope coefficients approach zero.

Since NA and DA are independent, the negative covariance of TA and CF is entirely due to the normal component of total accruals, while the positive covariance of TA and NI is entirely due to the discretionary component:

$$Cov(NA, CF) = -\sigma_{NA}^2 \quad (13)$$

$$Cov(DA, NI) = \sigma_{DA}^2 \quad (14)$$

$$Cov(DA, CF) = Cov(NA, NI) = 0. \quad (15)$$

2.2. Bayesian Estimates of Discretionary and Normal Accruals

In our framework, there are three sources of information on the unobservable profit NI^T : prior information, cash flow information CF and reported net income NI . The mean-adjusted cash flow $CF + \mu_{NA}$ and the mean-adjusted net income $NI - \mu_{DA}$ are informative

about NI^T because their conditional expected value is equal to NI^T (Eqs. (4) and (8)). Since the probability densities are all normal, the Bayesian posterior density of NI^T is also normal.⁵

$$NI^T | CF, NI \sim N(\mu^*, \sigma^{*2}) \quad (16)$$

with

$$\mu^* = \frac{1/\sigma_0^2 \cdot \mu_0 + 1/\sigma_{NA}^2 \cdot (CF + \mu_{NA}) + 1/\sigma_{DA}^2 \cdot (NI - \mu_{DA})}{1/\sigma_0^2 + 1/\sigma_{NA}^2 + 1/\sigma_{DA}^2} \quad (17)$$

and

$$\sigma^{*2} = \frac{1}{1/\sigma_0^2 + 1/\sigma_{NA}^2 + 1/\sigma_{DA}^2}. \quad (18)$$

The Maximum Likelihood estimator $\widehat{NI^T}$ of $NI^T | CF, NI$ is the posterior expected value μ^* of Eq. (17) which can be written as a weighted average of three estimates:

$$\widehat{NI^T} = \mu^* = w_1 \mu_0 + w_2 (CF + \mu_{NA}) + w_3 (NI - \mu_{DA}), \quad (19)$$

where the weights $w_1 = 1/\sigma_0^2 \cdot \sigma^{*2}$, $w_2 = 1/\sigma_{NA}^2 \cdot \sigma^{*2}$ and $w_3 = 1/\sigma_{DA}^2 \cdot \sigma^{*2}$ reflect the relative precision of the three sources of information (inverse of variance).

The Bayesian estimate of NI^T leads directly to the Bayesian estimates of normal and discretionary accruals. Given CF and NI , the posterior distributions of $NI - NI^T$ and

⁵For the standard result with one sample, see, e.g., Leonard and Hsu (2001). A second sample can be integrated via Bayesian updating. In doing so, the posterior distribution after considering the first sample serves as the new prior distribution to be combined with the second sample information. This updating leads to the specified solution.

$NI^T - CF$ are normal, so that the Maximum Likelihood estimators are:

$$\widehat{DA} | CF, NI = E [NI - NI^T | CF, NI] = NI - \widehat{NI^T} \quad (20)$$

$$\widehat{NA} | CF, NI = E [NI^T - CF | CF, NI] = \widehat{NI^T} - CF. \quad (21)$$

The following properties apply to these estimators:⁶

1. The unconditional variances are: $Var(\widehat{NI^T}) = \sigma_0^2 - \sigma^{*2}$; $Var(\widehat{DA}) = \sigma_{DA}^2 - \sigma^{*2}$ and $Var(\widehat{NA}) = \sigma_{NA}^2 - \sigma^{*2}$. The deduction of σ^{*2} reflects a shrinkage towards the prior expected value, which is a well-known characteristic of Bayesian estimates. Therefore, the variances of the estimates are smaller than the variances of the unobservable actual variables.
2. The estimates \widehat{DA} and \widehat{NA} have a positive covariance of $Cov(\widehat{DA}, \widehat{NA}) = \sigma^{*2}$ even though DA and NA are independent. The posterior uncertainty σ^{*2} about the size of NI^T reflects the remaining uncertainty about the decomposition of total accruals into normal and discretionary accruals. As a consequence, higher total accruals tend to be attributed in part to both components, which is the source of their covariance.
3. The covariances of discretionary and normal accruals with NI and CF are preserved in the Bayesian estimates: $Cov(\widehat{DA}, NI) = Cov(DA, NI) = \sigma_{DA}^2$; $Cov(\widehat{DA}, CF) = Cov(DA, CF) = 0$; $Cov(\widehat{NA}, CF) = Cov(NA, CF) = -\sigma_{NA}^2$ and $Cov(\widehat{NA}, NI) = Cov(NA, NI) = 0$.
4. The error term of a linear regression of total accruals TA on cash flow CF is pro-

⁶The variances and covariances of Properties 1. to 3. are straight-forward to derive. We prove Property 5. in Appendix A. Properties 4. and 6. to 9. can be derived analogously.

portional to mean adjusted \widehat{DA} . More precisely, total accruals can be represented as $TA = \alpha + \beta \cdot CF + \epsilon$, with constant α , error term ϵ and $\beta = -\sigma_{NA}^2 / \sigma_{CF}^2 = -w_1 / (w_1 + w_2)$. Then, $\widehat{DA} - \mu_{DA} = (w_1 + w_2) \cdot \epsilon$.

5. The error term of a linear regression of total accruals TA on net income NI is proportional to mean adjusted \widehat{NA} . More precisely, total accruals can be represented as $TA = \alpha + \beta \cdot NI + \epsilon$, with constant α , error term ϵ and $\beta = \sigma_{DA}^2 / \sigma_{NI}^2 = w_1 / (w_1 + w_3)$. Then, $\widehat{NA} - \mu_{NA} = (w_1 + w_3) \cdot \epsilon$.

6. The error term of a linear regression of \widehat{NA} on CF is proportional to mean adjusted \widehat{DA} . More precisely, the estimated normal accruals can be represented as $\widehat{NA} = \alpha + \beta \cdot CF + \epsilon$, with constant α , error term ϵ and $\beta = -\sigma_{NA}^2 / \sigma_{CF}^2 = -w_1 / (w_1 + w_2)$. Then, $\widehat{DA} - \mu_{DA} = (w_1 + w_2) / w_3 \cdot \epsilon$. Together with Property 4 this implies that the error term of regressing \widehat{NA} on CF is equal to w_3 times the error term of regressing TA on CF .

7. The error term of a linear regression of \widehat{DA} on NI is proportional to mean adjusted \widehat{NA} . More precisely, the estimated discretionary accruals can be represented as $\widehat{DA} = \alpha + \beta \cdot NI + \epsilon$, with constant α , error term ϵ and $\beta = \sigma_{DA}^2 / \sigma_{NI}^2 = w_1 / (w_1 + w_3)$. Then, $\widehat{NA} - \mu_{NA} = (w_1 + w_3) / w_2 \cdot \epsilon$. Together with Property 5 this implies that the error term of regressing \widehat{DA} on NI is equal to w_2 times the error term of regressing TA on NI .

8. The error term of a linear regression of \widehat{DA} on \widehat{NI}^T is proportional to mean adjusted TA . More precisely, the estimated discretionary accruals can be represented as $\widehat{DA} = \alpha + \beta \cdot \widehat{NI}^T + \epsilon$, with constant α , error term ϵ and $\beta = \sigma^{*2} / (\sigma_0^2 - \sigma^{*2}) = w_1 / (1 - w_1)$. Then, $TA - \mu_{DA} - \mu_{NA} = (1 - w_1) / w_2 \cdot \epsilon$.

9. The error term of a linear regression of \widehat{NA} on \widehat{NI}^T is proportional to mean adjusted TA . More precisely, the estimated normal accruals can be represented as $\widehat{NA} = \alpha + \beta \cdot \widehat{NI}^T + \epsilon$, with constant α , error term ϵ and $\beta = -\sigma^{*2} / (\sigma_0^2 - \sigma^{*2}) = -w_1 / (1 - w_1)$. Then, $TA - \mu_{DA} - \mu_{NA} = (1 - w_1) / w_3 \cdot \epsilon$.

Properties 4. and 5. are related to the cash flow and performance adjustments of total accruals commonly applied in the earnings management literature. In our model, estimated discretionary accruals correspond to total accruals after controlling for contemporaneous cash flows, and estimated normal accruals correspond to performance-adjusted total accruals. If we tried to control for cash flow and net income in successive regressions, we would switch back and forth between the estimates of normal and discretionary accruals (see Properties 6. and 7.).

Properties 1. to 9. and the covariance relationships (10), (11) and (12) derived in Section 2.1 do not depend on the assumption of normal distributions. They are valid for general distributions as long as the variances and covariances are defined and finite. However, outside the normal model, \widehat{NI}^T according to Eq. (19) is no longer the Maximum Likelihood Bayesian Estimate of NI^T .

The model could be extended in different ways. For example, income smoothing could be modelled by including a smoothing parameter α in Eq. (8) such that $NI - \mu_{DA} | NI^T \sim N((1 - \alpha) NI^T, \sigma_{DA}^2)$, which results in a lower precision of the earnings information and a lower weight of net income in the Bayesian estimate of true performance. We do not elaborate such extensions in this paper.

2.3. Illustrative Simulations

We run simulations based on realistic parameter estimates that are obtained in the later empirical part of this paper. The parameter values are (see Section 3.2): $\sigma_0 = 0.1056$; $\sigma_{CF} = 0.1133$; $\sigma_{NI} = 0.1315$; $\sigma_{DA} = 0.0783$; $\sigma_{NA} = 0.0410$; $w_1 = 0.106$, $w_2 = 0.702$, $w_3 = 0.192$; $\mu_0 = 0.02$; $\mu_{DA} = \mu_{NA} = 0$. A simulation of 1000 observations is shown in Figures 2 to 4. Figure 2 reveals how closely the estimates of true profitability, normal and discretionary accruals are related to the unobservable true values (Panels A-C). It also illustrates the positive association of the estimated accrual components (Panel D). Note that normal accruals are much more variable in our simulation setting than discretionary accruals.

Panels A and B of Figure 3 show that total accruals are negatively related to CF (slope $\hat{\sigma}_{NA}^2/\hat{\sigma}_{CF}^2 = -0.13$) and positively related to NI (slope $\hat{\sigma}_{DA}^2/\hat{\sigma}_{NI}^2 = 0.35$). Panel C shows the negative relation of estimated normal accruals to CF (with the same slope as in Panel A), Panel F the positive relation of estimated discretionary accruals to NI (with the same slope as in Panel B). \widehat{NA} and \widehat{DA} are unrelated to NI (Panel D) and CF (Panel E), respectively.

Figure 4 illustrates the positive association of CF and NI (slope coefficient of the blue regression line: $\hat{\sigma}_0^2/\hat{\sigma}_{CF}^2 = 0.87$). The blue points highlight ten observations, which we interpret as firm-years. For the same firm-years, the red triangles on the y-axis indicate the Bayesian estimate of true profitability, $\widehat{NI^T}$. The red triangles scatter around the exact same (blue) regression line, but they are much closer to this line than the blue points. This indicates a shrinkage of reported net income to less extreme values when cash flow, net income and prior information are combined to obtain a Bayesian estimate of true profitability. Total accruals correspond to the vertical distance of a blue point to the black 45 degree line. \widehat{DA} is the vertical distance of a blue point to the corresponding red triangle (positive if the blue point lies above the regression line and vice versa). \widehat{NA} is the vertical distance of the red

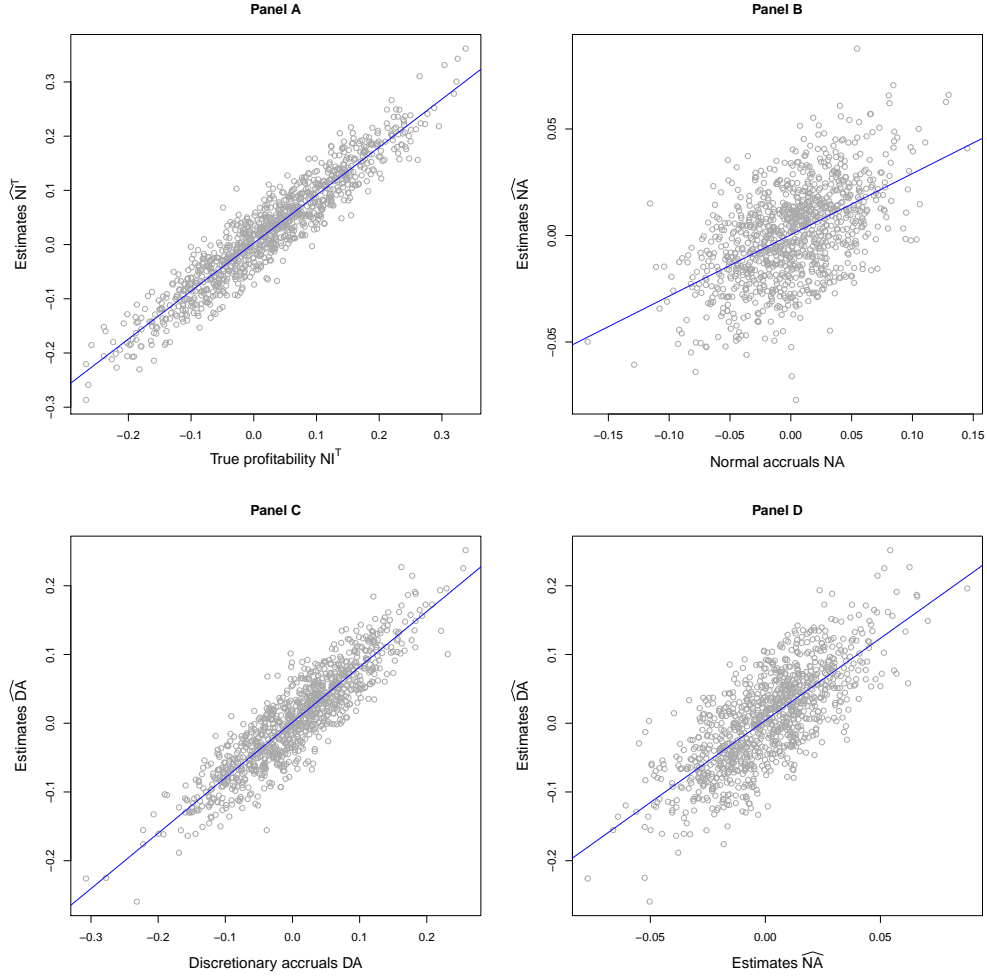


Figure 2: **Net income, normal and discretionary accruals: true values vs. estimates.**

Simulation of 1000 observations (stocks). Panels A, B and C show scatterplots of Bayesian estimates against true values NI^T , NA and DA . Panel D illustrates the positive association of estimated normal and discretionary accruals.

triangles to the black 45 degree line (positive for points above the line and vice versa). Thus, the Bayesian estimate of $\widehat{NI^T}$, which is illustrated by the positioning of the red triangles, determines the attribution of total accruals to the normal and discretionary components.

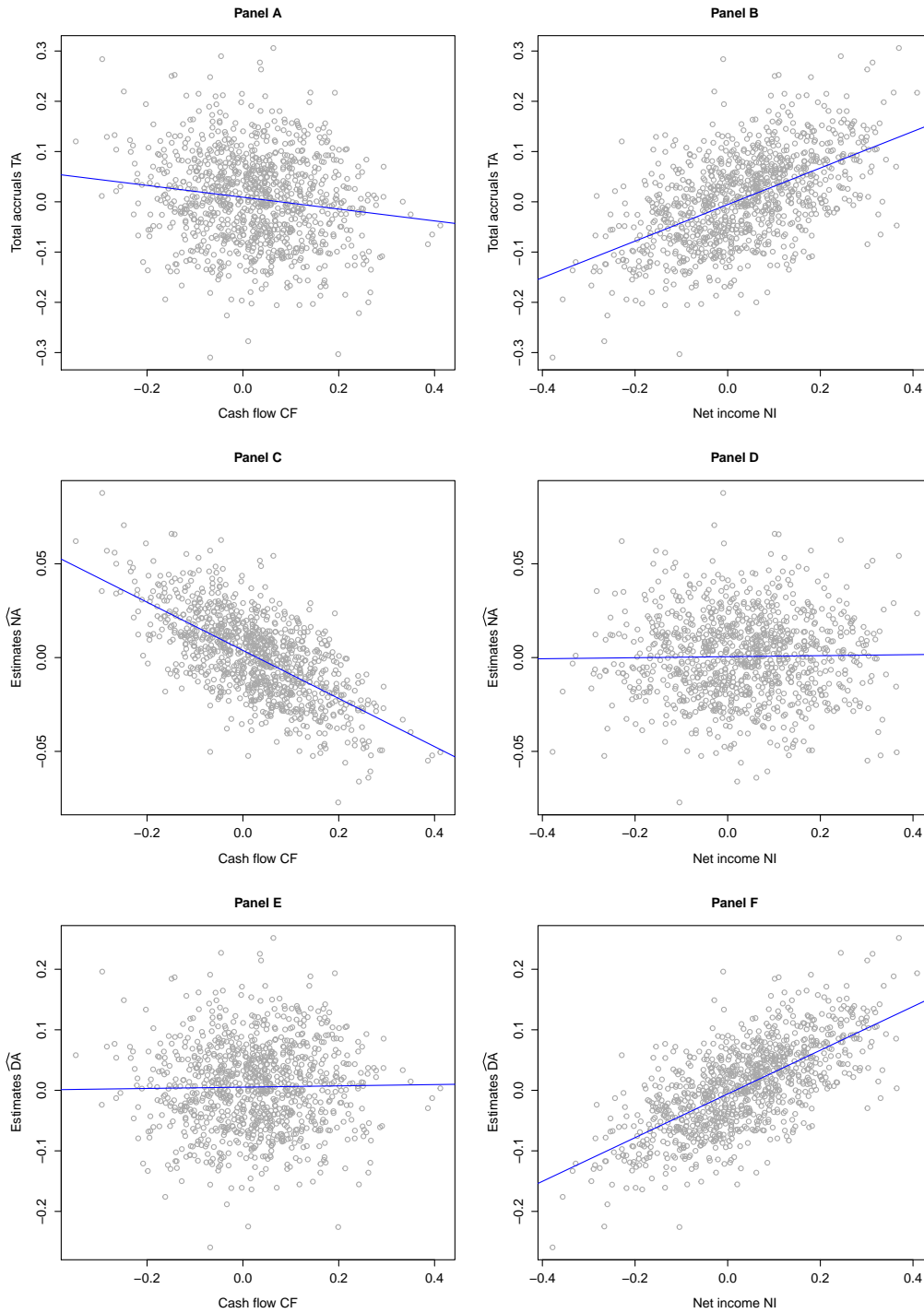


Figure 3: Association of (estimated) accruals, cash flow and net income. Simulation of 1000 observations (stocks). The left panels show the association of accruals to cash flow (CF), the right panels the association to net income (NI). The accrual measures are: total accruals in the upper panels, estimated normal accruals in the middle panels and estimated discretionary accruals in the lower panels. The figure shows that total accruals are negatively related to CF and positively related to net income. Normal accruals are negatively related to CF and unrelated to NI. Discretionary accruals are positively related to NI and unrelated to CF.

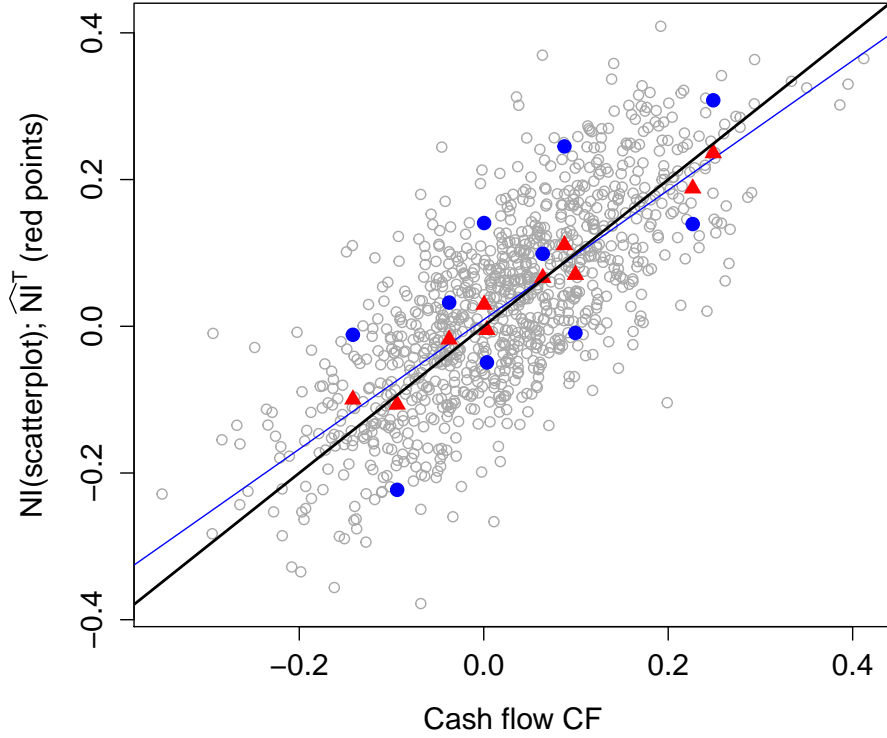


Figure 4: **Net income (NI) versus cash flow (CF).**

Simulation of 1000 observations (stocks). The blue points highlight 10 selected observations. For these observations, the red triangles indicate the estimated true profitability (y-axis) against CF. The blue line is the regression line, the black line the 45 degree line. Total accruals TA are the vertical distances of blue points to the black line. The estimated discretionary accruals are the vertical distances of blue points and red triangles. The estimated normal accruals are the vertical distances of the red triangles to the black line.

2.4. Detecting Earnings Management

2.4.1. Random sampling

When a firm engages in earnings management with a profit impact of EM , its managed net income is equal to $NI_{new} = NI + EM$.⁷ With unchanged cash flow CF , the new Bayesian estimate of NI^T according to Eq. (19) is $\widehat{NI^T}_{new} = \widehat{NI^T} + w_3 EM$. Thus, the new accrual estimates are:

$$\widehat{DA}_{new} = NI_{new} - \widehat{NI^T}_{new} = \widehat{DA} + (w_1 + w_2) EM \quad (22)$$

$$\widehat{NA}_{new} = \widehat{NI^T}_{new} - CF = \widehat{NA} + w_3 EM. \quad (23)$$

This means that only a share of $(w_1 + w_2)$ of managed earnings appears in discretionary accruals, while the remaining part is picked up by normal accruals. Therefore, earnings management can, in principle, be detected in total accruals, discretionary accruals or normal accruals. This raises the question of the most efficient test strategy. Under the null hypothesis of no earnings management, the ratio of mean accruals and accrual standard error for a random sample of firms follows a t-distribution so that a t-test for detecting earnings management can be applied.

When we base the test on total accruals, the full amount EM is compared to the standard deviation of total accruals so that the relevant ratio is:

$$z_{TA} = \frac{EM}{\sigma_{TA}} = \frac{EM}{\sqrt{\sigma_{DA}^2 + \sigma_{NA}^2}}. \quad (24)$$

⁷In the following, we consider earnings management as a pure accounting phenomenon and thus exclude real earnings management.

A test based on discretionary accruals will only capture a part of EM , but the standard deviation is smaller than in Eq. (24):

$$z_{DA} = \frac{(w_1 + w_2) EM}{\sigma_{\widehat{DA}}} = \frac{(w_1 + w_2) EM}{\sqrt{\sigma_{DA}^2 - \sigma^{*2}}}. \quad (25)$$

Finally, for normal accruals, we obtain:

$$z_{NA} = \frac{w_3 EM}{\sigma_{\widehat{NA}}} = \frac{w_3 EM}{\sqrt{\sigma_{NA}^2 - \sigma^{*2}}}. \quad (26)$$

A comparison of the three ratios reveals that $z_{DA} > z_{TA}$ and $z_{DA} > z_{NA}$ so that the most powerful test is based on discretionary accruals (see the derivation in Appendix B). Only when $\sigma_0^2 \rightarrow \infty$ so that $w_1 \rightarrow 0$, z_{TA} and z_{NA} are asymptotically equal to z_{DA} .

Following prior research, we run simulations to test how often earnings management of a given size infused in certain firm-years is detected (see, e.g., Dechow et al., 1995; Kothari et al., 2005; Dechow et al., 2012; Keung and Shih, 2014). We first create 10,000 simulated observations (firm-years).⁸ These remain the same for all further calculations. From these observations, we randomly draw 100 firm-years (treatment group) that are infused with earnings management of a fixed size (in percentage points of scaled net income). We use a two-sided t-test to test whether the mean discretionary accruals (Model DA), the mean normal accruals (Model NA) and the mean total accruals (Model TA) in the treatment group are different from zero (5% significance level). The estimates of DA and NA correspond to the Bayesian estimates \widehat{DA} and \widehat{NA} . We repeat the random drawing of the treatment sample 1,000 times. Table 1 reports the percentage of samples with a significant t-statistic that has

⁸This number of simulated firm-years is large enough so that earnings management of some firms does not have an effect on the estimated model parameters, for example the weights w_1 , w_2 and w_3 .

the right sign (1st number) and the percentage of samples with a significant t-statistic that has the wrong sign (2nd number). We consider earnings management between -3 and +3 percentage points as indicated at the top of the columns.

Table 1 shows that all three models, as expected, provide approximately 5% significant outcomes when in fact no earnings management is present ($EM = 0\%$). Positive and negative EM values are detected with similar likelihoods. Model DA is clearly superior to Model NA, which is in line with our theoretical considerations. However, Model DA is only marginally better than Model TA. The reason is that our parameter set puts a strong emphasis on cash flow information ($w_2 = 0.702$), so that the total accruals ($NI - CF$) are close enough to estimated discretionary accruals ($NI - \widehat{NI}^T$) to produce similar results in this setting of random sampling.⁹

Figure 5 highlights one sample of treated firm-years ($EM = 0$) in blue against the background of all 10,000 simulated firm-years. The figure is structured in the same way as Figure 3 (left panels: accruals vs. CF; right panels: accruals vs. NI; top / middle / bottom panels: accruals correspond to TA / \widehat{NA} / \widehat{DA}). Because of random sampling, all three types of accruals form a suitable basis for detecting earnings management, albeit with a clear advantage for discretionary accruals.

2.4.2. Sampling Related to Net Income

The situation changes if the firms suspected of earnings management cannot be considered a random sample. In particular, the indicator variable of earnings management might be related to profitability. To simulate this situation, we assume that the treated firm-years stem from the upper half of all observations sorted in descending order of net income (before

⁹If we halve the σ_0 parameter, weight w_2 decreases to 0.189 and Model DA is clearly superior to Model TA. Model NA performs much worse in this setting.

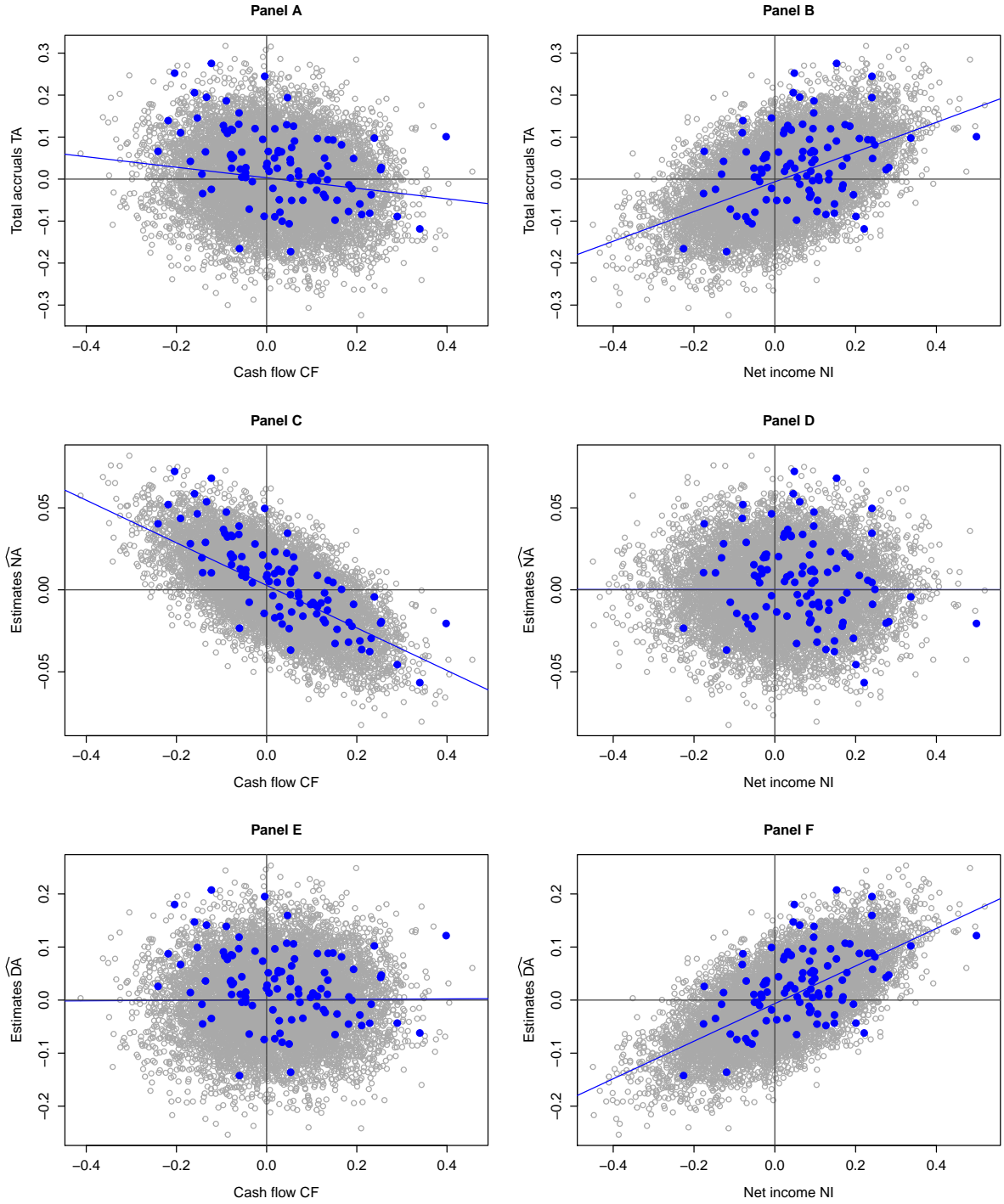


Figure 5: **Illustration of random sampling.**

The figure illustrates one sample of the simulation as described in Table 5. The grey scatterplot shows the 10,000 firm-years, the blue points highlight one random sample of 100 firm-years without earnings management. The panels are structured as in Figure 3.

Table 1: **Earnings management in simulated data: random sampling.**

We first create 10,000 simulated observations (firm-years). For each column in the table, we then repeat the following three steps 1,000 times: 1) randomly draw 100 firm-years (without replacement); 2) treat these 100 firm-years with earnings management as indicated in the column head; 3) run a t-test (two-sided) for the three models to test the null hypothesis of no earnings management.

The first number in each table cell indicates in what percentage of the 1,000 samples the null hypothesis was rejected on the 5% significance level with the right sign of the t-statistic. The second number indicates the percentage of significant cases with the wrong sign of the t-statistic. The size of infused earnings management is reported at the top of the columns.

	-3%	-2%	-1%	0%	1%	2%	3%
Model DA	92 / 0	60.8 / 0	19.7 / 0	NA / 5.4	18.8 / 0.3	64.3 / 0	92.1 / 0
Model NA	71.2 / 0	38 / 0	10.1 / 0.3	NA / 4.8	13.2 / 0.2	42.2 / 0	75.2 / 0
Model TA	91.4 / 0	58.6 / 0	17.7 / 0	NA / 5.6	19.6 / 0.2	62.8 / 0	91.4 / 0

the effect of earnings management). Table 2 and Figure 6 show the results in the same way as in the last section.

In this situation, a test based on discretionary accruals is heavily biased because the estimated discretionary accruals are positively related to net income (see Panel F in Fig. 6). Therefore, the selection of firm-years with high NI automatically results in high DA estimates. Even in the case of $EM = -3\%$, more than half of the t-statistics of Model DA is significantly positive. Model TA is strongly biased in the same direction. This is exactly the setting where a performance-adjustment is necessary. It consists of using the residuals of a regression of TA on NI or, equivalently, \widehat{DA} on NI (see Panels B and F). It is known from properties 5. and 7. in Section 2.2 that these residuals are proportional to \widehat{NA} . Therefore, the performance-adjustment means that we turn to Model NA (although it is less powerful under ideal conditions) in order to avoid the bias of Model DA. Panel D indeed illustrates that this sample selection leaves \widehat{NA} unbiased. The results of Model NA reported in Table 2 are similar to the results in Table 1 for random sampling.

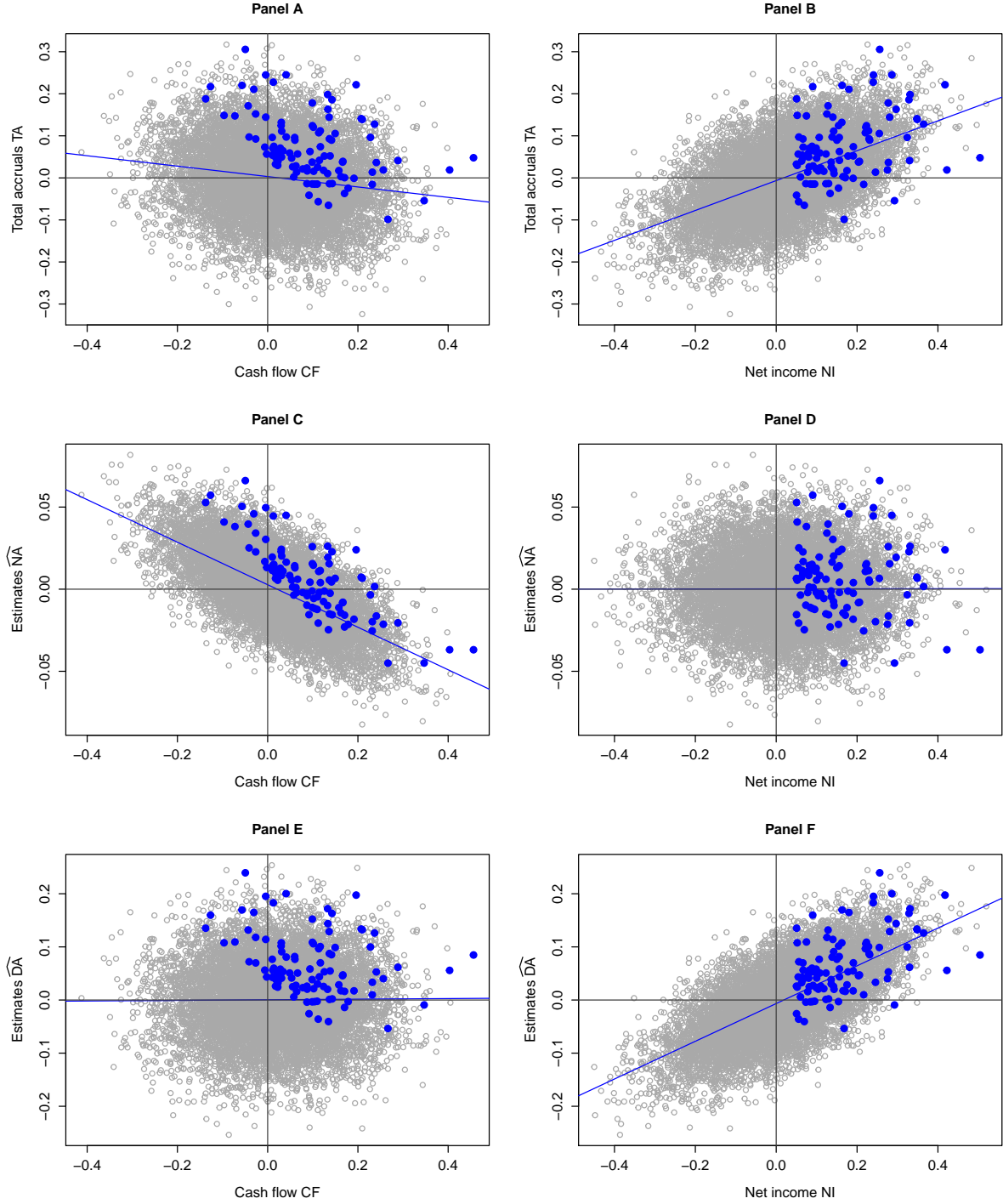


Figure 6: **Illustration of sampling among firms with high NI.**

The setup is the same as in Figure 5, but the treatment sample is now chosen from the upper half of net income. Again, no earnings management has been infused ($EM = 0$).

Table 2: **Earnings management in simulated data: sampling among firms with high NI.**

The setup is the same as in Table 1, but the treatment sample is now chosen from the upper half of net income.

	-3%	-2%	-1%	0%	1%	2%	3%
Model DA	0 / 56.7	0 / 94	0 / 99.8	NA / 100	100 / 0	100 / 0	100 / 0
Model NA	71.8 / 0	40.3 / 0	15.2 / 0.3	NA / 5.2	11.3 / 0.1	38.3 / 0.1	70.3 / 0
Model TA	0.3 / 14.3	0 / 55	0 / 89.9	NA / 99.8	100 / 0	100 / 0	100 / 0

2.4.3. Sampling Related to Estimated True Performance

We now assume that firms suspected of earnings management belong to the upper half of all firm-years sorted in descending order of \widehat{NI}^T . Since our estimate of true profitability is positively related to NI , we might consider a performance adjustment as in the last section appropriate. However, Table 3 reports extremely poor results for the corresponding Model NA. The model is so strongly biased downwards that in 44% of the cases with earnings management of $EM = +3\%$ the t-statistic is significantly *negative*. Model DA performs much better, but with a bias in the opposite direction. This time, total accruals perform remarkably well. Model TA's performance is similar to the results for random sampling in Section 2.4.1.

Figure 7 illustrates why this is the case. It is structured in the same way as Figures 5 and 6, but with an additional column of panels on the right for the relationship of the three types of accruals to \widehat{NI}^T . Panel F reveals the negative association of \widehat{NA} to \widehat{NI}^T . Thus, selecting treatment firm-years with high \widehat{NI}^T will result in negative mean \widehat{NA} values. The positive association of \widehat{DA} to \widehat{NI}^T as shown in Panel I is less pronounced so that Model DA is less strongly biased. To extract earnings management information from \widehat{NA} and \widehat{DA} , the performance effect has to be removed, which can be performed by a regression on \widehat{NI}^T . According to properties 8. and 9. in Section 2.2, the residuals of these regressions are

proportional to total accruals, which explains the good performance of Model TA. Panel C in Figure 7 confirms that TA is not related to \widehat{NI}^T (flat blue regression line) and therefore unbiased when treatment firm-years are selected with respect to \widehat{NI}^T .

These results suggest that adjusting for performance in earnings management studies is important. It might be difficult in practice to differentiate between patterns with respect to NI versus \widehat{NI}^T , but the implications strongly differ. Scatterplots as shown in Figure 7 can be helpful to identify the best procedure. It is important to note, however, that in practical applications, the observations of the treatment group will be affected by earnings management (other than our graphs with $EM = 0$), which makes it more difficult to identify patterns that are caused by the selection criteria. Consistent with the graphic diagnostic, we propose to apply a regression approach to identify the relevant performance adjustment setting. In the total sample, regressions of TA on \widehat{NI}^T , \widehat{NA} on NI , and \widehat{DA} on CF all (by construction) provide slope coefficients of zero. The corresponding accruals (TA , \widehat{NA} , or \widehat{DA}) are appropriate for a test of earnings management if the same slope coefficient of zero also applies to the treatment sample. Therefore, a simple diagnostic tool are regressions of TA on \widehat{NI}^T , \widehat{NA} on NI , and \widehat{DA} on CF for the sample of firm-years suspected of earnings management.

Table 3: **Earnings management in simulated data: sampling among firms with high estimates of true performance.**

The setup is the same as in Table 1, but the treatment sample is now chosen from the upper half of the Bayesian estimates of true performance.

	-3%	-2%	-1%	0%	1%	2%	3%
Model DA	52.4 / 0	14.3 / 0.1	1.3 / 4.2	NA / 28.3	72 / 0	96.4 / 0	99.7 / 0
Model NA	100 / 0	100 / 0	100 / 0	NA / 100	0 / 94.3	0 / 79.1	0 / 44
Model TA	91 / 0	56.9 / 0	20.4 / 0.1	NA / 4.8	20.4 / 0	60.3 / 0	90.9 / 0

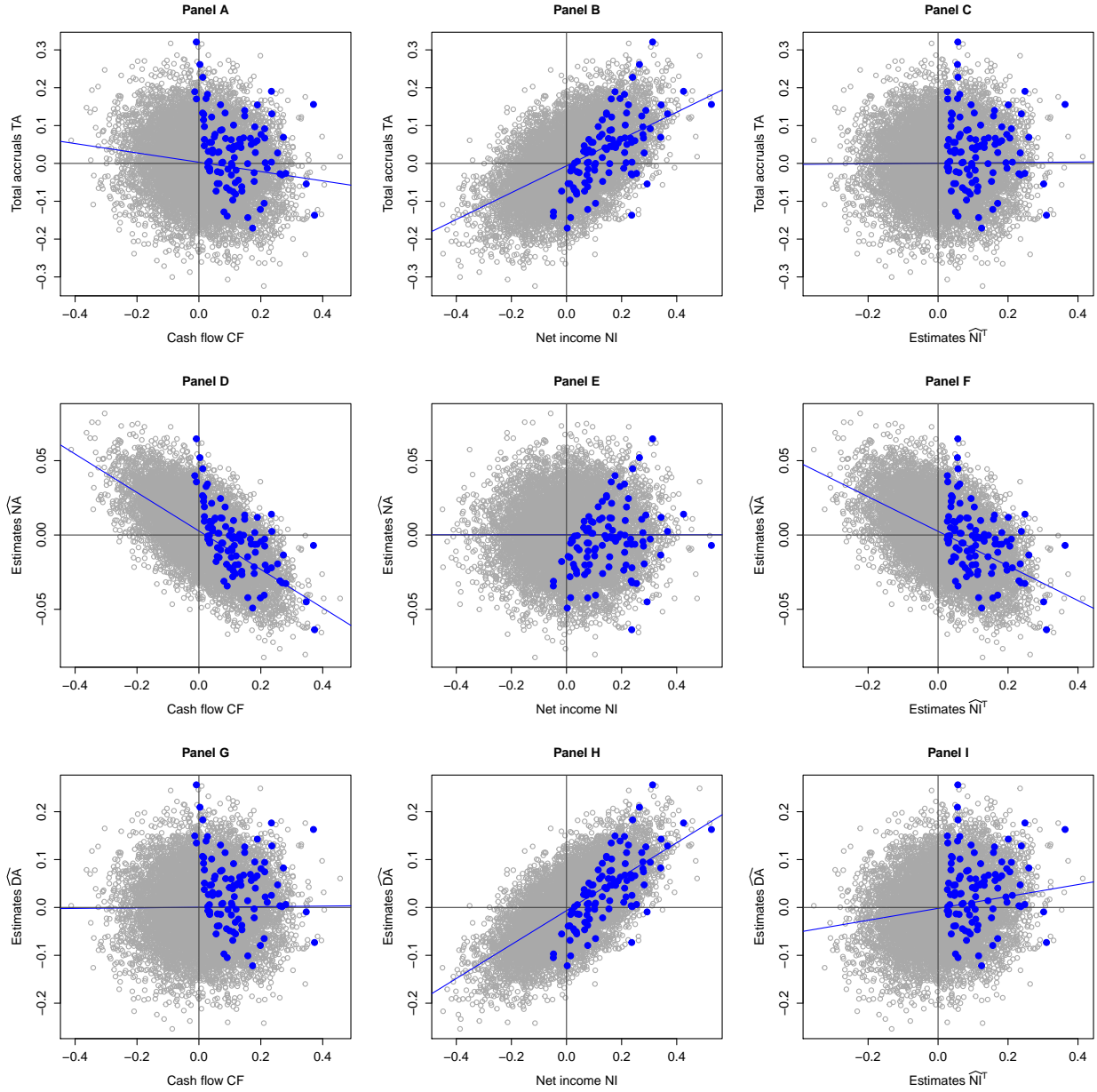


Figure 7: **Illustration of sampling among firms with high estimates of true performance.**

The setup is the same as in Figure 5, but the treatment sample is now chosen from the upper half of \widehat{NI}^T . Again, no earnings management has been infused ($EM = 0$).

3. Empirical Application

3.1. Data

In our empirical application of the model, we use Worldscope data for the U.S. from Refinitiv Datastream (former Thomson Reuters). Our investigation into how well artificially imposed earnings management is recognized (Section 3.4) is based on data from 2015 to 2019. We go further back to 2004 in order to show that the positive relation of total accruals to net income and the negative relation to cash flow is a persistent phenomenon that appears every year (Section 3.3). We exclude financial institutions with SIC codes ranging from 4400 to 5000 and 6000 to 7000. We also exclude firms without sales revenues and firm-years with net income of exactly zero.

The following positions must be available: net income (NI) (defined as operating and non-operating income after extraordinary items); cash flow from operating activities (CFO); depreciation and amortization; and total assets at the end of the prior fiscal year. Our cash flow measure CF is defined as CFO minus depreciation and amortization. Without subtracting these expenses, they would be part of normal accruals and would make the expected value μ_{NA} negative and firm-specific. For the Bayesian estimate of NI^T according to Eq. (19), $CF + \mu_{NA}$ is relevant. Here, it does not make a difference whether depreciation and amortization is included in μ_{NA} or in CF . Including it in CF has the advantage that the remaining μ_{NA} value can be considered to be the same for all firms. All variables (net income, cash flow, accruals) are scaled by lagged total assets.

Finally, we filter out observations with (scaled) CF or (scaled) NI below -0.5 or above $+0.5$. We consider these as outliers for the purpose of this study. If a firm loses or gains more than 50% of total assets in one year, there is no realistic chance to make an assessment

on earnings management without examining the circumstances of the individual case. The exact threshold is somewhat arbitrary. Scatterplots shown in the next section indicate that observations near the truncation threshold are rare and quite extreme. Nevertheless, 7.2% of all observations (2015-2019) are discarded due to this condition. We repeated the analysis with a threshold of ± 0.6 and obtain similar results.

3.2. Model Estimation and Descriptive Statistics

To estimate the parameters of our model, we proceed in three steps. We first estimate σ_{CF}^2 and σ_{NI}^2 from the pooled sample of firm-years from 2015 to 2019, resulting in estimates $\hat{\sigma}_{CF}^2$ and $\hat{\sigma}_{NI}^2$. In the second step, we run yearly regressions of NI on CF . According to Eq. (10), the slope coefficient of this regression is equal to σ_0^2/σ_{CF}^2 so that the estimated regression coefficients provide an estimate of σ_0^2 in each year. The mean over the sample years is our final estimate $\hat{\sigma}_0^2$. In the third step, we determine the implied variation of normal and discretionary accruals as $\hat{\sigma}_{DA}^2 = \hat{\sigma}_{NI}^2 - \hat{\sigma}_0^2$ and $\hat{\sigma}_{NA}^2 = \hat{\sigma}_{CF}^2 - \hat{\sigma}_0^2$.

In this way, we obtain the following estimates: $\hat{\sigma}_0 = 0.1056$; $\hat{\sigma}_{CF} = 0.1133$; $\hat{\sigma}_{NI} = 0.1315$; $\hat{\sigma}_{DA} = 0.0783$; $\hat{\sigma}_{NA} = 0.0410$; $\hat{\sigma}^* = 0.0343$. The corresponding weights in the Bayesian estimate of NI^T are: $w_1 = 0.106$, $w_2 = 0.702$, $w_3 = 0.192$. We assume a prior expected value of $\mu_0 = 0.02$. It is smaller than typical WACC estimates because our profit measure is net income (after interest expense) while the scaling variable is total assets (including liabilities). We set $\mu_{DA} = 0$ and determine μ_{NA} such that the mean value of $\widehat{DA} = NI - \widehat{NI}^T$ is zero, consistent with $\mu_{DA} = 0$.

Table 4 shows descriptive statistics of our main variables for pooled firm-year observations from 2015 to 2019. Figure 8 illustrates the relation of NI and CF for 1,000 observations selected randomly from the total sample of 9,859 observations. The blue line is the regression

line based on the total sample (slope coefficient of 0.87). Figure 9 is the empirical counterpart to Figure 3 for simulated data, showing the relation of the different accruals to CF and NI . In particular, Panels C and D confirm that the estimated normal accruals are negatively related to CF and unrelated to NI , while the estimated discretionary accruals are positively related to NI and unrelated to CF .

Table 4: **Descriptive statistics for pooled firm-years from 2015 to 2019.**

Note that the standard deviations (St.Dev.) of \widehat{NA} and \widehat{DA} differ from $\hat{\sigma}_{NA}$ and $\hat{\sigma}_{DA}$, respectively, as stated in Property 1. in Section 2.2.

Statistic	N	Mean	St. Dev.	Min	Pctl(25)	Pctl(75)	Max
NI	9,859	0.016	0.132	-0.499	-0.016	0.084	0.430
CFO	9,859	0.085	0.119	-0.499	0.047	0.145	0.714
CF	9,859	0.039	0.113	-0.499	0.002	0.095	0.396
TA	9,859	-0.023	0.089	-0.762	-0.049	0.013	0.852
\widehat{NI}^T	9,859	0.016	0.100	-0.449	-0.017	0.066	0.344
\widehat{NA}	9,859	-0.023	0.022	-0.193	-0.032	-0.014	0.202
\widehat{DA}	9,859	-0.000	0.071	-0.569	-0.021	0.030	0.650

3.3. Total Accruals, Net Income and Cash Flow

In this section, we report the stylized facts that are at the heart of this paper: the negative relation of total accruals to cash flow (Table 5) and the positive relation of total accruals to net income (Table 6). The slope coefficients are significant in each year of the sample period from 2004 to 2019. The relationship of TA to NI is particularly pronounced, with R^2 values mostly between 20% and 30%.

3.4. Detecting Earnings Management

Analogous to the study of simulated data in Section 2.4, we now examine how well artificially imposed earnings management is recognized. The imposed earnings management (EM) again ranges from -3% to $+3\%$. The number N of treated firm-years is 10, 25, 50

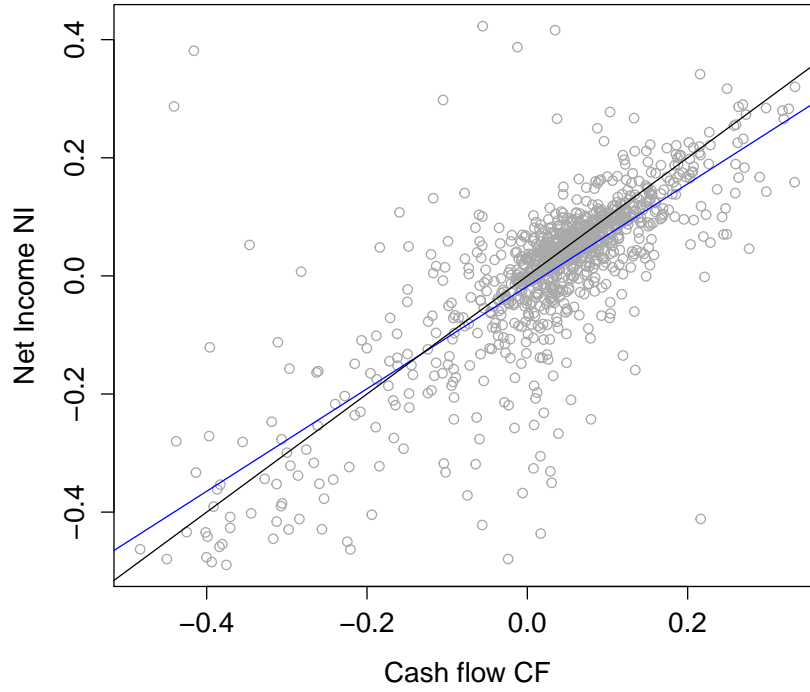


Figure 8: **Empirical relation of net income and cash flow.**

1,000 firm-years randomly sampled from the total sample of 9,859 firm-years (pooled data from 2015 to 2019). Blue: regression line based on all 9,859 firm-years; black: 45 degree line.

Table 5: **Cross-sectional relation of total accruals and cash flow for US firms.**

Linear regressions of TA on CF for each year from 2004 to 2019. t-statistics in brackets.

<i>Dependent variable: TA</i>								
	2004	2005	2006	2007	2008	2009	2010	2011
CF	-0.232*** (-18.0)	-0.241*** (-16.7)	-0.213*** (-14.0)	-0.191*** (-12.6)	-0.189*** (-8.4)	-0.229*** (-12.4)	-0.279*** (-17.2)	-0.238*** (-14.4)
N	2,366	2,352	2,366	2,280	1,905	2,132	2,155	2,024
R ²	0.121	0.106	0.076	0.065	0.036	0.067	0.120	0.093
<i>Dependent variable: TA</i>								
	2012	2013	2014	2015	2016	2017	2018	2019
CF	-0.189*** (-12.0)	-0.164*** (-10.5)	-0.161*** (-10.1)	-0.163*** (-8.6)	-0.108*** (-6.3)	-0.178*** (-10.3)	-0.147*** (-8.6)	-0.093*** (-5.9)
N	1,987	2,077	2,097	2,056	2,033	1,977	1,913	1,914
R ²	0.067	0.050	0.047	0.035	0.019	0.051	0.037	0.018

Note:

*p<0.1; **p<0.05; ***p<0.01

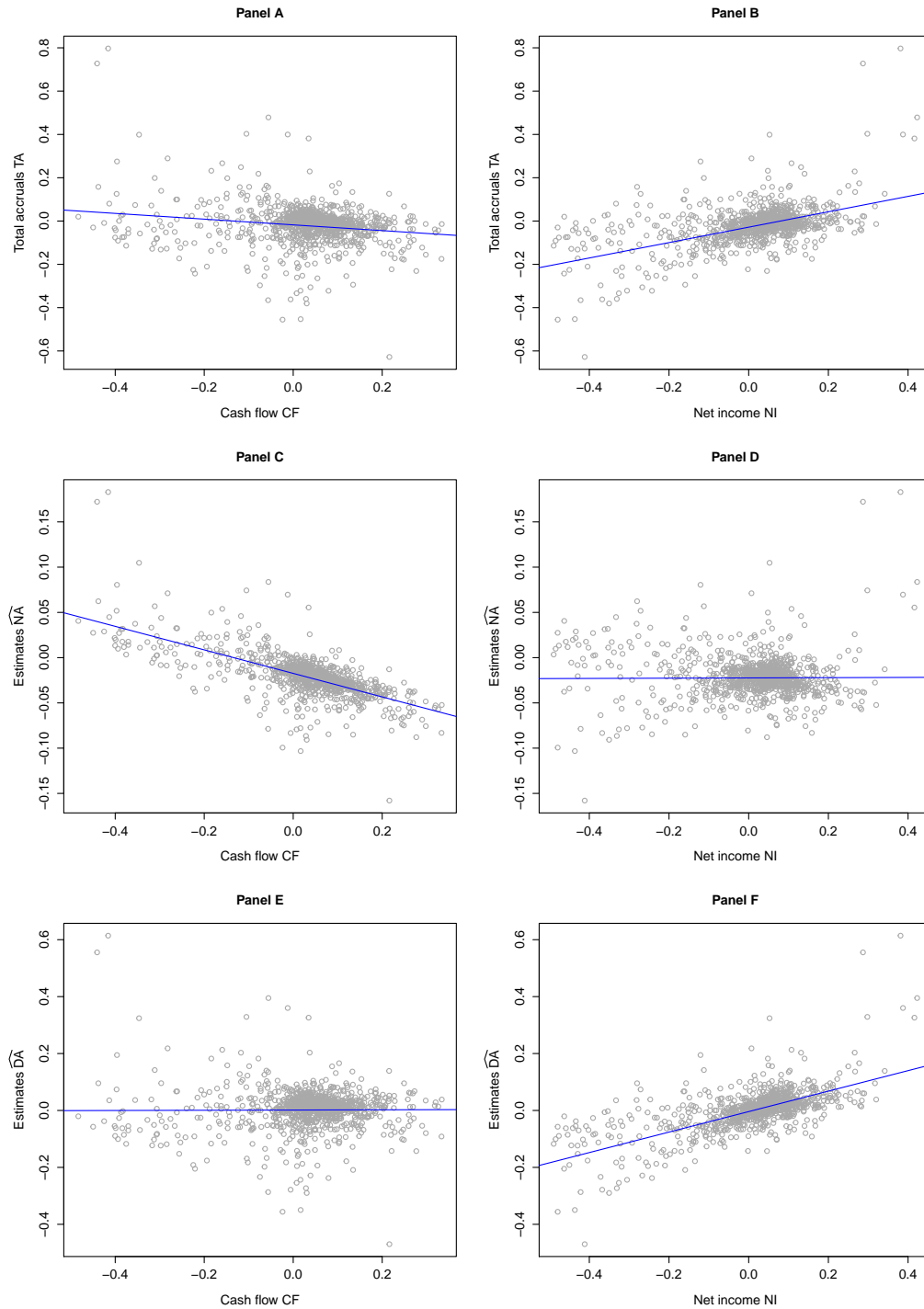


Figure 9: **Accrual components and their relation to cash flow and net income.** 1,000 firm-years randomly sampled from the total sample of 9,859 firm-years (pooled data from 2015 to 2019). Blue: regression line based on all 9,859 firm-years.

Table 6: **Cross-sectional relation of total accruals and net income for US firms.**
Linear regressions of TA on NI for each year from 2004 to 2019. t-statistics in brackets.

<i>Dependent variable: TA</i>								
	2004	2005	2006	2007	2008	2009	2010	2011
NI	0.216*** (16.5)	0.288*** (21.3)	0.327*** (25.1)	0.313*** (24.3)	0.498*** (35.9)	0.420*** (30.1)	0.338*** (22.6)	0.326*** (22.4)
N	2,366	2,352	2,366	2,280	1,905	2,132	2,155	2,024
R ²	0.103	0.161	0.210	0.206	0.404	0.298	0.192	0.198
<i>Dependent variable: TA</i>								
	2012	2013	2014	2015	2016	2017	2018	2019
NI	0.297*** (21.7)	0.308*** (23.7)	0.319*** (24.8)	0.418*** (31.7)	0.358*** (29.1)	0.350*** (25.6)	0.337*** (25.4)	0.299*** (24.7)
N	1,987	2,077	2,097	2,056	2,033	1,977	1,913	1,914
R ²	0.191	0.213	0.226	0.329	0.295	0.250	0.252	0.242

Note:

*p<0.1; **p<0.05; ***p<0.01

or 100. As before, we consider three types of sampling: random sampling among all firm-years (Table 7); sampling from firms with net income NI above the median (Table 8); and sampling from firms with estimated true performance \widehat{NI}^T above the median (Table 9).

Model DA is based on \widehat{DA} , which is equivalent to using the residuals of a regression of TA on CF or a regression of \widehat{NA} on CF (Properties 4. and 6. in Section 2.2). Model NA is based on \widehat{NA} , with analogous equivalence relations for regressions on NI (performance adjustment; Properties 5. and 7.). Model TA is based on TA , and we obtain the same results when regressing \widehat{DA} or \widehat{NA} on \widehat{NI}^T (modified performance adjustment). In all three models, we mean-adjust the accruals so that the mean (before imposing earnings management) of all firm-years is zero. We then apply a simple t-test with an expected accrual value of zero under the null hypothesis of no earnings management. We examine 1,000 samples and report the percentage of significant t-statistics with the right / wrong sign in each cell of Tables 7, 8 and 9.

The results confirm our findings from simulations. Random sampling is the least problematic case: Model DA is most efficient, but the other models are also unbiased. Model NA is the only suitable model when sampling with respect to NI , and Model TA is the only suitable model when sampling with respect to $\widehat{NI^T}$. The large discrepancies between the two types of performance adjustment are again remarkable given the fact that the two cases will be difficult to distinguish in practical applications when earnings management is not imposed and the sampling pattern is not evident.

Table 7: **Earnings management in empirical data: random sampling.**

	-3%	-2%	-1%	0%	1%	2%	3%
N = 10							
Model DA	18 / 0	7 / 0.2	1.6 / 0.9	NA / 6.3	12.4 / 0.2	19.5 / 0.1	31.7 / 0
Model NA	19.3 / 0.1	8.2 / 0.2	5.5 / 0.4	NA / 3.4	3.5 / 1	9 / 0.1	14.5 / 0.1
Model TA	21.2 / 0	8.1 / 0.1	2.2 / 0.4	NA / 6.4	10 / 0.3	18.9 / 0	30.6 / 0
N = 25							
Model DA	44.9 / 0	20.7 / 0.1	6.1 / 0.6	NA / 4.3	14.9 / 0.1	29.7 / 0	45.6 / 0
Model NA	30.3 / 0	17.8 / 0.1	8.3 / 0.4	NA / 3.6	7.1 / 0.8	13.9 / 0	25.1 / 0.1
Model TA	44.3 / 0	21.9 / 0.1	6.4 / 0.5	NA / 4.1	13.4 / 0.1	29 / 0	45.6 / 0
N = 50							
Model DA	71.7 / 0	36.6 / 0	9.4 / 0.3	NA / 3.9	15.3 / 0.1	42 / 0	65.1 / 0
Model NA	49.5 / 0	23.9 / 0	9.2 / 0.3	NA / 4.8	8.5 / 0.3	23.5 / 0	43.8 / 0
Model TA	71.1 / 0	36.1 / 0	10.8 / 0.3	NA / 4	13.8 / 0.1	40.7 / 0	63.8 / 0
N = 100							
Model DA	93.2 / 0	63.8 / 0	18.1 / 0	NA / 6.2	24.8 / 0	61.4 / 0	89 / 0
Model NA	74.7 / 0	41.1 / 0	13.7 / 0.5	NA / 5	13.4 / 0.3	43.6 / 0	73.3 / 0
Model TA	92.1 / 0	62.3 / 0	19.1 / 0	NA / 6	22.8 / 0	60.8 / 0	88.4 / 0

4. Integration of the Jones model

In the Jones (1991) model, total accruals are regressed on gross property, plant, and equipment (PPE) and the change in revenues (ΔREV), where all variables are deflated by lagged total assets. The fitted values of the regression are considered normal accruals

Table 8: **Earnings management in empirical data: sampling among firms with high NI.**

	-3%	-2%	-1%	0%	1%	2%	3%
N = 10							
Model DA	1.6 / 0.9	0 / 8	0.1 / 23.6	NA / 47.9	71.6 / 0	83.3 / 0	92.7 / 0
Model NA	25.7 / 0	18.5 / 0	9 / 0.1	NA / 6.1	2.3 / 1.8	6.5 / 0.1	11.3 / 0.4
Model TA	6.7 / 0.4	1.1 / 2.1	0.2 / 8.5	NA / 23.7	48.5 / 0	65.6 / 0	80.3 / 0
N = 25							
Model DA	1.7 / 2.8	0 / 22.7	0 / 62.9	NA / 89.4	98.3 / 0	99.8 / 0	100 / 0
Model NA	47.6 / 0	31.5 / 0	16.8 / 0.2	NA / 6.8	2.6 / 1.6	9.4 / 0.5	23.1 / 0.1
Model TA	6.4 / 0.4	0.8 / 5.2	0 / 25.5	NA / 57.7	85 / 0	96.8 / 0	99.1 / 0
N = 50							
Model DA	1 / 6.9	0 / 46.1	0 / 92.1	NA / 99.7	100 / 0	100 / 0	100 / 0
Model NA	63.2 / 0	43.6 / 0.1	19.6 / 0.1	NA / 7.1	4.9 / 0.6	17 / 0.1	44.3 / 0
Model TA	7.3 / 0.5	0.4 / 8.9	0 / 51.4	NA / 90.4	99.3 / 0	100 / 0	100 / 0
N = 100							
Model DA	0.3 / 15.6	0 / 83.2	0 / 99.9	NA / 100	100 / 0	100 / 0	100 / 0
Model NA	83.8 / 0	61.9 / 0	28.3 / 0	NA / 8.2	8.2 / 0.8	36.9 / 0	77.9 / 0
Model TA	5.5 / 0.5	0 / 23.2	0 / 85.1	NA / 100	100 / 0	100 / 0	100 / 0

Table 9: **Earnings management in empirical data: sampling among firms with high estimates of true performance.**

	-3%	-2%	-1%	0%	1%	2%	3%
N = 10							
Model DA	13.5 / 0	4.5 / 0.2	0.8 / 4.7	NA / 12.7	27.2 / 0	39.7 / 0	57.1 / 0
Model NA	71.1 / 0	55.9 / 0	46.9 / 0	NA / 27.6	0.1 / 14.3	0.1 / 5.5	1.4 / 2.6
Model TA	27.1 / 0	10.6 / 0.1	2.7 / 1.1	NA / 4.7	13.5 / 0	23.1 / 0	37.9 / 0
N = 25							
Model DA	32.3 / 0	8.2 / 0.5	1.1 / 3.6	NA / 18	42.5 / 0	65.4 / 0	80.1 / 0
Model NA	96.5 / 0	90.9 / 0	84.2 / 0	NA / 64.7	0 / 40.6	0.1 / 16.8	0.8 / 5.3
Model TA	61.6 / 0	26.2 / 0	8.2 / 0.3	NA / 5.5	17.6 / 0.1	42 / 0	64.6 / 0
N = 50							
Model DA	52.6 / 0	13.5 / 0.1	0.8 / 4.4	NA / 28.7	59.5 / 0	82 / 0	93.5 / 0
Model NA	99.9 / 0	99.6 / 0	97.8 / 0	NA / 88.2	0 / 65.9	0 / 34.5	0.2 / 8.9
Model TA	82.3 / 0	48.1 / 0	13.1 / 0.3	NA / 5.1	27.2 / 0	59.9 / 0	81.7 / 0
N = 100							
Model DA	81.1 / 0	26.9 / 0	1.3 / 4.6	NA / 36.8	78.4 / 0	96.5 / 0	99.7 / 0
Model NA	100 / 0	100 / 0	99.9 / 0	NA / 99.9	0 / 93.4	0 / 60.8	0.2 / 16.1
Model TA	98.9 / 0	78.4 / 0	22.7 / 0.3	NA / 5.9	33.7 / 0	78.2 / 0	96.4 / 0

while the residuals are the estimated discretionary accruals. The model can also be applied to working capital accruals instead of total accruals. It typically explains a small part of accruals, in the order of 5% R^2 .

In the terminology of our framework, the fitted values of the Jones model can be interpreted as expected normal accruals. Therefore, the residuals of the Jones model can be considered as modified total accruals, where expected normal accruals have been removed. Our framework can then be applied to these modified total accruals. We verify that they show the same cross-sectional patterns with respect to net income and modified cash flow (cash flow plus expected normal accruals according to the Jones model) as the original total accruals (see Appendix C for more details).

5. Summary and Conclusions

The starting point of this paper is the empirical observation that total accruals are positively related to earnings and negatively related to operating cash flows in the cross-section of firms. We propose a model in which these relationships arise from some degree of uncertainty about true firm performance combined with random deviations in cash flows and reported earnings from this true performance. Assuming normally distributed variables, we obtain Bayesian estimates of normal and discretionary accruals that optimally combine three sources of information about true performance and true accruals: prior information, cash flow information and earnings information.

In our model, earnings management is captured partly in estimated normal accruals and partly in estimated discretionary accruals. Therefore, in principle, earnings management can be detected in total accruals and its two components. With the random sampling of firms suspected of managing earnings, the use of discretionary accruals is most efficient,

but normal accruals and total accruals also obtain unbiased results. When sampling with respect to earnings, a performance adjustment is necessary, consisting of regressing the total accruals on earnings, which, in our model, is equivalent to using normal accruals. Since using normal accruals is (under ideal conditions) less efficient than using discretionary accruals, the performance adjustment is unavoidable but comes at a cost. When sampling with respect to the Bayesian estimate of true performance, a modified performance adjustment of normal accruals and discretionary accruals with respect to estimated true performance is necessary, which is equivalent to using total accruals. Our results suggest that the type of performance adjustment is crucial. We propose graphic diagnostics and a simple regression approach to identify the relevant setting.

Appendix A: Proof of Property 5. in Section 2.2

To show that

$$\widehat{NA} - \mu_{NA} = (w_1 + w_3) \epsilon.$$

we write the left-hand side as:

$$\begin{aligned} \widehat{NA} - \mu_{NA} &= \widehat{NI^T} - CF - \mu_{NA} \\ &= w_1 \mu_0 + w_2 (CF + \mu_{NA}) + w_3 (NI - \mu_{DA}) - CF - \mu_{NA} \end{aligned}$$

and insert on the right-hand side:

$$\begin{aligned} \epsilon &= TA - \alpha - \frac{w_1}{w_1 + w_3} NI \\ &= NI - CF - \alpha - \frac{w_1}{w_1 + w_3} NI, \end{aligned}$$

with

$$\begin{aligned} \alpha &= \overline{TA} - \frac{w_1}{w_1 + w_3} \overline{NI} \\ &= (\mu_{DA} + \mu_{NA}) - \frac{w_1}{w_1 + w_3} (\mu_0 + \mu_{DA}). \end{aligned}$$

Collecting terms related to NI , CF , μ_0 , μ_{DA} and μ_{NA} leads directly to the stated result.

Appendix B: Most efficient test with random sampling

In the following, we derive the property mentioned in Section 2.4.1 that the most powerful test for detecting earnings management in a setting with random sampling is based on discretionary accruals. The following applies: $w_1, w_2, w_3 > 0$; $w_1 + w_2 + w_3 = 1$; $\sigma^{*2} = w_1\sigma_0^2 = w_2\sigma_{NA}^2 = w_3\sigma_{DA}^2$; $\sigma_{DA}^2 - \sigma^{*2} > 0$; $\sigma_{NA}^2 - \sigma^{*2} > 0$. We first compare ratio z_{DA} as defined in Eq. (25) with ratio z_{TA} as defined in Eq. (24):

$$\begin{aligned}
z_{DA} &> z_{TA} \\
&\Leftrightarrow (w_1 + w_2) \sqrt{\sigma_{DA}^2 + \sigma_{NA}^2} > \sqrt{\sigma_{DA}^2 - \sigma^{*2}} \\
&\Leftrightarrow (w_1 + w_2)^2 (\sigma_{DA}^2 + \sigma_{NA}^2) > \sigma_{DA}^2 - \sigma^{*2} \\
&\Leftrightarrow \sigma^{*2} > \sigma_{DA}^2 - (w_1 + w_2)^2 (\sigma_{DA}^2 + \sigma_{NA}^2) \\
&\Leftrightarrow \sigma^{*2} > \sigma_{DA}^2 - (1 - w_3)^2 \sigma_{DA}^2 - (w_1 + w_2)^2 \sigma_{NA}^2
\end{aligned}$$

Multiplying out and using $\sigma^{*2} = w_1\sigma_0^2 = w_2\sigma_{NA}^2 = w_3\sigma_{DA}^2$, we obtain:

$$\begin{aligned}
\sigma^{*2} &> \sigma^{*2} \left(2 - w_3 - 2w_1 - w_2 - w_1 \frac{\sigma_{NA}^2}{\sigma_0^2} \right) \\
&\Leftrightarrow 1 > 2 - (w_1 + w_2 + w_3) - w_1 \left(1 + \frac{w_1}{w_2} \right) \\
&\Leftrightarrow 0 > -w_1 \left(1 + \frac{w_1}{w_2} \right)
\end{aligned}$$

This inequality is true since $w_1, w_2, w_3 > 0$.

Comparing z_{DA} with z_{NA} as defined in Eq. (26), we obtain:

$$z_{DA} > z_{NA}$$

$$\Leftrightarrow (w_1 + w_2)^2 (\sigma_{NA}^2 - \sigma^{*2}) > w_3^2 (\sigma_{DA}^2 - \sigma^{*2})$$

$$\Leftrightarrow (1 - w_3)^2 (\sigma_{NA}^2 - \sigma^{*2}) - w_3^2 (\sigma_{DA}^2 - \sigma^{*2}) > 0$$

$$\Leftrightarrow (1 - w_3)^2 \sigma_{NA}^2 - (1 - w_3)^2 \sigma^{*2} - w_3 \sigma^{*2} + w_3^2 \sigma^{*2} > 0$$

$$\Leftrightarrow (1 - w_3)^2 \sigma_{NA}^2 - (1 - w_3)^2 \sigma^{*2} - w_3 (1 - w_3) \sigma^{*2} > 0$$

$$\Leftrightarrow (1 - w_3) \sigma_{NA}^2 - (1 - w_3) \sigma^{*2} - w_3 \sigma^{*2} > 0$$

$$\Leftrightarrow (1 - w_3) \sigma_{NA}^2 - \sigma^{*2} > 0$$

$$\Leftrightarrow (1 - w_3) \sigma_{NA}^2 - w_2 \sigma_{NA}^2 > 0$$

$$\Leftrightarrow 1 - w_3 > w_2$$

$$\Leftrightarrow 1 - w_3 > 1 - w_3 - w_1$$

$$\Leftrightarrow w_1 > 0.$$

Appendix C: Jones (1991) model residuals as modified total accruals

We estimate the Jones (1991) model for pooled firm-years from 2004 to 2019. The dependent variable is working capital accruals as defined in (Dechow et al., 2012, p. 297). The independent variables are PPE and ΔREV . All variables are scaled by lagged total assets.

We obtain a highly significant slope coefficient for ΔREV of 0.061 and a significant negative coefficient for PPE of -0.006 (R^2 of 4.3%). We add the fitted values of the regression to cash flow CF to obtain a modified cash flow CF_{mod} . Modified total accruals TA_{mod} are thus equal to net income minus CF_{mod} , which corresponds to TA minus fitted values of the Jones regression. A regression of TA_{mod} on TA provides an R^2 value of 96.8%. The cross-sectional relation of modified total accruals to cash flow (Table 10) and net income (Table 11) is very similar to the prior results reported in Tables 5 and 6 in Section 3.3.¹⁰

Table 10: **Cross-sectional relation of total accruals and cash flow modified by the Jones model.**

<i>Dependent variable: TA(J)</i>								
	2004	2005	2006	2007	2008	2009	2010	2011
CF(J)	-0.256*** (-14.2)	-0.292*** (-14.2)	-0.229*** (-11.0)	-0.258*** (-12.1)	-0.241*** (-8.3)	-0.240*** (-9.2)	-0.252*** (-12.0)	-0.291*** (-12.3)
N	1,372	1,349	1,351	1,263	1,068	1,146	1,142	1,053
R ²	0.128	0.130	0.082	0.104	0.061	0.069	0.112	0.126
<i>Dependent variable: TA(J)</i>								
	2012	2013	2014	2015	2016	2017	2018	2019
CF(J)	-0.234*** (-11.1)	-0.225*** (-9.6)	-0.211*** (-9.2)	-0.240*** (-9.3)	-0.249*** (-10.7)	-0.245*** (-9.7)	-0.215*** (-8.6)	-0.167*** (-6.6)
N	1,052	1,102	1,158	1,103	1,058	1,018	973	987
R ²	0.105	0.078	0.068	0.073	0.097	0.085	0.070	0.043

Note:

*p<0.1; **p<0.05; ***p<0.01

¹⁰The number of observations is smaller owing to the additional data requirements for determining working capital accruals (current assets; current liabilities; short-term debt and cash).

Table 11: Cross-sectional relation of total accruals and net income modified by the Jones model.

<i>Dependent variable: TA(J)</i>								
	2004	2005	2006	2007	2008	2009	2010	2011
NI	0.257*** (14.2)	0.340*** (17.7)	0.346*** (19.6)	0.339*** (17.9)	0.485*** (24.6)	0.437*** (22.7)	0.295*** (14.9)	0.348*** (16.0)
N	1,372	1,349	1,351	1,263	1,068	1,146	1,142	1,053
R ²	0.129	0.189	0.221	0.202	0.362	0.310	0.163	0.197
<i>Dependent variable: TA(J)</i>								
	2012	2013	2014	2015	2016	2017	2018	2019
NI	0.274*** (13.7)	0.354*** (18.2)	0.359*** (19.3)	0.422*** (21.4)	0.341*** (16.6)	0.380*** (18.4)	0.360*** (17.6)	0.367*** (19.2)
N	1,052	1,102	1,158	1,103	1,058	1,018	973	987
R ²	0.151	0.232	0.243	0.294	0.208	0.249	0.242	0.273

Note:

*p<0.1; **p<0.05; ***p<0.01

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