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Incomplete Information and  
the Closed-End Fund Discount

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## Abstract

We model the closed-end fund discount/premium in a version of Merton's (1978) asset pricing model with incomplete information. In this economy, investors trade only assets which they "know about". The model generates a closed-end fund discount or premium, depending on risk-aversion parameters. The fund share price reverts to the net asset value on open-ending of the fund. The discount/premium is a result of two economic forces: (1) the fund manager's objective is to maximize expected utility of her fee income rather than the welfare of fund shareholders. Mis-alignment of objectives of the fund manager and shareholders results in discount/premium, and (2) for given risk aversion parameters, diversification benefits to investors determine the size of the discount/premium. Pontiff (1996) documents a positive relation between discounts and unhedgeable risk. This evidence along with other findings leads Pontiff to conclude that discounts appear to be a result of mispricing. Our model provides an alternative interpretation on the positive relation found by Pontiff based on the economic forces depicted above.

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# Incomplete information and the closed-end fund discount

**Abstract:** We model the closed-end fund discount/premium in a version of Merton's (1978) asset pricing model with incomplete information. In this economy, investors trade only assets which they "know about". The model generates a closed-end fund discount or premium, depending on risk-aversion parameters. The fund share price reverts to the net asset value on open-ending of the fund. The discount/premium is a result of two economic forces: (1) the fund manager's objective is to maximize expected utility of her fee income rather than the welfare of fund shareholders. Mis-alignment of objectives of the fund manager and shareholders results in discount/premium, and (2) for a given risk aversion parameters, the diversification benefit to investors determines the size of the discount/premium. Pontiff (1996) documents a positive relation between discounts and unhedgeable risk. This evidence along with other findings leads Pontiff to conclude that discounts appear to be a result of mispricing. Our model provides an alternative interpretation on the positive relation based on the economic forces depicted above.

The closed-end discount problem has been a puzzle to financial economists for a long time. Although many hypotheses have been put forth, they appear to have limited success in explaining stylized empirical facts. In this paper, we develop a rational model of the closed-end fund discount/premium in a version of Merton's (1978) asset pricing model with incomplete information.

Two strands of research have emerged in the literature. One strand of research has attempted to explain the puzzle within a traditional asset pricing model framework. The focus here is the possibility that the net asset value (nav) might be overstated due to agency cost, tax liabilities and the illiquidity of assets in the fund portfolio.<sup>1</sup> These hypotheses appear to have only partial success in explaining empirical evidence: namely, the closed-end fund share prices typically turn into a discount over 10% within 6-8 weeks from an initial public offering (IPO) price at a premium of about 10% (Weiss (1989)); discounts are persistent and their fluctuation appears to be mean-reverting (Thompson (1978)); discounts disappear on the open-ending of the fund (Bauer (1984)).

The second strand of research has proposed investor sentiment hypothesis based on Delong, Shleifer, Summers and Waldman (1990). Noting that individual investors hold major ownership of closed-end fund shares, Lee, Shleifer and Thaler (1991) argue that different individual sentiment explains the behavior of the closed-end fund discounts. Since individual investors trade a preponderance of closed-end fund shares and small firm stocks, they conjecture that same individual investor sentiment affect the price behavior of stocks in these two groups. Their empirical evidence appears to support this conjecture. While Lee, Shleifer and Thaler document a comovement between prices of closed-end fund and small firm stocks, the question still remains on whether this comovement

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<sup>1</sup> For literature surveys and detailed references to various hypotheses, see Dimson and Marsh (1999) and Lee, Shleifer and Thaler (1991).

reflects economic fundamentals or investor sentiment. Addressing this issue, Swaminathan (1996) presents evidence that the discounts forecast the future small firm stock returns. Swaminathan's finding suggests that there may be a rational explanation for the comovement of closed-end funds and small firm stocks. Furthermore, the investor sentiment hypothesis appears to be contradictory with the experience of UK closed-end fund discounts. While UK closed-end fund discounts have gone through similar patterns over time as those of US funds discounts, it is institutional investors rather than individual investors who own and trade a predominant portion of the UK closed-end fund shares.<sup>2</sup>

The closed-end fund discount problem, an old puzzle, still remains apparently a puzzle. In this paper, we attempt to provide a better understanding on the economic forces behind the puzzle. We introduce a closed-end fund to the simplest version of Merton's (1978) model, where investors know about only a subset of the available assets and they invest only in assets which they know about.<sup>3</sup>

We consider a two-period model with a risk-free asset, two risky assets and one closed-end fund. Investor  $I$  knows about only asset 1, investor 2 knows about only asset 2, and the fund manager knows about both assets 1 and 2. The fund manager forms a closed-end fund portfolio of the risk-free asset and risky assets 1 and 2. In return for her portfolio service, she collects, as management fee, a fraction of payoff from the fund portfolio at the end of period. In spirit of Merton's notion that all investors do not necessarily know about an asset, we assume that only investor  $I$  knows about the fund.

Given this set-up, we derive an equilibrium price for the closed-end fund share. The model shows that the fund share trades at a discount or a premium, depending on the risk aversion parameters of investor  $I$  and the manager. On open-ending the fund, fund share price equals to the nav.

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<sup>2</sup> Chen, Kan and Miller (1993) and Dimson and Minio-Kozerski (1998) also point out that the investor sentiment hypothesis is inconsistent with the experience of U.K. closed-end fund discounts.

There are two effects behind the discount/premium: the principal-agent problem effect and the diversification benefit effect. In establishing the fund portfolio, the fund manager is concerned with maximizing expected utility of her fee income rather than the welfare of investor  $I$ . Since investor  $I$  holds in equilibrium all the outstanding fund shares, a mis-alignment of objectives of the manager and investor  $I$  causes the fund share price to deviate from the nav. In a benchmark case of no mis-alignment of objectives, which is characterized by the risk aversion parameters of the manager and investor  $I$ , the fund share trades at nav. When investor  $I$  is more (less) risk averse than the benchmark risk aversion, a fund share trades at a discount (premium). For a given risk aversion parameter, the size of discount or premium is determined by the diversification benefit to investor  $I$  from holding the fund portfolio. The larger the discount, as expected, the smaller the diversification benefit becomes. The diversification benefit gets smaller in this model when the conditional variance of the nav rate of return given asset 1's rate of return gets larger; for instance, when the correlation coefficient of these two returns changes from 0 to 0.5.

Clearly one can interpret asset 1 as investor  $I$ 's portfolio of underlying risky assets, which he knows about, with no closed-end fund shares. Supposing investor  $I$ 's portfolio of underlying risky assets is a well-diversified portfolio, one can re-interpret the conditional variance of nav return given investor  $I$ 's portfolio return as the "unhedgeable" risk in Pontiff (1996). Pontiff provides evidence that the closed-end fund discount is increasing in unhedgeable risk, transaction costs, and decreasing in the amount of dividend. The evidence leads to conclude that closed-end fund discount is consistent with a noise trader model of asset pricing and that the discount appears to be a result of mispricing. Our

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<sup>3</sup> In Merton (1978), investors know about an asset when they know the mean and variance of the return process of the asset.

model provides an alternative interpretation on the positive relation between fund discounts and unhedgeable risk by pointing to a rational explanation based on the effects of principal-agent problem and diversification benefit.

The model in this paper is different in two aspects from the recent rational models of closed-end fund discounts proposed by Spiegel (1997) and Chordia and Swaminathan (1998). First, this model does not need noise trading, or unobservable supply shocks. Second, the fund manager in this model does play an economic role.

Section 1 of the paper describes the economy; it solves the optimization problems of investors and manager and derives the equilibrium prices. In section 2, we interpret the equilibrium price of fund shares and characterize economic forces affecting fund discounts. We also look into empirical implications of the model in this section. Section 3 contains concluding remarks.

## 1. The Model

Consider a version of the two-period asset pricing model with incomplete information by Merton (1987), where each investor knows about only a subset of the available securities. An investor is said to know about an asset when he knows the parameter values of return process of the asset. The investor confines his investment to assets which he knows about. We introduce a closed-end fund to the Merton model and explore the implications of the closed-end fund share price.

In the economy there are two risky assets, which pay  $\tilde{F}_k$  units of a single consumption goods at the end of the period,  $k = 1, 2$ . The random payoff  $\tilde{F}_k$  has normal distribution with mean  $\mu_k$ , variance  $\sigma_k^2$  and covariance  $\sigma_{12}$ . In addition to the risky assets, there is a risk-free asset with infinitely elastic supply, which pays  $R$  units of the consumption good.

There are three investors in the economy indexed by  $j$ ,  $j = 1, 2$  and  $m$  ( $m$  for fund manager). Following Merton, we assume that an investor trades only those assets which he knows about. Investors trade in a competitive capital market.<sup>4</sup> Each investor is endowed with the following information set:

- The payoff from the risk free asset,  $R$ , is common knowledge to all investors; investor 1 knows about only asset 1;
- Investor 2 knows about only asset 2; investor  $m$ , called the manager, knows about both assets 1 and 2. Therefore, investor 1 knows  $\mu_1$  and  $\sigma_1^2$ , investor 2 knows  $\mu_2$  and  $\sigma_2^2$  and the manager knows  $\mu_1$ ,  $\mu_2$ ,  $\sigma_1^2$ ,  $\sigma_2^2$  and  $\sigma_{12}$ .

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<sup>4</sup> Imagine an economy with three representative groups. In each group, there are a large number of identical agents. One can view investor 1, 2, and the manager in this model as three representative agents of each group of investors.



Prior to the beginning of the economy, the fund manager circulates a fund prospectus and sells the fund shares to the public at a certain price.<sup>5</sup> When our economy begins and capital markets are open, the fund manager forms her fund portfolio with the proceeds from the public offering. In return for her portfolio service, the manager collects a fraction,  $f$ , of the payoff from the fund portfolio at the end of the period. It is assumed that the manager does not trade, on her personal account, any risky assets or the fund shares.<sup>6</sup>

In the spirit of incomplete information, it is assumed that only investor 1 knows about the fund shares. That is, investor 1 knows the mean and variance of payoff from the fund shares as well as the covariance between the payoff of asset 1 and the fund shares. Therefore he will choose his portfolio to include the risk-free asset, risky asset 1 and the fund share. Investor 2 does not have any information about the payoff distribution of the fund shares. Therefore he will select his portfolio to hold the risk-free and the second risky asset.

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For simplicity, we assume here that proportion of each group is same. A different proportion of each group is easy to deal with, but it would not yield additional insights.

<sup>5</sup> Here we take the initial public offering (IPO) price as given. Typically closed-end fund shares are offered at premium, which turns into a discount > 10% within 120 days from the IPO (Weiss, 1989). Although this is an interesting issue, we do not address it in this paper.

<sup>6</sup> The manager's personal trading between risky assets and fund shares would eliminate any fund share discount or premium. Suppose she is allowed to trade in the risk-free and risky assets on a personal account, with no arbitrages between underlying risk assets and fund shares. One important question remains: why does the fund manager form the fund? One can consider this question in a more general set-up, where the fund manager chooses an optimal mix of her personal and fund portfolios. Let  $x_i^p$  and  $x_i^c$  denote manager's holdings of asset  $i$  for her personal account and for the closed-end fund portfolio. Assuming a zero personal wealth, the sum of payoff from her net zero wealth personal portfolio and her fee income is

$$\begin{aligned} & [x_1^p(\tilde{F}_1 - P_1R) + x_2^p(\tilde{F}_2 - P_2R)] + f[W_0^mR + x_1^c(\tilde{F}_1 - P_1R) + x_2^c(\tilde{F}_2 - P_2R)] \\ & = fW_0^mR + (x_1^p + fx_1^c)(\tilde{F}_1 - P_1R) + (x_2^p + fx_2^c)(\tilde{F}_2 - P_2R) \end{aligned}$$

It is clear from this expression that the manager cares only about  $(x_1^p + fx_1^c)$  and  $(x_2^p + fx_2^c)$ ; the allocation of asset holdings in personal and fund portfolio does not matter to her. (we study the case where she holds no risky assets in her personal portfolio.)

In a more general environment, such as the one suggested by Merton (see p. 506, 1987), multiple fund managers would invest only in stocks which they know about. Each individual investor in turn knows about some individual stocks and some mutual funds and confines his investments to them. Our model serves as the simplest case of introducing a closed-end mutual fund to an economy with incomplete information considered by Merton.<sup>7</sup>

Each investor is endowed with initial wealth of  $W_0^j$  at the beginning of the period.<sup>8</sup> Each investor's preference is represented by a negative exponential function of wealth in the form of  $U(W_1^j) = -\frac{1}{a_j} \exp(-a_j W_1^j)$ . Investors 1 and 2 maximize the expected utility of wealth from their portfolios; the manager maximizes expected utility of her fee income at the end of the period. It is assumed that the total numbers of outstanding risky assets are  $z_1$  for asset 1,  $z_2$  for asset 2, and 1 share for the fund shares. For clarification, we present the time line and events in Table 1.

	<b>Prior to Period 0</b>	<b>Period 0</b>	<b>Period 1</b>
Event	Closed-end fund initial public offering occurs	Capital markets are open & trading begins	Random payoffs are realized and paid out
Investor 1	buys fund shares at a certain price	Trades in risk-free assets, risky asset 1 and fund shares	Consumes payoffs
Investor 2		Trades in risk-free and risky asset 2	Consumes payoffs
Fund Manager	Circulates the fund prospectus.	Establishes the fund portfolio position in risky-free assets and	Collects and consumes fee income.

<sup>7</sup> Alternatively, one can invoke a story based on transaction costs to justify this feature of our model. Investor 1 has no transaction costs for the risk-free asset, risky asset 1 and the fund share, but huge transaction costs for risky asset 2. Investor 2 has no transaction costs for the risk-free asset and risky asset 2, but huge transaction costs for risky asset 1 and the fund share. The fund manager has no transaction costs for risky assets 1 and 2 and for managing other people's money, say, due to possession of technology in the back-office operation of the fund. The model under this transaction cost story yields the same results as those in the current model.

<sup>8</sup> Investor 1's wealth at the beginning of the period includes the current value of the fund shares that he purchased prior to the beginning of the economy.

	Sells fund shares to investor $I$ at a certain price	risky assets 1 and 2	Distributes the net payoff from the fund portfolio to investor $I$
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**Table 1: Time line, sequence of events and actions of investors**

We now fix some notations. Let  $c$  ( $c$  for closed-end fund share) denote the fund share so that  $\tilde{F}_c$  represents random payoff from the fund share with mean  $\mathbf{m}_c$ , variance  $\mathbf{S}_c^2$ , and covariance with asset 1's payoff  $\mathbf{S}_{1c}$ . Let  $x_k^j$  denote the number of risky asset  $k$  held by investor  $j$ , where  $k=1, 2, c$ , and  $j=1, 2, m$ . Let  $P_k$  denote price of one unit of asset  $k$  and  $g_k$  denote the expected excess payoff for asset  $k$ , i.e.,  $g_k \equiv E(\tilde{F}_k - P_k R) \equiv \mathbf{m}_k - P_k R$ ,  $k = 1, 2, c$ . Armed with these notations, we now consider the portfolio choice problems of the manager and investors  $1$  and  $2$ .

### 1.1 Portfolio problem of the manager

Let  $\tilde{F}_c$  denote the random payoff from the fund portfolio. The manager maximizes the expected utility of her fee income,  $\mathbf{f} \tilde{F}_c$ , with her optimal choice of holdings of asset 1,  $x_1^m$ , and holdings of asset 2,  $x_2^m$ . The payoff from the fund portfolio is,

$$\tilde{F}_c = W_0^m R + x_1^m (\tilde{F}_1 - P_1 R) + x_2^m (\tilde{F}_2 - P_2 R) \quad (1)$$

Her optimal demand is given by

$$x_1^m = \frac{\mathbf{S}_2^2 g_1 - \mathbf{S}_{12} g_2}{a_m \mathbf{f} s_1} \quad \text{and} \quad x_2^m = \frac{-\mathbf{S}_{12} g_1 + \mathbf{S}_1^2 g_2}{a_m \mathbf{f} s_1} \quad (2)$$

where  $s_1 \equiv \mathbf{S}_1^2 \mathbf{S}_2^2 - (\mathbf{S}_{12})^2$  and  $g_k \equiv E(\tilde{F}_k - P_k R)$ .

### 1.2 Portfolio problem of investor $1$ and $2$

The portfolio problem of investor 2 is simple, since he invests only in risky asset 2 and the risk-free asset. His optimal demand for risky asset 2,  $x_2^2$ , is given by,

$$x_2^2 = \frac{g_2}{a_2 \mathbf{S}_2^2} \quad (3)$$

Investor 1 can invest in risky asset 1, the fund share and the risk-free asset. He knows the mean and variance of the payoff from asset 1 and the fund share, as well as the covariance between the payoff from asset 1 and the fund share. He does not know, however, the composition of the fund manager's portfolio.<sup>9</sup> In view of equation (1), moments of payoff from the fund shares, before paying out the fee to the fund manager, are:

$$\mathbf{S}_c^2 \equiv \text{var}(\tilde{F}_c) = (x_1^m \mathbf{S}_1)^2 + (x_2^m \mathbf{S}_2)^2 + 2x_1^m x_2^m \mathbf{S}_{12}, \quad (4)$$

$$\mathbf{S}_{1c} \equiv \text{cov}(\tilde{F}_1, \tilde{F}_c) = x_1^m \mathbf{S}_1^2 + x_2^m \mathbf{S}_{12} \quad (5)$$

$$\mathbf{m}_c \equiv E(\tilde{F}_c) = W_0^m R + x_1^m g_1 + x_2^m g_2, \quad (6)$$

Investor 1 optimally chooses his demand for asset 1,  $x_1^1$ , and the fund share,  $x_c^1$ , to maximize his expected utility of wealth at the end of the period,  $\tilde{W}_1^1 = W_0^1 R + x_1^1 (\tilde{F}_1 - P_1 R) + x_c^1 [(1-f)\tilde{F}_c - P_c R]$ . Here  $P_c$  denotes price of the closed-end fund share. Investor 1's optimal demands are given by,

$$x_1^1 = \frac{(1-f)^2 \mathbf{S}_c^2 g_1 - (1-f) \mathbf{S}_{1c} [(1-f)\mathbf{m}_c - P_c R]}{a_1 (1-f)^2 s_2}, \quad (7)$$

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<sup>9</sup> It will be shown that this knowledge does not allow investor 1 to back out  $\mathbf{m}_2$ ,  $\mathbf{S}_2^2$  and  $\mathbf{S}_{12}$ . As shown in equation (14), the mean of fund payoff ( $\mathbf{m}_c$ ) is linear in its variance ( $\mathbf{S}_c^2$ ). However, even when  $\mathbf{S}_{12} = 0$ , investor 1 cannot infer  $\mathbf{m}_2$  and  $\mathbf{S}_2^2$ , since, in that case, the covariance between the fund's payoff with asset 1's payoff ( $\mathbf{S}_{1c}$ ) is independent of  $\mathbf{m}_2$  and  $\mathbf{S}_2^2$  (see equation (13)).

$$x_c^1 = \frac{-(1-f) \mathbf{s}_{1c} g_1 + \mathbf{s}_1^2 [(1-f) \mathbf{m}_c - P_c R]}{a_1 (1-f)^2 s_2} \quad (8)$$

where  $s_2 \equiv \mathbf{s}_1^2 \mathbf{s}_c^2 - (\mathbf{s}_{1c})^2$ .

### 1.3 The equilibrium

The equilibrium for the fund share market obtains when investor 1 holds the entire outstanding supply of one share. Markets for risky assets 1 and 2 reach equilibrium when the combined demand for asset 1 of investor 1 and the fund manager equals  $z_1$  and the combined demand for asset 2 of investor 2 and the fund manager equals  $z_2$ . The market clearing conditions are:

$$x_c^1 = 1 \quad (9)$$

$$x_1^1 + x_1^m = z_1 \quad (10)$$

$$x_2^2 + x_2^m = z_2 \quad (11)$$

### 1.4 Equilibrium prices

Before deriving prices for the fund share and risky assets 1 and 2, it is useful to simplify the moments for the payoff from the fund share, i.e., equations (4) – (6). We do so by substituting the manager's demand for asset 1 and 2, i.e., equation (2), to obtain the following equations:

$$\mathbf{s}_c^2 \equiv \text{var}(\tilde{F}_c) = \frac{\mathbf{s}_2^2 g_1^2 - 2 \mathbf{s}_{12} g_1 g_2 + \mathbf{s}_1^2 g_2^2}{(a_m \mathbf{f})^2 s_1} \quad (12)$$

$$\mathbf{s}_{1c} \equiv \text{cov}(\tilde{F}_1, \tilde{F}_c) = \frac{g_1}{a_m \mathbf{f}} \quad (13)$$

$$\begin{aligned}
\mathbf{m}_c &\equiv E(\tilde{F}_c) = W_0^m R + \frac{\mathbf{s}_2^2 g_1^2 - 2\mathbf{s}_{12} g_1 g_2 + \mathbf{s}_1^2 g_2^2}{a_m \mathbf{f} s_1} \\
&= W_0^m R + a_m \mathbf{f} \mathbf{s}_c^2
\end{aligned} \tag{14}$$

In order to obtain the price of the fund share, we first substitute equation (12) – (14) into investor 1's demand for the fund share, (8). Then we use the market clearing condition (9). The price of the fund share is given by,

$$P_c = (1 - \mathbf{f}) \left\{ NAV - \frac{s_2}{R \mathbf{s}_1^2} [a_1 (1 - \mathbf{f}) - a_m \mathbf{f}] \right\} \tag{15}$$

where  $s_2 \equiv \mathbf{s}_1^2 \mathbf{s}_c^2 - (\mathbf{s}_{1c})^2$ . Here we use the fact that  $W_0^m$  is equal to the net asset value of the fund.

Substituting equation (12) – (15) into investor 1's demand for asset 1, (7), simplifies  $x_1^1$  as,

$$x_1^1 = \left( \frac{-\mathbf{s}_{1c}}{a_1 \mathbf{s}_1^2} \right) [a_1 (1 - \mathbf{f}) - a_m \mathbf{f}] \tag{16}$$

We now obtain equilibrium prices of asset 1 and 2. Substituting demands for risky assets 1 and 2 by the three agents, i.e., equation (2), (3) and (16) into the market clearing conditions (10) and (11), yields two equations linear in  $P_1$  and  $P_2$ . Solving the equations for  $P_1$  and  $P_2$  yields:

$$P_1 = \frac{1}{R} \left\{ \mathbf{m}_1 - \frac{a_1 a_m \mathbf{s}_1^2 [(a_m \mathbf{f} s_1 + a_2 \mathbf{s}_1^2 \mathbf{s}_2^2) z_1 + a_2 \mathbf{s}_2^2 \mathbf{s}_{12} z_2]}{a_1 a_m (\mathbf{s}_{12})^2 + (a_1 + a_m)(a_m \mathbf{f} s_1 + a_2 \mathbf{s}_1^2 \mathbf{s}_2^2)} \right\} \tag{17}$$

$$P_2 = \frac{1}{R} \left\{ \mathbf{m}_2 - \frac{a_2 a_m \mathbf{s}_2^2 [a_1 \mathbf{s}_1^2 \mathbf{s}_{12} z_1 + [a_1 (\mathbf{s}_{12})^2 + (a_1 + a_m) \mathbf{f} s_1] z_2]}{a_1 a_m (\mathbf{s}_{12})^2 + (a_1 + a_m)(a_m \mathbf{f} s_1 + a_2 \mathbf{s}_1^2 \mathbf{s}_2^2)} \right\} \tag{18}$$

## 2. Interpretation

### 2.1 Fund share price

$P_c$  in equation (15) represents fund share price after paying out the management fee. The financial press typically quotes fund share prices before paying out the fee. Let  $P_c^*$  denote the share price before the fee. The fund manager collects a fraction  $\mathbf{f}$  of the fund portfolio payoff at the end of period 1. The present value of the fee income should be equal to  $\mathbf{f}^* \text{nav}$ . Therefore,  $P_c^*$  equals  $P_c + \phi^* \text{NAV}$ , and, from (15), one can write  $P_c^*$  as,

$$P_c^* = \left\{ NAV - \frac{(1-\mathbf{f})s_2}{R\mathbf{S}_1^2} [a_1(1-\mathbf{f}) - a_m\mathbf{f}] \right\} \quad (19)$$

where  $s_2 \equiv \mathbf{S}_1^2 \mathbf{S}_c^2 - (\mathbf{S}_{1c})^2$ . Eliminating NAV in (19) using (14) and the fact the nav equals  $W_0^m$  yields,

$$P_c^* = \frac{\mathbf{m}_c}{R} - \text{Risk Adjustment} \quad (20)$$

$$\text{where Risk Adjustment} \equiv \frac{(1-\mathbf{f})}{R} \left\{ a_1(1-\mathbf{f})\mathbf{S}_c^2 - \frac{(\mathbf{S}_{1c})^2}{\mathbf{S}_1^2} [a_1(1-\mathbf{f}) - a_m\mathbf{f}] \right\}.$$

Equation (20) has the usual interpretation: the fund share price is equal to the “risk-neutral” price less a “risk adjustment” term, which represents compensation to investor  $I$  for holding a risky asset. The risk neutral price is the expected payoff from fund portfolio discounted at the risk-free rate. The risk adjustment is proportional to  $(1 - \mathbf{f})$ , the actual ownership of the fund portfolio by investor  $I$ ; the risk adjustment is decreasing (increasing) in the square of the covariance between the payoff of asset 1 and the fund portfolio,  $(\mathbf{S}_{1c})^2$ , for a positive (negative) value of  $[a_1(1-\mathbf{f}) - a_m\mathbf{f}]$ . The risk adjustment

is closely related to the diversification benefit that investor  $I$  derives from holding the fund portfolio, as will be seen in section 2.4.

In this two-period model framework, one can view the final payoff as the sum of the dividend and any residual value of capital assets that produce the dividend. At the end of the period, the fund manager pays out dividends to investor  $I$ , and the fund share price is equal to the residual value of capital assets. Since the fund liquidates at the end of the period, the fund share price at an open-ending is equal to the nav in this model.

## 2.2 Closed-end fund discount/premium

Equation (19) states that the closed-end fund discount/premium is represented by the second term in the right-hand-side, i.e.,  $-\frac{(1-f)s_2}{R\mathbf{S}_1^2} [a_1(1-f) - a_m f]$ , a negative quantity being a discount, and a positive quantity being a premium. Here,  $s_2 \equiv \mathbf{S}_1^2 \mathbf{S}_c^2 - (\mathbf{S}_{1c})^2$  is the determinant of the covariance matrix of payoff from asset 1 and the fund share. A positive definite covariance matrix implies that  $s_2$  is positive. The term  $(1-f)$  is also positive since  $f$  is typically less than 5%. Whether the fund share is traded at a discount or premium depends on the sign of  $[a_1(1-f) - a_m f]$ : for  $a_1 > \frac{a_m f}{1-f}$ , the fund share price is at a discount from nav, for  $a_1 < \frac{a_m f}{1-f}$  at a premium and for  $a_1 = \frac{a_m f}{1-f}$ , zero discount/premium. For a given value of  $[a_1(1-f) - a_m f]$ , the discount/premium is determined as the negative of the product of  $[a_1(1-f) - a_m f]$  and  $\frac{(1-f)s_2}{R\mathbf{S}_1^2}$ . We interpret below the first term,  $[a_1(1-f) - a_m f]$ , as the effect arising from the principal-agent problem between investor  $I$  and the manager,



and the second term,  $\frac{(1-f)s_2}{RS_1^2}$ , as a measure of the diversification benefit to investor 1 from holding the fund portfolio.

### 2.3 Principal-agent problem effect

In forming the fund portfolio, the manager's objective is to maximize the expected utility of her fee income. The manager does not care about how the fund portfolio affects investor  $I$ 's welfare. Therefore, the fund portfolio optimal for the manager is not necessarily optimal for investor  $I$ . We call this mis-alignment of objectives of investor  $I$  and the manager as the “principal-agent” problem. In general this principal-agent problem causes the fund share price to deviate from the nav, except for a special case. In the special case of  $a_1(1-f) = a_m f$  there happens to be no principal-agent problem and there is no discount/premium. To see this, recall the maximization problems of the manager and investor  $I$ <sup>10</sup>:

$$\text{Manager:} \quad \text{Max } E[ \exp( -a_m f \tilde{F}_c ) ]$$

$$\text{Investor } I: \quad \text{Max } E\left\{ \exp[ -a_1(1-f)\tilde{F}_c - a_1 x_1^1 \tilde{F}_1 ] \right\}$$

When  $a_1(1-f) = a_m f$  and  $x_1^1 = 0$ , these two maximization problems are identical. In this case, the manager's portfolio is exactly the one which investor  $I$  would have formed if he knew about both assets 1 and 2. Indeed, when  $a_1(1-f) = a_m f$ , investor  $I$  optimally sets  $x_1^1 = 0$ , as implied by equation (16). With no principal-agent problem, investor  $I$  trades the fund share at the nav.

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<sup>10</sup> We ignore the constant terms in the maximization problems.

When investor  $I$  is more risk averse than the benchmark risk aversion, i.e.,  $a_1 > \frac{a_m \mathbf{f}}{1-\mathbf{f}}$ , he requires a compensation for holding fund shares and hence fund shares trade at a discount relative to the nav. Conversely, when investor  $I$  is less risk-averse than the benchmark risk aversion, fund shares trade at a premium.

## 2.4 Diversification benefit effect

Consider first how investor  $I$  optimally combines risky asset 1 and the fund share for his portfolio choice problem. For the fund share, he does not have any choice in terms of quantity of the fund share he holds, since in equilibrium he is to hold all the outstanding fund shares. The fund share is priced to induce investor  $I$  to hold willingly the entire supply.

Although the fund portfolio is imposed on investor  $I$ , it does offer investor  $I$  a diversification benefit in the sense that it allows him a wider investment opportunity set than the one he has in the absence of the fund. In choosing optimal quantity of asset 1, investor  $I$  takes into account this diversification benefit, which hinges on the covariance between the payoffs from asset 1 and the fund portfolio,  $\mathbf{s}_{1c}$ . Equation (16) shows that investor  $I$ 's equilibrium holdings of asset 1 depend on  $\mathbf{s}_{1c}$ . Suppose a positive value for  $[a_1(1-\mathbf{f}) - a_m \mathbf{f}]$ <sup>11</sup>. Then investor  $I$  holds a short (long) position in risky asset 1 when the payoff from asset 1 is positively (negatively) correlated with the fund portfolio payoffs.<sup>12</sup> In this simple model,  $\mathbf{s}_{1c}$  affects investor  $I$ 's holdings of asset 1 in a symmetric manner; that

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<sup>11</sup> Since the case of a negative value for  $[a_1(1-\mathbf{f}) - a_m \mathbf{f}]$  is the mirror image of the case of a positive value, we discuss only the case of positive value in the remainder of our paper.

<sup>12</sup> When  $\mathbf{s}_{1c}=0$ , equation (16) shows that investor  $I$  does not hold any asset 1. In fact, in this very special case, the fund manager ends up holding the entire stock of risky assets 1 and 2. Investor  $I$  holds only the fund portfolio and

is, his holdings of asset 1 corresponding to, say,  $\mathbf{s}_{1c}=0.4$  are equal to his holdings corresponding to  $\mathbf{s}_{1c} = -0.4$ , except for change in sign.

Investor 1's behavior here is a result of his attempt to reduce variation of his wealth at the end of the period. We look at the effect of investor 1's holdings of asset 1 on the variance of his wealth. Note that one can write  $\text{var}(W_1^1)$  using equations (7) and (8) as,

$$\begin{aligned} \text{Var}(\tilde{W}_1^1) &= \frac{1}{a_1} (g_1 x_1^1 + g_c x_c^1) \\ &= \frac{1}{a_1} \left\{ \frac{-(\mathbf{s}_{1c})^2}{a_1 \mathbf{s}_1^2} [a_1 (1 - \mathbf{f}) - a_m \mathbf{f}] + g_c x_c^1 \right\} \end{aligned} \quad (21)$$

The second equality in (21) obtains by using equation (13) and (16). Assume a positive value for  $[a_1(1 - \mathbf{f}) - a_m \mathbf{f}]$ . Equation (21) shows that investor 1 chooses his holdings of asset 1 as to reduce the variation in his wealth at the end of period. We call the extent to which the variation in his wealth is reduced as the “diversification benefit” to investor 1. The first term inside the curly bracket of equation (21) represents the diversification benefit. It depends on the covariance of payoffs from asset 1 and the fund portfolio. The larger the diversification benefit gets, the more valuable the fund portfolio becomes, and the narrower the discount gets. One can see from equation (20) that the diversification benefit reduces the risk adjustment in (20), and leads to a higher fund share price.

In summary, we interpret closed-end fund discount/premium as the result of two effects. The principal-agent problem effect determines whether a discount or a premium prevails depending on the

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investor 2 holds only the risk-free asset. Zero correlation between asset 1 and the fund portfolio implies that asset 1's payoff has a perfectly negative correlation with asset 2's payoff. The fund portfolio becomes a risk-free asset, and both risky assets 1 and 2 are priced as risk-free asset. Clearly, the fund share in this case trades at nav.

sign of  $[a_1(1-f) - a_m f]$ . For a positive value of  $[a_1(1-f) - a_m f]$ , a discount results, and the smaller  $(S_{1c})^2$ , the smaller the diversification benefit from the fund portfolio, and the larger discount prevails.

## 2.5 Empirical implication

It is more useful to consider equation (15) in terms of rates of return in exploring the empirical implication of our model. Let  $\tilde{R}_1 \equiv \frac{\tilde{F}_1}{P_1}$  denote the rate of return on asset 1 and  $\tilde{R}_{nav} \equiv \frac{\tilde{F}_c}{NAV}$  denote

the nav rate of return on the fund portfolio. Using these notations, one can rewrite equation (20) as,

$$\frac{P_c^*}{NAV} = \left\{ 1 - \frac{(1-f)NAV}{R} \text{var}(\tilde{R}_{nav}) \times [1 - \text{corr}(\tilde{R}_1, \tilde{R}_{nav})^2] \times [a_1(1-f) - a_m f] \right\} \quad (22)$$

Equation (22) says that for a positive value of  $[a_1(1-f) - a_m f]$ , the closed-end fund discount, defined here as  $\frac{-(P_c^* - NAV)}{NAV}$ , is proportional to  $\text{var}(\tilde{R}_{nav}) \times [1 - \text{corr}(\tilde{R}_1, \tilde{R}_{nav})^2]$ , which is the conditional variance of the nav rate of return given asset 1's return. The lower the squared correlation between nav return and return on investor  $I$ 's portfolio, the smaller the diversification benefit, resulting into a larger discount for positive values of  $[a_1(1-f) - a_m f]$ .

One can also interpret asset 1 as investor  $I$ 's portfolio of many risky assets, which he knows about. Suppose this portfolio is reasonably well-diversified. Equation (22) implies a larger discount for funds which have high conditional variance in nav returns given this well-diversified portfolio's returns.<sup>13</sup> One can characterize such funds as having high unhedgeable risk in the sense that it is hard to replicate

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<sup>13</sup> Again, we discuss the case of a positive value for  $[a_1(1-f) - a_m f]$ .

such funds with available securities. The model then implies that funds with high unhedgeable risk will have large discounts.

The result here provides an alternative interpretation of Pontiff (1996). He presents evidence that closed-end fund discounts are likely to get larger for closed-end funds which are hard to replicate (i.e., high unhedgeable risk), pay smaller dividends and require large transaction costs. He concludes that this evidence is consistent with a noise trader model of asset pricing and that the fund discount appears to be a result of mispricing in the market. Our model suggests that the positive relation between the discount and unhedgeable risk does not necessarily lead to a view that the discount is a form of mispricing. The discount can occur as a result of rational behavior in a market with incomplete information.

### **3. Conclusion**

In this paper, we show that a closed-end fund discount or premium can occur in a rational asset-pricing model with incomplete information using a version of Merton's (1987) model. The model does not assume any noise trading or unobservable supply shock. We derive the equilibrium price for the fund share, and characterize two economic forces behind the discount/premium: the principal-agent problem effect and the diversification benefit effect. The share price reverts to the nav on an open-ending of the fund. The model suggests these two effects could explain the positive relation between discounts and unhedgeable risk documented in Pontiff (1996). The implication is that this positive relation does not necessarily lead to the conclusion that the fund discounts appear to be a result of mispricing.

With the current one-period model framework, we are not able to say anything about the time-series behavior of the fund discount/premium. Also, we do not look into why investors purchase fund

shares at premium at the time of the IPO despite the observation that the premium typically turns into a discount of about 10% within a few weeks. We leave these as future research topics.

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