A Simple Model of Corporate Bailouts in a Globalized Economy

Article *in* Scandinavian Journal of Economics · February 2019

DOI: 10.2139/ssrn.3360437

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A simple model of corporate bailouts in a globalized economy*

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Abstract

This paper explores how globalization influences the decision of governments to rescue inefficient domestic firms when bailouts affect firms' markups. We develop a model of international trade where immobile domestic enterprises (DOEs) compete with foreign enterprises (FOEs) in an oligopolistic market. The decision to bail out DOEs leads to lower corporate tax revenues if FOEs are immobile whereas tax revenues might increase if FOEs are mobile. Interestingly, the mobility of FOEs makes governments more prone to rescue inefficient domestic firms because tax competition reduces the opportunity cost of a bailout policy in terms of public good provision.

Keywords: bailout of manufacturing firms, tax competition; trade costs; firm mobility.

JEL classification: F12; F15; D21; H25.

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*We are grateful to participants at the 3rd SEBA GATE workhop (Chengde), 59th annual RSAI conference (Ottawa), 4th Lisbon Meetings on Game Theory and Applications, GATE workshop on Public Policies and Spatial Economy (Lyon and Saint-Etienne), seminars in Dijon, Geneva and Rennes. Jacques-François Thisse, Carl Gaigné, Frédéric Robert-Nicoud, Cécile Detang Dessendre, Maire-Laure Breuillé, Simon Lapointe and Stephanie Fürer are acknowledged for helpful comments and suggestions. We are also grateful to the Editor and two anonymous referees for providing very useful comments, which have significantly improved our paper.

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1 Introduction

There are many examples of governments saving ailing firms from bankruptcy in recent history. The most emblematic example of private corporate bailout is certainly the federal bailout of the Big Three US automakers in 2009, but many rescue plans have been decided in other countries and industrial sectors over the past decades. Not only firms operating in the private sector receive bailout money; oftentimes, State-Owned Enterprises (SOEs) are concerned. SOEs have come back to the forefront as key players in the global trade and are often granted advantages that can affect export competition.[1]

Usually, bailouts are granted to firms – public or private – with structurally high production costs.[2] Regarding the 2009 US auto bailout, Goolsbee and Krueger (2015, p. 6) underline that when including the legacy costs or retirees, average labor costs for the Big Three were almost 45 percent higher than in the transplants. In addition, a surprisingly large share of labor compensation for the Big Three automakers was a fixed cost. There is also a long-standing literature showing that SOEs are often more labor intensive and less efficient than foreign firms operating in the same sectors (Dewenter and Malatesta, 2001; Gupta, 2005; Golden et al., 2008 among others). SOEs often incur high fixed costs reflecting bureaucratic overheads and legacy costs inherited from the past such as the financing of civil servant pension schemes (Kowalski et al., 2013).[3] Finally, manufacturing firms that benefit from rescue plans are often large firms, so that their survival or bankruptcy may induce important distortions.[4] Wollman (2018) provides empirical evidence for such distortive effects by exploring the consequences of the 2009 rescue plan of US automobile industry. Using data on the commercial vehicle segment of the US automotive industry, he finds that if the government had not rescued automakers, markups would have drastically increased while output would have dropped.[5] Those potential distortions also concern

[1] In the business year 2010-2011, 10% of the 2000 world's largest firms on the Forbes Global list were identified as SOEs. These firms compete with private firms in the global market place and are present in a wide range of manufacturing sectors. The only manufacture of motor vehicles accounts for nearly 12% of world trade and displays a share of SOEs as high as 20%.

[2] Those high cost firms are even more vulnerable in the context of economic downturns, hence encouraging governments to avoid them going bankrupt.

[3] Despite the fact that progress has been made in the recent period to separate commercial activities from the provision of public services, SOEs still have to bear higher fixed and variable costs than private firms.

[4] In Europe, such distortions on prices and market shares legitimate that all state aids for rescue and restructuring must be authorized by the European Commission, sometimes by requiring compensatory measures.

[5] Importantly, Wollman (2018) shows that this conclusion remains valid whether the alternative sce-

ailing SOEs, given their rising importance in world markets. As emphasized in Kowalski et al. (2013, p.13), private owned enterprises frequently find themselves competing with SOEs, both domestically and internationally. Given that firms in international markets are often more productive (Helpman, Melitz and Yeaple, 2004), "SOE-related distortions in contestable international markets may be associated with higher welfare costs as compared to distortions in closed markets" (Kowalski and Perepechay, 2015, p.15).[6]

The purpose of this paper is to study how the decision of policy makers to rescue ailing firms may be shaped by economic integration taking these potential distortive effects into account. To the best of our knowledge, this question has been mostly ignored by the literature. Most of the long-standing Soft Budget Constraint (SBC) literature looks at identifying the key determinants of financial bailouts in a closed economy framework (Kornai, Maskin and Roland, 2003). Politicians may be motivated to grant subsidies to companies in financial difficulties for electoral reasons (Robinson and Torvik, 2001). Policy-makers may also be concerned by a too big to fail argument when failures cause negative spillovers and disruptive systemic effects on the rest of the economy (Kornai et al., 2003; Goolsbee and Krueger, 2014).

We enrich this existing literature by analyzing how two driving forces of economic integration - namely firm mobility and trade integration - influence the costs and benefits of a bailout. On the one hand, the decision for policymakers to rescue firms in financial trouble might be motivated by the fear that the bankruptcy of ailing firms could induce distortive effects on the goods markets, which would harm consumers by raising prices. Interestingly, this adverse effect of corporate bankruptcy on consumers' might be exacerbated when trade costs are low in so far as it strengthens competition on the goods market. On the other hand, the opportunity cost of a bailout in terms of public good provision is linked to the intensity of international tax competition. Indeed, whether rescuing ailing firms is likely or not to engender tax revenue losses depends on the ability of governments to raise tax revenues, which ultimately depends on whether firms are mobile or not. Surprisingly, despite the public policy implications of this question, we are not aware of existing studies that provide an analysis of the possible impact of economic integration on the frequency of such corporate bailouts.

To the best of our knowledge, the only empirical contribution on the determinants of corporate bailouts is that of Faccio, Masulis and Mc Connell (2006). Besides the

nario were bankruptcy of ailing firms, or their acquisition by a competitor.

[6]Interestingly, the (late) Trans-pacific partnership (TPP) included provisions aimed at making competition between SOEs and private enterprises as fair as possible.

importance of political connections between firms and the governments, they show that larger firms, or firms that have been owned by the government in the past, are more likely to be bailed out. On the theoretical side, contributions are also very scarce. An important contribution by Qian and Roland (1998) explores the determinants of the SBC in the context of federal economies. They show that by inducing fiscal competition among local governments, factor mobility increases the opportunity costs of bailout and then serves as a commitment device (Qian and Roland, 1998, p. 1143). However, their model is designed to analyze bailout within the framework of a perfectly integrated national economy. More recently, Alexeev and Jang (2010) develop a SBC model using a Melitz' (2003) framework of international trade with heterogeneous monopolistically competitive firms. The authors show that by raising the level of effort made by firms in order to cut their marginal production costs, trade liberalization reduces the number of firms eligible for a bailout and therefore moderates the inefficiencies introduced by the SBC[7]. However, the bailout policy is considered as exogenous. In addition, the authors ignore the market distortions that bailout policies are reputed to generate by assuming that rescue plans do not "significantly affect the aggregate price index and aggregate profit in the economy" (Alexeev and Jang, 2010, p. 452) .

In contrast, we develop a two country model of international trade where, given the existence of ailing domestic firms that compete with foreign firms, the government has to decide whether or not to adopt a bailout policy. We consider that there are two kinds of enterprises: Foreign Owned Enterprises (FOEs) and Domestic Owned Enterprises (DOEs). Following Gaigné and Wooton (2011), the former are mobile and owned by investors residing in a third country while the latter are domestic and immobile. They all compete on the same oligopolistic market, but they differ in their production costs. Importantly, FOEs have low production costs and are always profitable while DOEs are high cost firms and are profitable only when they benefit from a financial support on the part of their government. Both kinds of firm export part of their production to the foreign market and are subject to trade costs. Governments levy a source-based corporate tax on the profits of FOEs. Those tax revenues are used to produce a public good which benefits the domestic households. Governments may also use part of their tax revenues to cancel out the deficit of DOEs through a subsidy (bailout policy). Governments are thus faced with two sequential choices: in a first stage they have to decide whether to bail out firms

[7] In Alexeev and Jang (2010), the marginal cost of production is endogenized by the level of effort made by the firm after entry and prior to production. Thus, the marginal cost is a decreasing function of the effort level.

or not and, in a second stage, they choose their corporate tax in a context where countries compete with each other to attract foreign firms. Those firms decide in which country to settle at the third stage of the game. For sake of simplicity, we assume benevolent governments and disregard political benefits or employment issues that might, in the real world, also matter for governments.[8] Therefore, the decision whether or not to rescue ailing firms results from a trade-off between the costs and benefits of a bailout versus laissez-faire policy. The cost of a bailout is composed of both the financial cost of the subsidies granted to the DOEs and a potential opportunity cost in terms of tax revenues forgone, whereas the benefits result from abating the distortive effects that would harm consumer's surplus under the alternative scenario of a bankruptcy.

Our central result is that the mobility of FOEs makes governments more prone to adopt a bailout policy. To obtain this result, we proceed in two steps. First, we analyze what would be the decision of government with respect to inefficient domestic firms if competing FOEs were immobile as well. In that case, the tax base elasticity is equal to zero so that governments can tax away all the profits of foreign firms. If the DOEs go bankrupt, the profits made by FOEs are higher, which allows the governments to raise higher tax revenues. However, there is also an incentive to bail out DOEs because their survival raises the consumers surplus through a competition effect on the good market. In the end, the bailout decision depends on the country size. Below a certain population size, governments decide to rescue firms if and only if the fixed cost of production for DOEs is low enough and trade cost are high enough. By contrast, governments of large countries always decide to let DOEs go bankrupt because tax revenues arising from FOEs and the related benefits in terms of public good provision are magnified by the countries' size.

Second, we solve the model when FOEs are mobile and governments compete over their corporate taxes to attract firms. We first show that corporate taxes are higher under the bailout scenario if country size is large enough. The intuition behind this result is that an increase in the corporate tax reduces the tax attractiveness of the country. This reduces the intensity of competition between local DOEs and mobile FOEs, which in turn limits the level of deficit that governments will have to cover through the rescue aid. In other words, governments may strategically compete less fiercely for FOEs in order to protect local DOEs from competition and ensure that the financial cost of the bailout is acceptable. Importantly, this incentive to set higher tax rates is strengthened for larger

[8] However, we can extend our model, without changing qualitatively our result by assuming that governments also aim at defending extra salaries in DOEs (see section 5).

countries because the positive impact of a foreign capital outflow on operating profits of DOEs (and by incidence the reduction of the financial cost of the bailout) is increasing with country size. This provides governments with an additional incentive to rescue firms. In the end, we show that whatever country size, governments decide to bailout DOEs if and only if their level of fixed cost is lower than a certain threshold. Finally, we verify that the minimum level of fixed cost below which rescuing DOEs becomes welfare enhancing is higher when FOEs are mobile than when they are immobile. Therefore, whatever the country size, the presence of mobile FOEs always makes governments more prone to adopt a bailout policy.

The rest of the paper is organized as follows. In section 2, we set up a basic model with trade but assuming that FOEs are immobile. This section is used as a benchmark. Then, in section 3 we introduce the possibility for FOEs to relocate and for governments to set their corporate tax strategically in order to analyze how firm mobility influences governments' incentives to rescue ailing firms. Section 4 provides some extensions and robustness checks to our model. Section 5 concludes.

2 The Model

The economy consists of two countries, labelled $i = A, B$, equally populated by $L_A = L_B = L/2$ immobile individuals.[9] There are two production factors, labor and capital. Each individual inelastically supplies one unit of labor and works in its country of residence. There are two private sectors: a manufacturing and a traditional sector. In the manufacturing (M) sector, firms produce a homogeneous good, labelled x, in an oligopolistic market using both labor and capital that may be owned domestically or by foreign investors living in a third country. In the traditional (T) sector, firms produce the numeraire commodity z under conditions of perfect competition, using labor exclusively.[10]

2.1 Firms

The M sector is characterized by a fixed number of $m + n$ oligopolistic firms. They produce an industrial good for both the domestic and foreign markets. The two countries are then bilateral trading partners. Exporting this good involves a per-unit cost of τ units

[9] Voluntarily, we restrict our study to the symmetric case. This allows us to control for any comparative advantage that would intervene in a bailout decision.

[10] Both sectors are always active in each one of the two countries.

of numeraire. Each of these firms possesses one unit of "knowledge capital" so that firms in the manufacturing sector enjoy increasing returns to scale. This factor is indispensable for production, but available in limited quantity such that at most $m+n$ firms engage in production. Importantly, we assume that there are two types of firms depending on the origin of capital ownership. There are m foreign enterprises (Foreign-Owned Enterprises or FOEs) based in a third country and prepared to invest by setting up a single production plant in country A or B, and n domestic firms (hereafter Domestic-Owned Enterprises or DOEs) whose capital owners are either the State or residents (or a mix of both) of the country in which they are settled.

Through these simple modelling assumptions, we want to capture how governments behave with respect to inefficient DOEs that compete with foreign firms in an international oligopolistic market. Importantly, DOEs and FOEs differ with respect to their level of profitability and their ability to relocate their production. Firstly, foreign firms are mobile across countries in that foreign investors invest their capital freely in country A or B. By contrast, domestic firms are immobile and we assume that they are equally distributed across countries. Secondly, we assume that DOEs are less productive.[11] Their marginal cost is equal to c units of labor in DOEs, with $c > 0$. By contrast, the marginal cost in FOEs is lower and normalized to zero. Thus, DOEs compete with firms producing at a lower marginal cost. Moreover, DOEs bear an additional fixed cost equal to Θ units of the numéraire.[12] We assume that because of these higher marginal and fixed costs, DOEs encounter financial difficulties. They will go bankrupt, unless the governments bail them out. We model this bailout in a simple way by assuming that it takes the form of a lump-sum subsidy S such that firms make non-negative profits, allowing them to survive and sell on both the domestic and foreign markets. Put differently, we consider bailouts that take the form of temporary liquidities (such as rescue aid in the EU) rather than a restructing plan aimed at restoring the profitability of the firm by requiring a technological change that minimize the production costs.

In the T sector, firms produce the numeraire commodity z under perfect competition using workers only. Specifically, one unit of labour is required to produce one unit of output. Moreover, this good is assumed to be freely traded. Therefore, free trade in the

[11] In case those DOEs are (at least partially) owned by the government, this assumption is consistent with the empirical evidence showing that SOEs are often less productive and more labor intensive than private enterprises (Dewenter and Malatesta, 2001, and Gupta, 2005).

[12] In case those DOEs are owned by the government, this cost reflects the bureaucratic overhead including corporate governance supervision by the public administration or the management of civil servants directly or indirectly devoted to the activity of the firm.

numeraire good equalises the wage to one in each country. Finally, we consider that the T sector is always active in both countries so that labor mobility across sectors equalizes wages to unity in each country.

2.2 Governments

The public sector in each country is represented by a benevolent government levying a per unit tax t_i on profits made by foreign firms of the M-industry. This tax is levied on the source principle, which means that governments only tax firms located in their country. By assumption, DOEs make negative profits in the absence of government intervention. Therefore they are not subject to the corporate tax.

Tax revenues from FOEs are recycled in a public good provision G_i for domestic households, using the numéraire as the only input so that the marginal rate of transformation between the public good and the numéraire is unity. Let θ indicate whether the government rescues DOEs through a subsidy ($\theta = R$ for "rescue") or whether it lets them go bankupt ($\theta = F$ for "failure"). The budget constraint of the country i's government writes:

$$G_i^F = t_i^F m_i^F \text{ in case of laissez-faire policy} \tag{1}$$

$$G_i^R + \frac{n}{2} S_i^R = t_i^R m_i^R \text{ in case of bailout policy} \tag{2}$$

with m_i^θ the number of foreign firms settled in country i, and S_i^R the amount of rescue aid defined hereafter. Note that the level of public good provision under a bailout policy is reduced by the amount of expenditures in subsidies. Therefore, there might be a trade-off between two destinations of public expenditures, one (the bailout subsidy) sustaining the economic activity, the other (the public good) directly benefiting to consumers. Note also that the level of tax will directly depend on the bailout decision.

Given the mobility of foreign firms, the two governments not only decide whether or not to rescue ailing DOEs, they also compete for the investment of foreign-owned firms. Therefore, our model shares common features with Gaigné and Wooton (2011), who analyse tax policies with respect to mobile vs. immobile firms when governments can set different tax rates on different tax bases.

2.3 Consumers

Individuals share identical preferences given by a quasi-linear utility function:

$$u_i^\theta = a x_i^\theta - \frac{1}{2}\left(x_i^\theta\right)^2 + z_i^\theta + G_i^\theta \tag{3}$$

where x_i^θ and z_i^θ stand for the consumption of the manufacturing good and the numeraire, respectively, whereas G_i^θ stands for the public good supply in country i. Private good consumption is impacted by the decision of governments to bailout DOEs, through its influence on the market structure and on the equilibrium prices.

The budget constraint for a representative consumer in each country i is then:

$$1 + \bar{z} = z_i^\theta + p_i^\theta x_i^\theta$$

where the wage is equal to unity, p_i^θ is the price of the good produced in the M-sector, and $\bar{z}$ is the individual endowment in the numeraire.

2.4 Sequence of events

Our model contains various forms of interactions embedded in the following sequential game (see Figure 1):

Stage 1 [*Bailout or laissez-faire policy*]. Governments decide independently whether to rescue ailing firms through a full tax exemption and a subsidy (*bailout policy*), or to let them go bankrupt (*laissez-faire policy*)[13]. When doing so, they perfectly anticipate the tax competition outcome, the location of foreign firms and the resulting market outcome.

Stage 2 [*Taxation of foreign firms*]. Governments simultaneously and non-cooperatively choose the level of lump-sum tax on FOEs according to the source principle, given the decision taken at stage 1 and anticipating the impact of their tax choice on the location of foreign firms as well as the market outcome.

Stage 3 [*Location of foreign firms*]. Foreign capital owners decide whether to invest in country A or B given the observed levels of taxation in each country and anticipating the market outcome. The location equilibrium is defined by the share of foreign firms located in country A, that is δ^θ.

Stage 4 [*Production and consumptio*n]. Surviving firms make their output choices and consumption takes place. The remaining firms go bankrupt. This results in an equilibrium price p^θ in the M sector.

The sequence of events from stage 2 to stage 4 is standard in tax competition models (Haufler and Wooton, 2010; Ottaviano and Van Ypersele, 2005; Gaigné and Wooton,

[13] Because DOEs are immobile, there is no reason to consider that governments behave non-cooperatively in the first stage of the game.

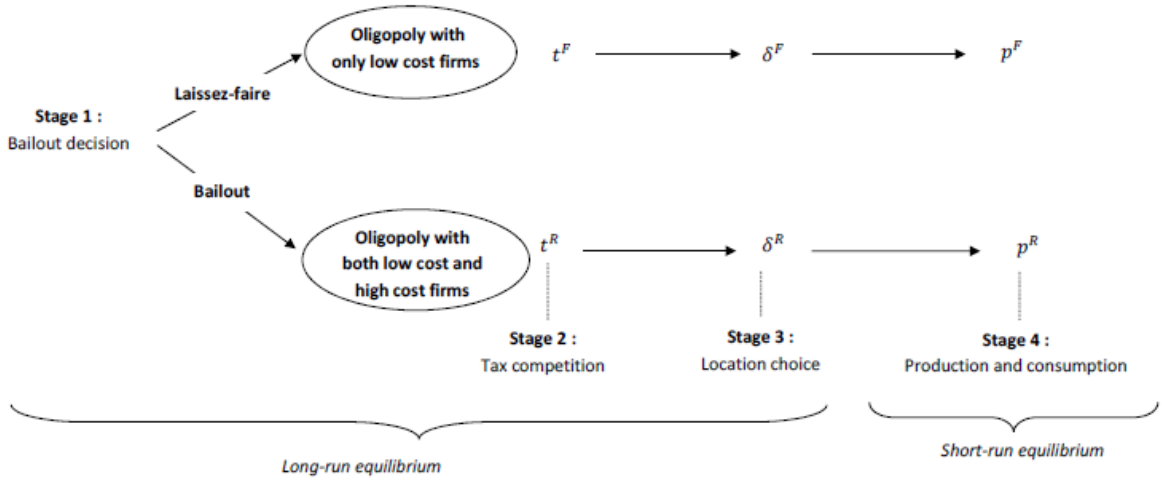

Figure 1: Structure of the game

2010). We incorporate the bailout decision at stage 1 of the game because one may consider that a bailout decision should be less frequent than tax adjustments. In section 5, we describe the results under alternative timing of events. The solution to such a sequential game is given by a subgame perfect Nash equilibrium that we obtain by backward induction beginning with the last stage of the game.[14]

3 Bailout vs. Laissez faire with immobile foreign firms

In this section, we analyze how production and consumption choices made in the last stage are influenced by the laissez-faire vs. bailout decision, when FOEs are immobile and equally distributed across countries (that is, $m_i^\theta = m/2$ in each country, for all θ). This benchmark case corresponds to a situation where governments have no attractiveness concerns when evaluating the opportunity of granting a rescue aid to DOEs. The impact of firm mobility is evaluated in section 4.

3.1 Consumption and production in the private sector

Utility maximization leads to the individual inverse demand function with respect to the manufacturing good:

$$p_i^\theta = a - x_i^\theta \tag{4}$$

[14] Because of the symmetry of our model, the equilibrium outcome is the same in each country.

Aggregating the demand over all consumers yields market demand curves for each country i in the oligopolistic industry :

$$X_i^\theta = \frac{L}{2}\left(a - p_i^\theta\right)$$

whereas the demand for numéraire good is residual.

Now let us analyse output decisions in the manufacturing sector. We assume manufacturing firms compete in quantities. Before describing their output choices, two comments are in order.

Firstly, it is worth stressing that the number of competitors on the market, and thus the toughness of competition is closely related with the bailing-out decision of governments (see Table 1). Neglecting this market structure effect would lead to an incomplete picture of the criteria entering a bailing-out decision.

	$\boldsymbol{\theta = Failure}$	$\boldsymbol{\theta = Rescue}$
Surviving firms	m low cost FOEs	m low cost FOEs n high cost DOEs
Dying firms	n high cost DOEs	0

Table 1: Number and type of firms

If governments decide to adopt a laissez-faire policy ($\theta = F$), DOEs go bankrupt and there will be oligopolistic competition among m FOEs. If governments decide to bail DOEs out ($\theta = R$), there will be oligopolistic competition among $m+n$ asymmetric firms: m high-cost DOEs, and n low-cost FOEs. Therefore, unlike Alexeev and Jang (2010), our model accounts for the effects of a bailout policy on the market outcomes in so far as each configuration described above will lead to specific prices and quantities.

Secondly, the level of trade liberalization shapes output decisions. Indeed, the cost incurred by a firm for exporting each unit of the manufacturing good is equal to τ units of the numeraire.[15]

We are now equipped to describe the product market outcome. Firms are able to segment their markets, choosing the quantities to sell on their domestic and export markets independently. Let $x_{ii}^{k,\theta}$ and $x_{ij}^{k,\theta}$ denote the output choices made by a firm of type k located in country i to sell its good in the domestic and foreign country, respectively. Production directly depends on the type of firm – ($k = FOE$, or $k = DOE$) – through its influence on the marginal cost. Moreover, the output choices are influenced indirectly

[15] This captures all frictions making bilateral trade costly including transport costs or administrative barriers to the free mobility of goods between countries.

by the bailout decision θ through its impact on the number of surviving firms and the toughness of competition.

Following Gaigné and Wooton (2011) and Haufler and Wooton (2010), the total operating profit of a FOE equals the before-tax return to the required unit of knowledge capital, and is defined as:

$$\Pi_i^{FOE,\theta} = p_i^{FOE,\theta} x_{ii}^{FOE,\theta} + \left(p_j^{FOE,\theta} - \tau\right) x_{ij}^{FOE,\theta} \tag{5}$$

Because DOEs bear an additional fixed cost in numéraire, the before-subsidy return to the required unit of knowledge capital is equal to their total operating profit minus the additional fixed cost. As we are interested in the behavior of governments with respect to firms making losses, we assume that before the taxation and bailout decisions,

$$\Pi_i^{DOE} - \Theta < 0$$

with

$$\Pi_i^{DOE,\theta} = \left(p_i^{DOE,\theta} - c\right) x_{ii}^{DOE,\theta} + \left(p_j^{DOE,\theta} - c - \tau\right) x_{ij}^{DOE,\theta}$$

Therefore, DOEs will go bankrupt without any intervention from the government. In order to rescue these firms, the government has to offer a minimum amount of subsidy S such that after the susbidy is granted, the return to capital become non-negative, that is $S = \Theta - \Pi_i^{DOE}$. This assumption is consistent with the conventional wisdom in most countries. For example, the guidelines on state aid for the rescue and the restructuring of firms in difficulties in the European Union include a principle of proportionality (aid limited to the minimum).[16]

Let us now analyse the production outcome, for given taxes and bailout policies. In the short-run, the corporate tax has no impact on the production outcome because it is lump-sum.[17] By contrast, the production outcome varies depending on the attitude of the governments with respect to their ailing DOEs. Maximizing before-tax profits with respect to quantities given the inverse demand function (4), we get the following output levels for a FOE:

$$x_{ii}^{FOE,\theta} = \frac{L}{2} p_i^{\theta} \text{ and } x_{ij}^{FOE,\theta} = \frac{L}{2}\left(p_j^{\theta} - \tau\right) \tag{6}$$

[16] See the Guidelines on State aid for rescuing and restructuring non-financial undertakings in difficulty: *"Rescue aid must be restricted to the amount needed to keep the beneficiary in business for six months..."* (European Commission, 2014/C 249/01).

[17] However, corporate taxation will introduce a distorsion in the long-run when foreign investors will decide whether to invest in country A or B.

Output levels for a DOE are given by:

$$x_{ii}^{DOE,R} = \frac{L}{2}\left(p_i^R - c\right) \text{ and } x_{ij}^{DOE,R} = \frac{L}{2}\left(p_j^R - c - \tau\right) \tag{7}$$

in case of a bailout whereas $x_{ii}^{DOE,F} = x_{ij}^{DOE,F} = 0$ if the government does not rescue the firms.

Equilibrium prices are obtained by inserting the equilibrium output choices (6) and (7) in the market clearing conditions. The equilibrium prices are as follows:

$$p_i^F \equiv \frac{a + \frac{m}{2}\tau}{m+1} \tag{8}$$

$$p_i^R \equiv \frac{a + cn + \tau\left(\frac{m}{2} + \frac{n}{2}\right)}{m+n+1} \tag{9}$$

Then, we determine the trade feasibility conditions ensuring that exporting is always profitable for all types of firms whatever the decision of the government. We obtain the following condition for the foreign firms located in i:

$$x_{ij}^{FOE,F} > 0 \text{ iif } \tau < \tau^F = 2\frac{a}{m+2}$$

and the same condition applies for firms located in country j. The trade feasibility condition ensuring that domestic firms export is given by:

$$x_{ij}^{DOE,R} > 0 \text{ iif } \tau < \tau^R = 2\frac{a - c\left(m+1\right)}{m+n+2}$$

and the same condition applies for firms located in country j. Hereafter, we assume that $a > a_{\min} \equiv c\left(m+1\right)$ so that this trade condition is compatible with the presence of trade costs. We check that the trade condition is more restrictive in the case of bailout ($\tau^R < \tau^F$). Starting from the highest possible level of trade cost (that is, τ^F), trade liberalization will first allow low-cost FOEs to export and it is only once trade costs reach a lower threshold value (that is, τ^R) that high-cost DOEs rescued by the government will also begin to export. Given our focus on the attitude of governments with respect to DOEs that are active on the global market, we will consider in what follows that the most restrictive trade condition applies, that is $\tau < \tau^R$. Importantly, those conditions are unchanged when FOEs are mobile because the symmetry of the model gives rise to the same spatial allocation of firms across countries at the equilibrium. We summarise these conditions by the following assumption.

Assumption 1 *We assume that trade costs are lower than* $\tau^R = 2\frac{a - c\left(m+1\right)}{m+n+2}$, *with* $a > a_{\min} \equiv c\left(m+1\right)$. *Therefore, both FOEs and DOEs have an exporting activity.*

Expressions of the market prices (8) and (9) reveal the presence of the distorsive effect arising from a bailout policy. On the one hand, the average marginal cost is higher under bailout policy because surviving firms bear a higher marginal cost. On the other hand, a bailout decision strengthens competition by raising the number of surviving firms as compared to the laissez-faire decision. The latter effect dominates. We verify that under Assumption 1, $p_i^R < p_i^F$. Thus, the decision to rescue inefficient domestic firms exerts a downward pressure on prices by contributing to a higher number of competitors on the market, even though they are less productive.

Importantly, this result is consistent with the findings of Wollman (2018) regarding the impact of the US auto industry bailout in 2009. This is, to our knowledge, the only paper that analyzes the consequences of a government rescue plan from the perspective of industrial organization by comparing this outcome with a counterfactual such as bankruptcy or acquisition. By estimating a structural model of the US commercial vehicle market, the author shows that the exit of GM and Chrysler or their merger into a rival (Ford or PACCAR) would have resulted in major markup increases due to the higher market power of surviving firms.[18]

From eq. (6), (7), (8) and (9), we obtain the following equilibrium operating profits:

$$\Pi_i^{FOE,F} = \frac{L}{4}\frac{\Lambda}{(m+1)^2},\ \Pi_i^{FOE,R} = \frac{L}{4}\frac{\Gamma}{(m+n+1)^2} \text{ and } \Pi_i^{DOE,R} = \frac{L}{4}\frac{\Omega}{(m+n+1)^2}$$

where, Λ, Γ and Ω are positive bundles of parameters defined in appendix A.1. We check that:

$$\Pi_i^{FOE,F} > \Pi_i^{FOE,R} > \Pi_i^{DOE,R} > 0$$

Intuitively, the operating profits made by FOEs are higher than those made by DOEs rescued by the government. This results from the higher marginal cost in DOEs as compared to FOEs. In addition, observe that the operating profits made by FOEs are always higher if the governments do not rescue DOEs, because the exit of DOEs relaxes competition in the M sector and raises the equilibrium price.

3.2 Tax policy

Each government is benevolent and maximises the overall welfare of residents given by the sum of the total consumer surplus and the utility from public good provision:

$$W_i^\theta = \frac{L}{2}\left(CS_i^\theta + G_i^\theta\right) \tag{10}$$

[18] In addition, they show that the possibility for surviving firms to introduce or withdraw products can be decisive for the bailout decision (see Wollman, 2018).

The individual consumer's surplus in each country is given by:

$$CS_i^\theta = \frac{1}{2}\left(a - p_i^\theta\right)^2 \tag{11}$$

while public good provision is given by

$$G_i^F = t_i^F m \tag{12}$$

in case of laissez-faire policy and

$$G_i^R = t_i^R m - \frac{n}{2} S \tag{13}$$

in case of rescue policy, with $S = \Theta - \Pi_i^{DOE}$.

Observe that a bailout decision influences both the consumer's surplus, through its impact on competition in the manufacturing sector, and public good provision through its influence on tax revenues and the financial cost of the bailout.

Given the timing of events, the decision whether or not to rescue ailing firms (θ) is considered as given when governments set their tax policy. Moreover, they anticipate the impact of their tax policy on the outcome on the product market when deciding upon their tax policy. Recall that we assume in this section that FOEs are immobile, so that there are no strategic interactions. Substituting the budget constraint (1) in (10) and differentiating the country's welfare expression with respect to its own tax, we obtain the following first order condition for any country i:

$$\frac{dW_i^\theta}{dt_i^\theta} = \frac{L}{2}\frac{dG_i^\theta}{dt_i^\theta} = \frac{L}{2} m_i^\theta > 0 \tag{14}$$

When FOEs are immobile, the consumer surplus is independent of taxes so the tax policy influences the national welfare through its impact on public good provision only. The immobility of FOEs leads to a corner solution such that governments grab all their profits. We therefore obtain the following symmetric equilibrium corporate tax on FOEs in each country:

$$\hat{t}^F = \Pi^{FOE,F} \text{ and } \hat{t}^R = \Pi^{FOE,R}$$

Intuitively, in line with the results of Gaigné and Wooton (2011), governments tax away the profits made by FOEs because their tax base elasticity is equal to zero and the return to capital invested in FOEs accrue to individuals living in a third country.[19]

[19] Of course this is an extreme case and the maximum corporate tax rate is never confiscatory in the reality. However, assuming that governments are faced with constitutional and more generally institutional constraints, which prevent them from taxing away all the profits, would not change anything to our reasoning.

Importantly, the decision of the government to bailout DOE or not impacts the level of profits extracted by the government. As the decision will induce different number of competitors, the resulting operating profits and the level of equillibrium taxes vary. Unsurprisingly, we check that the most competitive configuration, which is the one resulting from the decision to bailout DOEs, implies a lower tax level. Indeed, we have in each country:

$$\hat{t}^F - \hat{t}^R = \frac{1}{4} Ln \frac{\tau^R (m+n+2) - \tau}{(m+1)^2 (m+n+1)^2} \Xi > 0 \tag{15}$$

with $\Xi = 2(a(2m+n+2) + cn(m+1)) - (2m+n+2)\tau > 0$ for admissible values of trade costs.

Proposition 1 *Assume that FOEs cannot relocate and that assumption 1 holds. Then,*

i) regardless of their bailout policy, the governments tax away all the FOEs' profits;

ii) equilibrium corporate taxes are lower when governments decide to rescue the DOEs than when they choose to let them go bankrupt.

Finally, we check that $d\left(\hat{t}^F - \hat{t}^R\right)/d\tau < 0$, meaning that trade integration tends to increase the tax gap between t_i^F and t_i^R . Therefore, the decision not to rescue DOEs is becoming more and more profitable in terms of public good provision.

3.3 Bailout decision

We now analyze the rescue vs. laissez-faire decision taken by the government at the 1st stage of the game. By doing this, the government anticipates the impact of its decision on the tax outcome and the short-run equilibrium.

The sum spent to rescue each ailing firm is considered as exogenous by the government. The intuition is that the government engages the minimum amount of subsidy that just allows the firm to survive. This assumption is in line with the general consensus according to which governments are incited not to give too much money to the ailing firm in order to limit a potential moral hazard problem, which is well known in the literature about the soft budget constraint (see Kornai, Maskin and Roland, 2003).

Therefore, the government decides *whether or not* to rescue ailing DOEs, given that rescueing the firm would involve an individual subsidy $S^R = \Theta - \Pi_i^{DOE}$. The government is assumed to be benevolent, so it decides to rescue DOEs if and only if it leads to a higher national welfare compared to a laissez-faire decision. The variation in welfare that would result from a bailout decision is given by:

$$W^R - W^F = \frac{L}{2}\left[\left(CS^R - CS^F\right) + \left(G^R - G^F\right)\right] \tag{16}$$

Thus, the difference in national welfare between a bailout and a laissez-faire decision depends on the sum of the difference in consumers' surplus and the difference in public good provision.

The consumer surplus is higher under a bailout policy, thanks to its negative impact on the equilibrium price level. However, public good provision is higher under a laissez-faire policy because : i) corporate tax revenues are higher and ii) the budget devoted to public good provision is not reduced by subsidies going to DOEs. Therefore, each government is faced with a trade-off between welfare gains of the bailout in terms of consumers surplus and the cost of the bailout in terms of reduced public good provision.

Inserting all equilibrium variables in the above welfare differential, we obtain that:

$$\hat{W}^R \gtreqqless \hat{W}^F \text{ iif } \Theta \lesseqqgtr \hat{\Theta} = \frac{\hat{\Psi}\tau^2 + \hat{\Upsilon}\tau - \hat{\Sigma}}{(m+1)^2 (m+n+1)^2}$$

where $\hat{\Psi}$, $\hat{\Upsilon}$ and $\hat{\Sigma}$ are positive bundles of parameters given in appendix A.2.

Clearly, $\hat{\Theta}$ is increasing with the level of trade costs, and is negative when trade costs are equalized to zero. The maximum value of $\hat{\Theta}$ is therefore given by $\hat{\Theta}\Big|_{\tau=\tau^R}$. In appendix A.2, we show that

$$\hat{\Theta}\Big|_{\tau=\tau^R} \gtrless 0 \text{ for all } L \lessgtr \hat{L} = \frac{a\left(2m + n\left(2m+1\right) + 2m^2\right) + c\left(m+1\right)\left(2m-n\right)}{a\left(-2m + mn - 2\right) + c\left(m+1\right)\left(6m + mn + 2m^2 + 2\right)}$$

When $L < \hat{L}$, the threshold value $\hat{\Theta}$ is positive for all trade costs higher than $\hat{\tau}$, the second root of $\hat{\Psi}\tau^2 + \hat{\Upsilon}\tau - \hat{\Sigma}$.[20] If however, $L \geq \hat{L}$, we verify that $\hat{\Theta} \leq 0$ for all admissible values of trade costs so that the inequality $\Theta > \hat{\Theta}$ is checked and governments are incited to let DOEs fail.

We summarize those results in Proposition 2.

Proposition 2 *Assume that FOEs are immobile and assumption 1 holds. Then:*

i) For $L < \hat{L}$, each government decides to rescue local DOEs if and only if $\hat{\tau} < \tau \leqq \tau^R$ and $\Theta < \hat{\Theta}$. It lets them go bankrupt otherwise.

ii) For $L \geq \hat{L}$, each government let local DOEs go bankrupt whatever the level of trade costs and fixed costs.

In sum, there are two necessary (but not sufficient) conditions for a rescue plan to be decided. First, high trade costs render the rescue more likely as $d\hat{\Theta}/d\tau > 0$. This is partly due to the fact that the consumer surplus is less sensitive to a variation of the

[20] The first root is negative.

number of competitors when bilateral trade is facilitated. As a consequence, the welfare gain of a bailout policy decreases and the laissez-faire policy may become the best policy option. In addition, the level of fixed costs, that captures a part of the inefficiency of DOEs, matters through its influence on the financial cost of the bailout. Fixed costs have to be low enough for a bailout policy to arise because it limits the detrimental impact on public good provision.

However, both conditions are not sufficient for a bailout to occur. Country size (L) also matters through its effect on public good provision, which might be different depending on whether bailout or laissez-faire is chosen.[21] Indeed, recall that the operating profits of FOEs and the level of corporate taxation grow with L. Thus, a larger market size contributes to lower the financial cost of a bailout (see equation 13). However, the market size also increases the opportunity cost of the bailout in terms of tax revenues. Indeed, as illustrated by (15), the government's ability to extract rent from FOEs, which is captured by $\hat{t}^{\theta} = \Pi^{FOE,\theta}$, rises more rapidly with the market size when $\theta = F$ than when $\theta = R$, because competition is less intense in the former case. Part ii) of Proposition 2 suggests that the latter effect dominates. In the end, it is never optimal to rescue DOEs from the point of view of a large country (that is, when $L \geq \hat{L}$) because the opportunity cost of a bailout in terms of tax revenues losses is too high.

4 Bailout vs laissez-faire decision with mobile foreign firms

We now assume that FOEs are mobile in order to evaluate the impact of firm mobility on the behavior of governments. By contrast, DOEs remain immobile. Therefore, internationally mobile FOEs and immobile DOEs compete in the oligopolistic sector (see Gaigné and Wooton, 2011). As it is standard in the tax competition literature (see. Wilson, 1999), we assume that the location choice occurs after governments have set their taxes, so that the tax base elasticity becomes negative. Therefore, the mobility of FOEs will influence the fiscal policy and the bailout decision that are taken in the long-run, by anticipating their impact on the location of FOEs.

In what follows, we solve the game by backward induction. Section 4.1 analyses the short-run outcome and the location decision. Then, we analyse how this location pattern influences the tax decision (section 4.2) and, *in fine*, the bailout decision (section 4.3.).

[21] By contrast, the difference in individual consumer's surplus is unaffected by country size.

4.1 Location decision

Let $\delta = m_A/m$ stand for the proportion of FOEs that locates in country A. In the short run, equilibrium prices are now expressed as follows:

$$p_A^F \equiv \frac{a + \left(1 - \delta^F\right) m\tau}{m+1}$$

$$p_A^R \equiv \frac{a + cn + \tau\left(\left(1 - \delta^R\right) m + \frac{n}{2}\right)}{m+n+1} \tag{17}$$

and symmetric expressions hold for country B.

Observe that the prices now depend on the endogeneous spatial distribution of FOEs. The higher the share of FOEs located within a country, the lower the price level in that country ($dp_A^\theta/d\delta^\theta < 0$), and therefore the higher the consumer's surplus. This effect is proportional to the level of trade costs, because trade costs act as a protection barrier with respect to competition in the foreign market. When they decline, prices in each country become less responsive to the number of local firms.

Moreover, competition is fiercer when governments rescue DOEs, so that equilibrium prices are less responsive to a change in the location of firms:

$$\frac{dp_A^F}{d\delta^F} < \frac{dp_A^R}{d\delta^R} < 0$$

Recall that governments impose a lump-sum tax on profits made by foreign firms, so that the after-tax profit made by FOEs located in country i is given by $\Pi_i^{FOE,\theta} - t_i^\theta$. Foreign investors decide to invest in the most profitable country. Therefore, the equilibrium location δ^θ is governed by the spatial difference in after-tax profit, that is:

$$\Delta^\theta = \left(\Pi_A^{FOE,\theta} - t_A^\theta\right) - \left(\Pi_B^{FOE,\theta} - t_B^\theta\right)$$

After inserting equilibrium prices and quantities in (5), we obtain:

$$\begin{aligned}
\Delta^F &= t_B - t_A - L\tau^2 \frac{\left(2\delta^F - 1\right) m}{m+1} \\
\Delta^R &= t_B - t_A - L\tau^2 \frac{\left(2\delta^R - 1\right) m}{m+n+1}
\end{aligned}$$

The location equilibrium, in case of failure or bailout respectively, can be defined as the share of FOEs located in country A such that $\Delta^\theta = 0$, that is:

$$\begin{aligned}
\delta^F &= \frac{1}{2} - \frac{1}{2}\frac{m+1}{Lm\tau^2}\left(t_A - t_B\right) \\
\delta^R &= \frac{1}{2} - \frac{1}{2}\frac{m+n+1}{Lm\tau^2}\left(t_A - t_B\right)
\end{aligned} \tag{18}$$

These location equilibria are the result of two forces.

The first one is standard and depicts a pro-competitive effect. When a country hosts new firms, incumbent firms face more competitors in their domestic market and fewer in the other one. Thus, the domestic price falls while it rises in the other market. Because domestic sales generate more revenues in the presence of trade costs, this effect acts as a dispersion force ($d\Delta^{\theta}/d\delta_A^{\theta} < 0$). Interestingly, the effect is amplified when the number of incumbent firms is low. Hence, the pro-competitive effect is stronger when governments decide not to rescue DOEs.

The second force results from the impact of the tax wedge $t_A - t_B$ on the location choice.[22] A unilateral rise in corporate taxation in country i leads to an outflow of firms ($d\delta^{\theta}/dt_A^{\theta} < 0$). Moreover, as competition is fiercer when $\theta = R$, FOEs are more responsive to positive tax variations and the tax base erosion effect is stronger ($d\delta^R/dt_A^R < d\delta^F/dt_A^F < 0$). Thus, by influencing the tax responsiveness of firms, our model displays a relationship between the bail out decision and the ability to collect tax revenues. It is worth stressing that the market size measured by L reduces this tax base erosion effect, and this downward effect is amplified when governments decide to bail out DOEs (see. 18). As demonstrated below, this property which recalls the key role of the market size for tax policy in presence of mobile firms, will influence the policy choice of governments with respect to ailing firms.

Finally, we also verify that the tax base erosion effect is getting stronger when trade costs fall because prices become less and less responsive to the spatial distribution of firms. In other terms, gradual trade integration weakens the pro-competitive effect, increasing the weight of taxes in the capital location choice.

4.2 Tax policy

When deciding upon their tax policies, governments anticipate the repercussion of their choice on the location of FOEs. As the corporate tax base is now elastic, governments choose their tax policy non-cooperatively and independently. Substituting the budget constraint (1) in (10) and differentiating each country's welfare expression with respect to its own tax, we derive the following first order conditions:

[22] Despite their lump-sum form, taxes distort investment choices through the spatial distribution of FOEs.

$$\frac{dW_i^F}{dt_i^F} = \frac{L}{2}[\underbrace{-\frac{dp_i^F}{dt_i^F}\left(a - p_i^F\right)}_{\text{Consumer surplus effect (-)}} + \underbrace{m(\delta_i^F + t_i^F \frac{d\delta_i^F}{dt_i^F})}_{\substack{\text{Public good provision effect} \\ \text{arising through tax revenues (+/-)}}}] \tag{19}$$

$$\frac{dW_i^R}{dt_i^R} = \frac{L}{2}[\underbrace{-\frac{dp_i^R}{dt_i^R}\left(a - p_i^R\right)}_{\text{Consumer surplus effect (-)}} + \underbrace{m(\delta_i^R + t_i^R \frac{d\delta_i^R}{dt_i^R})}_{\substack{\text{Public good provision effect} \\ \text{arising through tax revenues (+/-)}}} + \underbrace{\frac{n}{2}\frac{d\Pi_i^{DOE}}{dt_i^R}}_{\substack{\text{Public good provision effect} \\ \text{arising through the amount of subsidies (+)}}}] \tag{20}$$

Whatever the attitude of governments with respect to ailing firms ($\theta = R$ or F), an unilateral tax rise exerts two – qualitatively – identical effects on the national welfare.

The first term captures the negative effect of taxation on the consumer's surplus. Following an unilateral tax rise, some owners of FOEs will relocate their production in the other country. This relocation relaxes the degree of competition on the domestic market, thereby rising the equilibrium price ($dp_i^\theta/dt_i^\theta = 1/2L\tau \ \forall \ \theta$)[23]. Nevertheless, recall that the level of prices are lower and the consumer surplus initially higher when $\theta = R$. Therefore, the negative consumer surplus effect resulting from a positive tax variation is then stronger in the case of a bailout policy. Thus, focusing on the consumer surplus, governments have a stronger incentive to set a low level of taxation if they decide to rescue DOEs.

The second term captures the standard taxation effect on public good provision. For a given spatial distribution of FOEs, a marginal increase in t_i^θ raises tax revenues and the resulting level of public good provision. This provides governments with an incentive to raise taxation, that is not impacted by the bailout decision at the symmetric equilibrium ($\delta_i^R = \delta_i^F = 1/2$). Nevertheless, this effect can be counterbalanced by a tax base erosion effect due to the relocation of some FOE in the other country. The latter effect is stronger if governments rescue DOEs because of fiercer competition on the goods market ($d\delta^R/dt_A^R < d\delta^F/dt_A^F < 0$). Therefore, because of a higher tax responsiveness, governments rescueing DOEs are more incited to set a low level of taxation.

In sum, and as expected, these two first effects identified through the FOC illustrate that capital mobility assumption sharply affects the way tax choices are influenced by

[23] Note that this effect is not influenced by the bailout decision, because the stronger price competition effect that could arise from the bankruptcy of FOEs ($dp_A^F/d\delta < dp_A^R/d\delta < 0$) is exactly compensated by the weaker tax base erosion effect ($d\delta^R/dt_A^R < d\delta^F/dt_A^F < 0$).

the bail-out decision. Deciding to rescue ailing firms impacts the market structure and the competitive pressure, two elements that governments should internalise when they set their optimal level of taxation on mobile firms. Summarising those two first terms of the FOC, it appears that the incentive to set a low tax is stronger for governments if they decided, at the first stage of the game, to rescue the DOEs. The intuition is that fiercer competition on the goods market both raises the consumer surplus and the tax base elasticity.

But this is not the end of the story. It is worth stressing that when governments decide to rescue DOEs, they implement a regime of tax discrimination as only mobile firms are taxed. In a similar framework, Gaigné and Wooton (2011) define the various tax equilibria resulting from such a configuration. They show that being all competitors on the same market, the tax choice on a category of firms impacts the profitability of the other one. Here, when making the bailout choice in the first stage, governments cannot ignore that their tax policy with respect to FOEs will influence the degree of competition faced by the DOEs, their operating profits, the cost of the bailout and by incidence, the level of public good provision. This effect is captured by the third component of (20). Following a marginal positive variation of t_i^R, the relocation of some FOEs in the foreign country relaxes competition on the domestic market. As a consequence, prices and operating profits of DOEs increase ($d\Pi_i^{DOE}/dt_i^R > 0$). This reduces the level of deficit of DOEs, and the amount of subsidy necessary to rescue them. In other terms, the tax policy acts here a as protection instrument, which relaxes competition faced by DOEs and restores tax revenues in order to invest in public goods. Clearly, this new effect moderates the two effects mentioned above that lead governments to set lower taxes when $\theta = R$. Importantly, this incentive is more important for large countries because the positive impact of a foreign capital outflow on operating profits of DOEs is increasing with country size. To sum up, following a positive variation of tax, the reduction of the bailout cost is higher, the larger the market size. This finding will crucially matter to understand the attitude of governments with respect to ailing firms.

Solving the first order condition for each government and crossing the reaction functions, we obtain the following symmetric Nash tax equilibrium:

$$\begin{aligned}
\tilde{t}^F &= \frac{1}{2} m\tau \frac{\tau + 2L\tau + 2Lm\tau - 2a}{(m+1)^2} \\
\tilde{t}^R &= \frac{1}{2}\tau \frac{(m+n+L(2m+n)(m+n+1))\tau - 2(am+an-cn)}{(m+n+1)^2}
\end{aligned}$$

In what follows, we restrict ourselves to the case where the resulting tax equilibrium

allows governments to provide public goods. In other terms, we only retain positive tax equilibria for the rest of the analysis. As we can see from the above expressions, taxes are increasing with country and market size (L). Thus, $\tilde{t}^F$and $\tilde{t}^R$ are positive when L is large. Formally, we obtain:

$$\begin{aligned} \tilde{t}^F &> 0 \text{ when } L > \tilde{L}^F = \frac{1}{2}\frac{2a-\tau}{\tau(m+1)} \\ \tilde{t}^R &> 0 \text{ when } L > \tilde{L}^R = \frac{2am+2an-2cn-m\tau-n\tau}{\tau(2m+n)(m+n+1)} \end{aligned}$$

with $\tilde{L}^F > \tilde{L}^R > 0$ for $\tau < \tau^R$. Therefore, a sufficient condition for both $\tilde{t}^F$ and $\tilde{t}^R$ to be positive is that $L > \tilde{L}^F$.

Whatever the attitude of governments with respect to DOEs, equilibrium taxes are increasing with L. There exists a vast literature on tax competition showing that larger countries set a higher tax on mobile capital, based on different theoretical backgrounds. The fact that larger countries set higher corporate taxes was first shown in the standard tax competition literature that assumes perfectly competitive factor and product markets (see Bucovetsky, 1991, and Wilson, 1991). Indeed, the cost of capital is less sensitive to tax changes in a large country than in a small one because the former has a market power on the world capital market, and then has an incentive to increase its tax rate in order to push down the world net return on capital. More recently, Han et al. (2014) showed that a small country set a lower tax due to its limited institutional capacity in providing infrastructure services. Moreover, in a similar framework than ours assuming increasing returns to scale and trade costs, Baldwin and Krugman (2004), Ottaviano and Van Ypersele (2005) as well as Haufler and Wooton (2010) demonstrate that the need for governments to cut taxes and sustain the attractiveness of their country decreases with market size.[24] Indeed, the country size reduces the tax base erosion that follows a positive tax variation (see. 18).

We enrich this existing literature by showing that besides the latter effect, governments that in the first stage of the game decided to rescue DOEs have an additional incentive to increase taxation which is proportional to the country size. As explained above, governments may be less prompt to improve their tax attractiveness, because the relocation of some foreign firms in the country would deteriorate the operating profits of DOEs and this would raise the financial cost of the bailout. As this effect is amplified when the population is larger, it contributes to explain why the condition on the market

[24]There is also empirical evidence that, on average, large countries set relatively higher tax rates than smaller ones (Exbrayat and Geys, 2014; Azémar et al., 2015).

size for $\tilde{t}^{\theta} > 0$ is more restrictive when $\theta = F$ (that is $L > \tilde{L}^F > \tilde{L}^R$).

Whatever the bailout decision at stage 1, it is straightforward to check that $\tilde{t}^F$ and $\tilde{t}^R$ are strictly increasing with τ according to the above conditions. Put differently, the tax equilibrium arising from the two configurations reacts in the same way to trade integration. Following a decline in trade costs, mobile firms are more responsive to tax variations, tax competition becomes fiercer and the resulting levels of taxation are lower.

Finally, observe that the tax equilibrium is no more restricted to corner solutions where governments grab all the profits of FOEs. The level of equilibrium taxes remains lower than operating profits when trade costs are not too high. To be precise, $\tilde{t}^F < \Pi^{FOE,F}$ if only if and only if $\tau < \tilde{\tau}^F$ and $t^R < \Pi^{FOE,R}$ if and only if $\tau < \tilde{\tau}^R$, with the values of $\tilde{\tau}^F$ and $\tilde{\tau}^R$ being given in Appendix A.3. In what follows, we exclude the corner solution of a full taxation of profits, a highly unrealistic configuration in a context of mobile firms. Conditions for an interior tax equilibrium are summarised by the following assumption.

Assumption 2 *When FOEs are mobile, we assume that* $\tau < \min\left[\tilde{\tau}^F, \tilde{\tau}^R\right]$ *and* $L > \tilde{L}^F$. *This makes it sure that governments partially tax away FOEs' profits, whatever their decision with respect to ailing firms at stage 1 of the game.*

The tax differential between the bailout and the no-bailout case is given by:

$$\tilde{t}^R - \tilde{t}^F = \frac{n\tau}{2\left(m+1\right)^2\left(m+n+1\right)^2}\Xi \tag{21}$$

where $\Xi = 2\left[a\left(mn+m^2-1\right)+c\left(m+1\right)^2\right] - \tau\left[L\left(m-1\right)\left(m+1\right)\left(m+n+1\right)+m\left(m+n\right)-1\right]$.

The sign of this tax gap is a priori ambiguous. We show that $\tilde{t}^R \gtrless \tilde{t}^F$ when $L \gtrless \tilde{L}$ (cf. Proof in the appendix A.4), with

$$\tilde{L} = \frac{2\left(m+1\right)\left(c-a+am+cm\right)+2amn-\tau\left(m^2+nm-1\right)}{\tau\left(m-1\right)\left(m+1\right)\left(m+n+1\right)} \in \left[\tilde{L}^F; +\infty\right]$$

Remember that $\hat{t}^R < \hat{t}^F$ when FOEs were assumed to be immobile. Thus, as expected, tax decisions can be influenced by the assumption of mobility of FOEs. Taxes become higher under a bailout policy as soon as the level of population is high enough, thereby reversing the sign of the tax differential compared to a situation of firm immobility. This is the direct consequence of the last term of equation (20), we discussed above. To avoid a stronger deterioration of the operating profits of DOEs and limit the cost of the bailout, governements raise taxation to reduce the country's attractiveness. This effect is more important for larger countries. This explains why $\tilde{t}^R$ becomes higher than $\tilde{t}^F$ when country size is higher than $\tilde{L}$.

This leads us to formulate the following proposition.

Proposition 3 *Assume that FOEs are mobile and assumption 2 holds. Then,*

i) the equilibrium taxes correspond to an interior solution such that $0 < \tilde{t}^F < \Pi^{FOE,F}$ *and* $0 < \tilde{t}^R < \Pi^{FOE,R}$*;*

ii) equilibrium corporate taxes are higher when governments decide to rescue the DOEs than when they choose to let them go bankrupt ($\tilde{t}^R > \tilde{t}^F$) *if and only if the country size is large enough* ($L > \tilde{L}$)*.*

A consequence from this proposition is that when FOEs are mobile, public good provision may be higher under a bailout policy. A necessary (but not sufficient) condition is that country size is large ($L > \tilde{L}$) so that taxes are higher under a bailout policy. In addition, the fixed cost beared only by DOEs must be low enough to reduce the impact of the financial cost of the rescue plan on public good provision. Formally, we verify that:

$$\begin{aligned} G^F &< G^R \text{ when } L > \tilde{L} \text{ and } \Theta < \frac{m}{n}\left(\tilde{t}^R - \tilde{t}^F\right) + \Pi^{DOE,R} = \bar{\Theta} \\ G^F &\geqslant G^R \text{ if } L \leqslant \tilde{L} \text{ or if } L > \tilde{L} \text{ and } \Theta \geq \bar{\Theta} \end{aligned}$$

4.3 Bailout decision

We now analyze the government's trade-off between the bailout and the laissez-faire policies, given its impact on equilibrium taxes, the location equilibrium, production and consumption choices. The model being symmetric, the equilibrium location that results from the above equilibrium taxes is also symmetric. It is also independant of the governments' decision with respect to ailing firms, that is: $\delta^F = \delta^R = 1/2$.

Each government compares the national welfare under a bailout vs. laissez-faire decision. Again, the variation in welfare that would result from a bailout decision is given by the sum of the difference in consumers' surplus and the difference in public good provision (16). As in the benchmark case with immobile FOEs, the attitude of governments with respect to ailing firms will strongly depend on their country size.

Let us first consider a large country, with $L > \tilde{L}$. If the fixed cost level borne by ailing DOEs is not too high (that is, $\Theta < \bar{\Theta}$), then the public good provision is improved by a rescue policy because $\tilde{t}^R > \tilde{t}^F$ and the financial cost of the bailout is limited. In that case a bailout policy both raise the consumers surplus and improve public good provision, so that governments decide to rescue DOEs. In addition, it is worth mentioning that the increase in consumers' surplus that results from the bailout is getting stronger as trade costs fall ($d\left(CS^R - CS^F\right)/d\tau < 0$). Stronger competition in the manufacturing sector makes prices more responsive to the level of trade costs ($dp^R/d\tau > dp^F/d\tau > 0$),

so that the decline in prices due to a switch between a laissez-faire and a bailout policy is increasing with trade integration. If however, fixed costs are large and higher than $\bar{\Theta}$, governments are confronted with a trade-off between sustaining a higher number of competitors that lower the consumer's price and reducing public good provision.

Let us now consider a small country, with $L \leq \tilde{L}$. Public good provision in those countries is always higher under a laissez-faire policy so that governments are confronted with the same trade-off as governments of large countries when their DOEs suffer large fixed costs. The result of this trade-off finally depends of the level of fixed costs. Specifically,

$$\tilde{W}^R \gtrless \tilde{W}^F \text{ if and only if } \Theta \lessgtr \frac{\tilde{\Psi}\tau^2 + \tilde{\Upsilon}\tau + \tilde{\Sigma}}{(m+1)^2 (m+n+1)^2} = \tilde{\Theta} > 0$$

with $\tilde{\Theta} > \bar{\Theta}$ for all admissible values of trade costs.

$\tilde{\Psi}$, $\tilde{\Upsilon}$ and $\tilde{\Sigma}$ are bundles of parameters given in appendix A.5. $\tilde{\Psi}$ and $\tilde{\Upsilon}$ can take a positive or negative values, whereas we ckeck that $\tilde{\Sigma} > 0$. Thus, $\tilde{\Theta}$ is positive for zero trade costs. We also verify that $\tilde{\Theta}$ is positive for the highest possible level of trade costs τ^R. As far as $\tilde{\Psi}$ can be negative, we can conclude that $\tilde{\Theta}$ can be positive for all admissible values of trade costs ($\tau \in \left[0; \tau^R\right]$).[25]

We summarize the attitude of governments with respect to ailing firms by Proposition 4.

Proposition 4 *Assume that FOEs are mobile, and that both assumptions 1 and 2 hold. Whatever the country size, governments decide to rescue ailing DOEs if and only if* $\Theta < \tilde{\Theta}$ *whereas they let them go bankrupt otherwise.*

The mobility of FOEs therefore influences the attitude of governments with respect to ailing DOEs, especially for large countries. When FOEs are immobile, those countries always let DOEs go bankrupt because the opportunity costs of a bailout in terms of tax revenue losses (related with the fact that governments grab all profits) are magnified by country size. The mobility of FOEs engenders tax competition. Contrary to the benchmark case with immobile firms, tax competition gives rise to a higher level of tax revenues in larger countries when the governments rescue DOEs, which provides governments with an additional incentive to rescue DOEs.

[25] We cannot fully characterize the property of the function $\tilde{\Theta}(\tau)$ without making additionnal assumptions regarding the number of FOEs and DOEs. Nevertheless, we can show that the only configuration under which $\tilde{\Theta}$ can be negative is if and only if i) $\tilde{\Psi} > 0$, ii) $\tilde{\Upsilon} < 0$ and iii) τ is neither too low nor too high (see Appendix A.5). Under these very specific circumstances, governments would adopt a laissez-faire policy whatever the level of fixed cost. This does not call into question the conclusion in Proposition 5.

Finally, let us analyze the impact of firm mobility on the behavior of governments with respect to ailing firms. By comparing $\tilde{\Theta}$ and $\hat{\Theta}$, we obtain:

$$\tilde{\Theta} - \hat{\Theta} = \frac{m}{n}\left(\left(\tilde{t}^S - \tilde{t}^H\right) - \left(\hat{t}^S - \hat{t}^H\right)\right)$$

and we check that:

$$\tilde{\Theta} > \hat{\Theta} \text{ for all } \tau \in \left[0, \tau^R\right]$$

By summarizing the results in Propositions 2 and 4, we thus verify that the mobility of FOEs either relaxes the condition ensuring that rescuing DOEs is welfare enhancing (when $L < \hat{L}$), or it leads governments to move from an unconditional laissez-faire policy to a potential bailout policy conditional on the fixed cost level (when $L \geqslant \hat{L}$).

This lead us to formulate the following proposition.

Proposition 5 *Assume that assumptions 1 and 2 hold. Whatever the size of their country, governments are more prone to bail out DOEs if FOEs are mobile.*

To understand why firm mobility makes governments more prone to adopt a bailout policy, we can compare the welfare variation that results from a bailout vs. laissez-faire policy if FOEs are immobile and if there are mobile. Because the spatial distribution of FOEs is always symmetric in our model, the only welfare component that depends on the mobility of FOEs is the impact of the bailout on corporate tax revenues arising from FOEs[26]. Thus, the mobility of foreign firms makes governments more prone to adopt a bailout policy because tax competition reduces the opportunity cost of the bailout policy in terms of public good provision.

5 Discussion

Although the model we develop is simple, the mechanisms behind the main results are at work under various alternative assumptions.

One might wonder if our results remain valid if in the sequence of events, the bailout decision is the second (rather than the first) decision to be taken by governments. Governments would first set their tax policy (stage 1), then foreign investors would make their location choice (stage 2), and governments would decide wether or not to rescue ailing firms (stage 3) given the decisions at these previous stages. Under this alternative

[26] At the subgame perfect Nash equilibrium, neither the gains from a bailout in terms of consumer's surplus, nor the direct cost of the bailout are affected by firm mobility.

sequence of events, at the 2nd or 3rd stages of the game (depending on wether we consider mobile or immobile FOEs) governments will opt for a laissez faire policy if the fixed cost of DOEs is higher than a threshold $\dot{\Theta}$ and this threshold would depend on taxes through their influence on the consumer surplus and public provision. At the 1st stage of the game, government anticipate this result when they define their tax policy, but the tax outcome would result from the same trade-off as those described in our benchmark as location choices are always made after tax policy decisions. Therefore, once evaluated at the subgame perfect equilibrium, the bailout decision would be identical to the one described in our benchmark.

The level of employment and labor income might be another important reason why governments are enclined to rescue some firms. In our baseline framework, even though DOEs firms are more labor intensive, this has no impact on the bailout decision because in the end, all workers are employed and earn the same wage. The reason is that even if these firms go bankrupt, their employees will find a new job in the numéraire sector. While introducing unemployment in the model would complicate the analysis too much, one approach to modelling this bailout motive could be to assume that workers employed in the modern industry (therefore, in DOEs) receive a wage premium over what can be earned in the numeraire sector (wage in sector M equal to α, with $\alpha > 1$). Said differently, this industry offers better jobs. In that case, labor market clearing is such that workers first seek for a job in sector M and, if they do not find, they look for a job in sector T.[27] Therefore, the national welfare under a bailout policy would include a component with a wage premium multiplied by the number of individuals employed in the M sector. Specifically, the welfare differential would write $W^R - W^F = CS^R - CS^F + \frac{L}{2}\left(G^R - G^F\right) + L^M\alpha$, with L^M the number of individuals employed in the M sector.[28] Thus, this concern about labor income will provide an additional incentive to bailout ailing firms in order to raise the level of agregate income. However, this does call into question the other mechanisms behind the bailout decision and that operate through the impact of the government's decision on the consumers surplus and public good provision. Indeed, there are no income effects in our model, so the demand function is still given

[27] We assume, that total labor supply is always higher than total labor demand coming from the M industry so that there are always some workers employed in the T sector. Thus, the number of jobs in the T sector (L^T) is equal to the difference between national labor supply and national labor demand coming from DOEs.

[28] L^M is equal to total employment in DOEs. Each firm employs Φ workers as a fixed cost and c production workers by unit of the good procuded. Therefore, $L^M = \frac{n}{2}\left(c\left(x_{ii}^{DOE,R} + x_{ij}^{DOE,R}\right) + \Phi\right)$. We check that for admissible values of parameters, $L^M < \frac{L}{2}$ so that the traditionnal sector remains active.

by (4). While the wage premium exerts an additional upward pressure on prices under a bailout decision, we verify that the downward pressure on prices due to the survival of firms still make the price level lower under a bailout decision than under a laissez-faire policy. Therefore, our main result regarding the incentive to bailout in order to raise the consumer surplus remains valid, as well as results regarding the tax equilibrium.[29] The main difference under this alternative modelling strategy concerns the impact of trade integration on the incentives to bailout. At the equilibrium, trade liberalization raises total output in the M sector, and therefore improves total labor income under a bailout policy by increasing the level of employment in the high-wage sector. Interestingly, this effect completes the story of the baseline model by showing that trade liberalization may exert an additional force that makes governments more enclined to bailout corporate firms in order to raise the national labor income.

6 Conclusion

This paper aims at analyzing the effects of both trade integration and capital mobility on the propensity for governments to bailout inefficient domestic firms. Emphasis has been laid on how bailouts affect price competition and markups in oligopolistic markets. On the one hand, bailouts entail costs in terms of public good provision but on the other hand, they may allow to preserve competition, which benefits domestic consumers. The latter channel has been ignored by the existing theoretical literature on soft budget constraint, which often explains such a phenomenon by electoral reasons and the "too big to fail" argument. In addition, we show that both the benefits and the costs of bailouts ultimately depend on the extent to which international market are integrated and firms are mobile. As explained in the paper, our point does not run encounter to standard explanations of bailouts but, rather, is a complementary explanation. Interestingly, empirical evidence in the US strongly suggests that recent bailouts in the car industry may have had huge effects on markups and outputs compared to a situation where GM and Chrysler would have gone bankrupt (Wollman, 2018). Quite surprisingly, our main result is that mobility of firms and international tax competition make governments more prone to bail out firms. In a sense, our paper contributes also to the literature dealing with the effects of mobility on disciplining governments (Cai and Treisman, 2005). Our results suggest that

[29] Even when FOEs are mobile, the tax outcome is unchanged. Indeed, total production in DOEs is independent on the location of FOEs because of the symmetry of the model. Therefore, at stage 1, the presence of a wage premium in the objective function does not affect tax choices.

globalization may relax discipline, nevertheless it could be welfare improving thanks to competition effects.

A Appendix

A.1 Operating profits

The equilibrium operating profits in each country write as follows:

$$\Pi^{FOE,F} = \frac{1}{4}L\frac{\Lambda}{(m+1)^2},\ \Pi^{FOE,R} = \frac{1}{4}L\frac{\Gamma}{(m+n+1)^2} \text{ and } \Pi^{DOE,R} = \frac{1}{4}L\frac{\Omega}{(m+n+1)^2}$$

where

$\Lambda = \tau^2(2m+m^2+2)+4a(a-\tau)$

$\Gamma = (m^2+2mn+2m+n^2+2n+2)\tau^2+4(a+cn)(a-\tau+cn)$

$\Omega = (m^2+2mn+2m+n^2+2n+2)\tau^2+4(a-c-cm)(a-c-cm-\tau)$

Λ, Γ and Ω are all positive under Assumption 1.

A.2 Bailout decision when FOEs are immobile

Assume that FOEs are immobile. Then, $\hat{W}^R \gtreqless \hat{W}^F$ if and only $\Theta \lesseqgtr \hat{\Theta}$, where:

$$\hat{\Theta} = \frac{\hat{\Psi}\tau^2+\hat{\Upsilon}\tau-\hat{\Sigma}}{(m+1)^2(m+n+1)^2}$$

and

$$\hat{\Psi} = \frac{1}{4}\begin{pmatrix} (m+1)(L(m+1)n^2+2L+3Lm^2+Lm^3+2Lm+2m) \\ +(L(m+2)(2m+2m^2+1)+2m+1)n \end{pmatrix}$$

$$\hat{\Upsilon} = ((m^2+nm-1)L-(2m+n+2mn+2m^2))a+c(m+1)^2(L+m+n)$$

$$\hat{\Sigma} = (a-a_{\min})\begin{pmatrix} (L(mn+m^2-1)-2m(m+n+1)-n)a \\ +c(m+1)(L+n+Lm(m+n+2)) \end{pmatrix}$$

We verify that $\hat{\Psi}$, $\hat{\Upsilon}$ et $\hat{\Sigma}$ are positive under Assumption 1 and for admissible values of parameters (that is, $L>4$, and $m>2$).

Therefore, $\hat{\Theta}$ is increasing in τ and its maximum value is given by:

$$\hat{\Theta}\Big|_{\tau=\tau^R} = (a-a_{\min})\frac{\left[a(2m-mn+2)-c(m+1)\left(6m+mn+2m^2+2\right)\right]L+(a(2m+1)-c(m+1))n+2m(m+1)(a+c)}{(m+1)^2(m+n+2)^2}$$

with $a-a_{\min}>0$ and $(a(2m+1)-c(m+1))n+2m(m+1)(a+c)>0$ under Assumption 1.

We further assume that $m > 2/(n-2)$, which means that the number of foreign firms is not too low. Then, the numerator is decreasing with population size, and becomes negative when:

$$L > \frac{a\left(2m + n\left(2m+1\right) + 2m^2\right) + c\left(m+1\right)\left(2m-n\right)}{a\left(-2m + mn - 2\right) + c\left(m+1\right)\left(6m + mn + 2m^2 + 2\right)} = \hat{L}$$

Therefore,

$$\hat{\Theta}\Big|_{\tau=\tau^R} \gtrless 0 \text{ when } L \lessgtr \hat{L}$$

A.3 Equilibrium taxes when FOEs are immobile

When FOEs are mobile, we have to ensure that Nash taxes correspond to an interior equilibrium so that profits made by FOEs are not entirely taxed.

Calculating the difference between Nash taxes and operating profits, we obtain:

$$\tilde{t}^R - \Pi^{FOE,R} = \frac{\left(2L(m-1)+3Lm^2+Ln^2+4Lmn+2m+2n\right)\tau^2+4(a(L-m-n)+cn+Lcn)\tau-4L(a+cn)^2}{4(m+n+1)^2}$$

$$\tilde{t}^F - \Pi^{FOE,F} = \frac{1}{4}\frac{\left(2m+L\left(3m^2+2m-2\right)\right)\tau^2+4a(L-m)\tau-4La^2}{(m+1)^2}$$

The numerator in the above expressions is a quadratic function with respect to trade costs.

Considering that the number of individuals in the economy is always higher than the number of surviving firms, we verify that $4\left(a\left(L-m-n\right)+cn+Lcn\right)$ and $4a\left(L-m\right)$ are positive. Therefore, $\tilde{t}^R - \Pi^{FOE,R}$ and $\tilde{t}^F - \Pi^{FOE,F}$ are increasing with trade costs.

Moreover, we check that these expressions are negative when $\tau = 0$ and positive at the maximum value of trade cost τ^R. Therefore, $\tilde{t}^R - \Pi^{FOE,R}$ and $\tilde{t}^F - \Pi^{FOE,F}$ are negative if and only if trade cost values are lower than the second root of the numerator, wich are respectively given by:

$$\tilde{\tau}^F = 2a\frac{\sqrt{(L^2(m+1)(3m-1)+m^2)}-(L-m)}{2m+L(3m^2+2m-2)}$$

and

$$\tilde{\tau}^R = 2\frac{\sqrt{(m+n+1)L\left((3m+n-1)(a+cn)^2L+2cn(a+cn)\right)+(am+an-cn)^2}-a(L-m-n)-cn(L+1)}{2L(m-1)+L(3m+n)(m+n)+2m+2n}$$

A.4 Equilibrium taxes when FOEs are mobile

When FOEs are mobile, the differences in corporate tax that would result from the decision not to rescue ailing firms is given by:

$$\tilde{t}^F - \tilde{t}^R = \frac{1}{2} n\tau \frac{\left(L(m-1)(m+1)(m+n+1)+mn+m^2-1\right)\tau - 2((m+1)(-a+am+cm+c)+amn)}{(m+1)^2(m+n+1)^2}$$

This expression is increasing with L. Therefore,

$$\tilde{t}^F > \tilde{t}^R \text{ when } L > \frac{2(m+1)(c-a+am+cm)+2amn-\tau\left(m^2+nm-1\right)}{\tau(m-1)(m+1)(m+n+1)} = \tilde{L}$$

and we check that $\tilde{L} > \tilde{L}^F$.

Therefore, under Assumption 2,

$$\tilde{t}^F \gtrless \tilde{t}^R \text{ when } L \gtrless \tilde{L}$$

A.5 Bailout decision when FOEs are mobile

Assume that FOEs are mobile. Then, $\tilde{W}^R \gtreqless \tilde{W}^F$ if and only $\Theta \lesseqgtr \tilde{\Theta}$, where:

$$\tilde{\Theta} = \frac{\tilde{\Psi}\tau^2 + \tilde{\Upsilon}\tau + \tilde{\Sigma}}{(m+1)^2 (m+n+1)^2}$$

with

$\tilde{\Psi} = (m+1)\left((m+1)\left(4m+2-m^2+n^2\right)L - 2m(m-2)\right) + n\left((6L-2)m^2 + (8L+2)m + 2L+1\right)$

$\tilde{\Upsilon} = (m+1)^2 (L+2m+n+Lm)\, c - a\left(L(m+1)^2 + 3m + n - m^2 n + 2mn + 2m^2 - m^3\right)$

$\tilde{\Sigma} = \left(L+2m+n+Lm^2+2Lm+2mn+2m^2\right)a - c(m+1)\left(L+n+Lm^2+2Lm\right)$

The sign of $\tilde{\Psi}$ and $\tilde{\Upsilon}$ cannot be derived analytically without making additional assumptions regarding the number of FOEs and DOEs.

However, we verify that that $\tilde{\Sigma} > 0$ under Assumption 1, so that $\tilde{\Theta}\Big|_{\tau=0} > 0$.

Evaluating the threshold value $\tilde{\Theta}$ at the maximum trade cost value, we obtain:

$$\tilde{\Theta}\Big|_{\tau=\tau^R} = \frac{a - a_{\min}}{(m+1)^2 (m+n+1)(m+n+2)^2}\kappa$$

$$\text{with } \kappa = \begin{pmatrix} a\left[2(m+1)(3m+n+mn+1)L + \left(4m^3+6m^2n+4m^2+2mn^2+5mn+n^2+n\right)\right] \\ -c(m+1)\left[\left(6(L-1)+2Ln\right)m^2 + \left(4(2L-1)+n(4L-1)\right)m + (n+1)(2L+n)\right] \end{pmatrix}$$

We check that $\tilde{\Theta}\Big|_{\tau=\tau^R} > 0$ under Assumption 1.

The sign of $\tilde{\Theta}$ over the whole interval of admissible trade cost values $\left[0;\tau^R\right]$ depends on the value of $\tilde{\Psi}$ and $\tilde{\Upsilon}$.

i) Assume that $\tilde{\Psi} < 0$ so that $\tilde{\Theta}$ is a hump-shaped function of trade cost.

Recall that $\tilde{\Theta}\Big|_{\tau=0}$ and $\tilde{\Theta}\Big|_{\tau=\tau^R}$ are positive.

Therefore, $\tilde{\Theta}$ is positive for all admissible trade values, whatever the sign of $\tilde{\Upsilon}$.

ii) Assume that $\tilde{\Psi} > 0$. We have to consider two cases to characterise the sign of $\tilde{\Theta}$ over the interval $\left[0; \tau^R\right]$.

If $\tilde{\Upsilon} > 0$, then $\tilde{\Theta}$ is increasing with trade costs and is positive for all admissible values of trade costs.

If $\tilde{\Upsilon} < 0$, then $\tilde{\Theta}$ reaches its minimum value for a positive value of trade cost $-\tilde{\Upsilon}/2\tilde{\Psi}$, that might lie within the interval $\left[0; \tau^R\right]$. Unfortunately, we cannot identify the sign of $\tilde{\Theta}\Big|_{\tau=-\tilde{\Upsilon}/2\tilde{\Psi}}$ analytically. Therefore, in that specific configuration ($\tilde{\Psi} > 0$ and $\tilde{\Upsilon} < 0$)[30], $\tilde{\Theta}$ can take negative values for intermediate levels of trade cost (between the two roots of $\tilde{\Psi}\tau^2 + \tilde{\Upsilon}\tau + \tilde{\Sigma}$), whereas $\tilde{\Theta}$ is positive for either low or high trade cost levels.

To summarize, $\tilde{\Theta}$ is positive for all admissible values of trade costs when i) $\tilde{\Psi} < 0$, or ii) $\tilde{\Psi} > 0$ and $\tilde{\Upsilon} > 0$. Otherwise (that is, if and only if $\tilde{\Psi} > 0$ and $\tilde{\Upsilon} < 0$), we verify that $\tilde{\Theta}$ is positive for either low or high trade cost level, whereas it might be negative for intermediate trade cost values.

References

Alexeev M. and Y. Jang (2010). Trade liberalization, heterogeneous firms and the soft budget constraint, *Journal of Comparative Economics* 38(4), 449-460.

Azémar, C., Desbordes R. and I. Wooton (2015). Country Size and Corporate Tax Rate : Rational and Empirics. CEPR Discussion Paper No. 10800

Bucovetsky, S. (1991). Asymetric Tax Competition, *Journal of Urban Economics*, 1991, 30, 167–181.

Baldwin, R., et Krugman, P. (2004). Agglomeration, Integration and Tax Harmonisation. *European Economic Review*, 48(1), 1–23.

Cai H. and Treisman D. (2005). Does Competition for Capital Discipline Governments? Decentralization, Globalization, and Public Policy. *The American Economic Review* 95(3), 817-830.

Dewenter K.L. and Malatesta P.H. (2001). State-Owned and Privately Owned Firms: An Empirical Analysis of Profitability, Leverage and Labor Intensity. *The American Economic Review* 91(1), pp. 320-334

Exbrayat, N. and B. Geys (2014). Trade integration and business tax differentials: Evidence from OECD countries. *International Tax and Public Finance* 21(2), 298-323.

[30] We verify that those inequalities are compatible.

Faccio M., Masulis R.W. and J. McConnell (2006). Political Connections and Corporate Bailouts, *The Journal of Finance* 61, 2597-2635.

Gaigné C. and I. Wooton (2011). The gains from prefebenefitial tax regimes reconsidered, *Regional Science and Urban Economics* 41, 59-66.

Goldeng, E., Grünfeld, L. and G. Benito (2008). The performance differential between Private and SOE: The role of ownership, management and market structure, *Journal of management Studies* 45(7), 1244-1273.

Goolsbee A.D. and Krueger A.B. (2015). A Retrospective Look at Rescuing and Restructuring General Motors and Chrysler. *Journal of Economic Perspectives* 29(2), 3–24

Gupta, N. (2005). Partial Privatization and Firm Performance. *The Journal of Finance* 60(2), pp. 987-1015.

Han, Y, Pieretti, P, Zanaj and B. Zou (2014). Asymmetric competition among nation states: A differential game approach. *Journal of Public Economics* 119, 71–79.

Haufler A. and I. Wooton (2010). Competition for firms in an oligopolistic industry: The impact of economic integration, *Journal of International Economics* 80, 239-248.

Helpman, E., Melitz M.J. and S.R. Yeaple. (2004). Export Versus FDI with Heterogeneous Firms. *The American Economic Review*, 94 (1), 300-316

Kornai J., Maskin E. and G. Roland (2003). Understanding the Soft Budget Constraint, *Journal of Economic Literature* 41(4), 1095-1136.

Kowalski P., Büge M., Sztajerowska M. and M. Egeland (2013). State-Owned Enterprises: Trade Effects and Policy Implications, OECD Trade Policy Papers, No. 147, OECD Publishing, Paris.

Kowalski, P. and K. Perepechay (2015). International Trade and Investment by State Enterprises, OECD Trade Policy Papers, No. 184, OECD Publishing, Paris.

Melitz M. (2003). The Impact of Trade on Intra-Industry Reallocations and Aggregate Industry Productivity, *Econometrica* 71(6), 1695-1725.

Ottaviano G.I.P. and T. Van Ypersele (2005). Market size and tax competition. *Journal of International Economics* 67 (1), 25-46.

Qian Y. and G. Roland (1998). Federalism and the Soft Budget Constraint. *The American Economic Review* 88(5), 1143-62.

Robinson J. and R. Torvik (2009). A political economy theory of the soft budget constraint. *European Economic Review* 53(7), 786-798.

Wilson, J. (1991). Tax Competition with Interregional Differences in Factor Endowments, *Regional Science and Urban Economics*, 1991, 21, 423–451.

Wilson, (1999). Theories of Tax Competition. *National Tax Journal*, 52(2), 269–304.

Wollman, T. (2018). Trucks without bailouts: equilibrium product characteristics for commercial vehicles. *The American Economic Review* 108(6), 1364-1406.