Contemporary Educational Psychology 65 (2021) 101967

Contents lists available at ScienceDirect

Contemporary Educational Psychology

journal homepage: www.elsevier.com/locate/cedpsych

Do emotions and prior performance facilitate the use of the learner-generated drawing strategy? Effects of enjoyment, anxiety, and intramathematical performance on the use of the drawing strategy and modelling performance☆

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ARTICLE INFO

Keywords:
Learning strategies
Word problem solving
Diagram
Anxiety
Enjoyment
Drawing strategy
Mathematics

ABSTRACT

The use of self-generated drawings has been found to be a powerful strategy for problem solving. However, many students do not engage in drawing activities. In this study, we investigated the effects of the enjoyment of the drawing strategy, anxiety about the drawing strategy, and prior intramathematical performance on the use of the drawing strategy and modelling performance. We explored the role of the drawing strategy as a mediator between emotions and modelling and whether intramathematical performance moderated the effects of emotions ($N = 220$, mean age 14.5 years). Enjoyment and anxiety with respect to generating drawings and intramathematical performance predicted the use of the drawing strategy. Enjoyment positively affected modelling performance indirectly via the use of the drawing strategy. Anxiety negatively affected modelling performance via the use of the drawing strategy for students with lower intramathematical performance. Our findings demonstrate that experiencing activating emotions (i.e., enjoyment and anxiety) with respect to strategies and prior intramathematical performance are important for strategy use and modelling performance. Implications for the theory of self-generated drawing and the control-value theory of achievement emotions and practical implications for training and supporting the drawing strategy are discussed.

1. Introduction

Although theoretical models (Fiorella & Mayer, 2015; Van Meter & Garner, 2005) and empirical findings have underlined the importance of using learner-generated drawings for problem solving (Hembree, 1992), students often do not create drawings when asked to solve mathematical problems. Indeed, the construction of visual external representations was already recommended by Pólya (1945) in his seminal work on problem solving. There is evidence of positive effects of teaching methods that focus on teaching students how to make a drawing (Csíkos, Szitányi, & Kelemen, 2012) or that include this strategy along with others to promote students' performance while they solve problems with a connection to reality (Verschaffel et al., 1999). Why do some students engage in drawing activities whereas others do not? Although many studies have investigated how cognitive factors affect strategy use, there is a need for research on how prior performance and non-cognitive factors such as emotions affect drawing strategy use and performance (Fiorella & Zhang, 2018; Van Meter, Aleksic, Schwartz, & Garner, 2006; Wu & Rau, 2019). We chose the activating emotions of enjoyment and anxiety because they have been assumed to affect students' use of learning strategies and have repeatedly been shown to be related to students' mathematical performance and to affect students' career choices and well-being (Brown, Brown, & Bibby, 2008; Ma, 1999; Pekrun, 2006; Putwain, Becker, Symes, & Pekrun, 2018; Ramirez, Shaw, & Maloney, 2018; Schukajlow & Rakoczy, 2016). Enjoyment of and anxiety about using the drawing strategy can affect the use of the drawing strategy, and these emotions can affect performance through the use of the drawing strategy as an intervening variable.

☆ The present study was funded by the German Research Foundation [Deutsche Forschungsgemeinschaft, grant number SSCHU 2629/3-1]; and the Swiss National Science Foundation [Schweitzer Nationalfond, grant number 100019E-164816/1].

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https://doi.org/10.1016/j.cedpsych.2021.101967

Available online 14 March 2021

This study was conducted in the field of modelling, which comprises the ability to solve problems that have a strong connection to reality (modelling performance). This ability can be clearly separated from the ability to solve problems without a connection to reality (intramathematical performance) (Niss, Blum, & Galbraith, 2007; Schukajlow, Krug, & Rakoczy, 2015). Solving modelling problems requires the problem solver to activate extra-mathematical knowledge, deal with vague conditions, make assumptions about real-world contexts, and sometimes go back and forth between the real world and mathematics in the process of constructing and refining mathematical models.

Our study is embedded in the "Visualization while solving modelling problems" project. The main goal of this project is to clarify the role of cognitive, strategic, motivational, and emotional factors in the construction and quality of visual representations and performance in solving real-world problems in mathematics. Results from two prior studies based on two different data sets demonstrated the importance of student knowledge of drawings for improving the quality of student-generated drawings and student performance (Rellensmann, Schukajlow, & Leopold, 2017, 2020). The main goal of this work is based on a new data set that includes ninth and tenth graders. This work investigates how enjoyment from using the drawing strategy, anxiety about using the drawing strategy, and intramathematical performance affect the use of the drawing strategy and whether the effects of the activating emotions of enjoyment and anxiety on the use of the drawing strategy differ in students with low and high intramathematical performance. The second goal is to determine whether the use of the drawing strategy increases modelling performance and whether the use of the drawing strategy serves as an intervening variable for the indirect effects of enjoyment, anxiety, and intramathematical performance on modelling performance. We also aim to determine whether the indirect effects of the activating emotions that may accompany the drawing strategy on modelling are different in students with low and high intramathematical performance. The third goal is to examine the overall (total) effects of enjoyment and anxiety about using the drawing strategy on modelling performance and how important prior intramathematical performance is for solving modelling problems.

2. Prior research, theoretical model, and hypotheses

2.1. Learner-generated drawing

Learner-generated drawing describes the process and the product of generating an illustration that corresponds to the objects and relations described in a task (Arcavi, 2003; Rellensmann, Schukajlow, & Leopold, 2017). In a meta-analysis of correlational and interventional studies in mathematics, Hembree (1992) found that making a drawing was more effective than other strategies such as using equations, guessing and testing, or checking one's work. Fiorella and Mayer (2016) analyzed different studies on the effectiveness of learner-generated drawings and found that 26 out of 28 studies found positive effects on learning performance for learner-generated drawings compared with a no-drawing condition across several domains. The effectiveness of learner-generated drawings was explained by the idea that generating drawings helps learners select relevant pieces of information, organize them into a coherent representation, and integrate prior knowledge from long-term memory (Fiorella & Mayer, 2016; Van Meter & Garner, 2005). Theories of learning focus on three reasons for the positive effects of constructing drawings on performance (Arcavi, 2003; Cox, 1999; Rellensmann et al., 2017; Van Meter & Firetto, 2013; Van Meter & Garner, 2005):

(1) Generating a drawing helps to organize and structure the information given in the problem,
(2) Students have more capacity in their working memory, and they can more easily make solution-related inferences when they generate a drawing beforehand, and
(3) The implicit information can be made visible in a drawing, and this information can be included as a further consideration while solving a problem.

Generating a drawing is a kind of learning strategy. Learning strategies comprise cognitive and behavioral activities during the learning process (Fiorella & Zhang, 2018). Researchers have applied the levels of processing approach to distinguish between surface-level and deep-level processing strategies (Marton & Säljö, 2005). Surface-level processing strategies refer to the text as it is presented to the learner. Learners try to memorize facts and rehearse the information they have read. Deep-level processing strategies refer to the meaning of the text. Learners try to understand the meaning by thinking about relations within the text, by organizing the information in a new way, and by connecting it with their prior knowledge (elaboration). Generating a drawing, imagining, and summarizing are examples of deep-level processing strategies because they go beyond the text and draw the learners' attention toward representing the situation described in the text (Leopold & Leutner, 2012).

Solving modelling problems mainly includes the ability to perform the demanding process of mathematizing, which allows mathematics to be apples to the real world (Niss et al., 2007). Students are required to deeply process the text that describes the modelling problem in order to recognize the underlying mathematical structure of the problem. There are different models that describe students' activities while solving modelling problems (Blum & Leiss, 2007; Galbraith & Stillman, 2006; Verschaffel, Greer, & De Corte, 2000). Let us consider the Kite modelling problem (Fig. 1) as an example that illustrates the benefits of drawing for solving geometrical modelling problems. To solve the Kite modelling problem, students construct a model of a situation that includes two individuals, a piece of cord, a kite, and the positions of the people and the kite. The height of the kite is unknown, and it can be calculated, for example, by using the Pythagorean theorem and adding an estimate of the person's (Lucas') height. By simplifying, structuring, and idealizing the problem, students make assumptions such as that the piece of cord is straight and has a length of 100 m and that Lucas holds the cord about 1 m above the ground. Then the simplified model of the situation can be translated into a mathematical model that consists of a right-angled triangle and a line segment that has a distance of 1 m from one angle of the rectangle to the ground (see Fig. 1). After calculating one leg of the triangle by applying, for example, the Pythagorean theorem and adding the distance of 1 m, students can then calculate a mathematical result of 61 m. This result should then be interpreted and validated. The answer might be that the altitude above the ground is about 60 m, which seems to be a realistic result that takes into account the length of the cord and the distance between the two people.

While solving the Kite modelling problem, students can organize and structure the distances by generating a drawing. They can infer from the drawing that a right-angled triangle is an important part of the mathematical model. In the drawing, they can identify the importance of including the distance that is implicitly given in the problem between the ground and one angle in the triangle for solving the problem. The many studies on drawing have emphasized the idea that students who generate a drawing show better performance in mathematics than students who do not generate a drawing (Hembree, 1992; Uesaka & Manalo, 2017). Fewer studies have investigated which factors determine whether students use drawings to solve problems. Do intramathematical performance and the emotions anxiety and enjoyment influence a student's decision to create a drawing while solving a mathematical modelling problem, and how important is intramathematical performance for modelling?

2.2. The role of intramathematical performance

Students' intramathematical performance is one important prerequisite for solving modelling problems. In a case study of 18 students from lower secondary school, the activation of appropriate prior knowledge

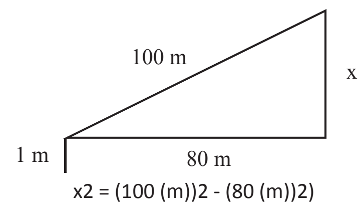
The Kite	Mathematical model
Lucas got a red kite for his birthday. It is 1 m long and 50 cm wide. He flies his kite with his friend Susanne. The distance between them is 80 m. The kite’s cord is 100 m long. Susanne is standing exactly under the kite. At what altitude is the kite flying at this moment?	

Fig. 1. A Sample Modelling Problem Called the Kite (Böckmann & Schukajlow, 2018) and an Illustration of the Mathematical Model.

was found to be important from the very beginning of the modelling process because appropriate knowledge can help people understand the modelling problem, simplify and organize the situation model, and construct a mathematical model (Krawitz & Schukajlow, 2018). Further, after constructing a mathematical model, students apply mathematical procedures to get a mathematical result. In this stage of the modelling process, they need to activate intramathematical knowledge (measured as performance) to calculate a mathematical result. At the end of the modelling process, students interpret what their mathematical result means in the real world and validate it. Intramathematical performance might also be helpful in this stage of the modelling processes. A positive relation between intramathematical performance and modelling performance was found in another study (Schukajlow et al., 2015), and thus, we expected the same in the present study.

Intramathematical performance has been suggested to be a key factor in the use of strategies (Winne & Hadwin, 1998), and researchers have repeatedly called for investigations of its role in future studies (Wu & Rau, 2019). Students with low prior performance can be expected to use less sophisticated strategies in their problem-solving-related activities because they have a smaller number of appropriate domain-specific procedures and schemata they can rely on and will therefore have less working memory capacity available to select appropriate strategies (Taub, Azevedo, Bouchet, & Khosravifar, 2014). A case study on solving modelling problems revealed that students with low prior performance often did not use a drawing strategy to solve problems because they did not recognize that making a drawing could help them solve a modelling problem (Schukajlow, 2011). Further, low-achieving students were found to have deficits in applying deep-level processing strategies in prior studies (Kramarski, Weisse, & Kololshi-Minsker, 2010). Some researchers have suggested that an explanation for this result might be that low-achieving students have been taught to focus to a great extent on the memorization of mathematical procedures (Kramarski et al., 2010). An analysis of boundary conditions for learning by drawing revealed that students must have sufficient prior knowledge in the domain in order to produce an accurate drawing (Fiorella & Zhang, 2018). In prior research, students’ grades in mathematics were found to be positively related to their self-reported use of deep-level processing strategies such as the drawing strategy (Ahmed, van der Werf, Kuyper, & Minnaert, 2013). On the basis of these theoretical considerations, we expected a positive relation between prior intramathematical performance and drawing strategy use. Because intramathematical performance predicts the use of the drawing strategy, and the use of drawings is positively related to modelling performance, we expected indirect effects of prior intramathematical performance on students’ modelling performance via the use of the drawing strategy as an intervening variable.

2.3. The roles of enjoyment and anxiety as achievement emotions

Emotions are defined as complex phenomena that include affective, cognitive, physiological, motivational, and expressive parts (Pekrun & Linnenbrink-Garcia, 2014). According to the control-value theory of achievement emotions, learning-related emotions emerge from control and value appraisals (Pekrun, 2006). If students ascribe a positive value to a learning activity or outcome and perceive a high level of control over this activity (they expect to perform well), they will enjoy engaging in this activity. If students perceive low levels of control (they expect to fail) and ascribe high value to a learning activity or outcome, they are likely to feel anxious. Two important underlying dimensions of emotions are valence and activation (Pekrun, 2006). Enjoyment has a positive valence, whereas anxiety has a negative valence. Whereas deactivating emotions impede learning activities, activating emotions are assumed to activate students and to predict their engagement with the learning materials. Enjoyment and anxiety are activating emotions that influence learning-related activities and performance. Pekrun (2006) suggested that enjoyment can improve engagement by facilitating the use of “flexible, creative learning strategies,” whereas anxiety can improve engagement by increasing the use of more rigid strategies such as rehearsal. Thus, enjoyment is believed to have an overall positive effect on performance. The role of mathematical anxiety for performance, however, has yet to be clarified and seems to depend on other factors such as level of mathematical anxiety (high vs. low), students’ prior experiences with mathematics, students’ ability to regulate emotions, and teachers’ responsiveness to emotional needs (Aldrup, Klusmann, & Lüdtke, 2020; John, Nelson, Klenczar, & Robnett, 2020; Ramirez et al., 2018; Wang, Oh, Malanchini, & Borriello, 2020). But most studies have found a negative relationship between anxiety and performance.

Emotions can be related to different objects (Schukajlow, Rakoczy, & Pekrun, 2017). The objects of emotions vary from general life experiences to a specific type of problem. An area of mathematics (e.g., geometry), a topic (e.g., intercept theorem), or a learning activity (e.g., making a drawing to solve a problem) are examples of objects in mathematics. Some initial empirical evidence has confirmed the importance of distinguishing between the objects of emotions. The levels of enjoyment related to life, school, learning, and strategies were found to be distinct factors with low to moderate relations with each other (Goetz, Hall, Frenzel, & Pekrun, 2006). Further, the enjoyment of different objects can have different levels of power in predicting learning- and achievement-related outcomes. When the relevant object was described more specifically, the impact of a particular emotion could be better distinguished. The enjoyment of solving modelling problems was demonstrated to be more strongly related to modelling performance than the enjoyment of mathematics (Schukajlow & Krug, 2014). The importance of differentiating between objects of emotions has also been identified in research on anxiety in which researchers have distinguished between different objects of anxiety such as anxiety about life or test anxiety (Carey, Devine, Hill, & Szűcs, 2017; Hembree, 1990; Kazelskis et al., 2000). In this study, we chose the activity of using drawings to solve problems as the object of students' emotions. By addressing students' emotions about using drawings to solve problems, we expected to learn more about the factors that predict the use of the drawing strategy to solve modelling problems.

Do emotions influence the use of the drawing strategy to solve problems? According to the control-value theory of achievement, enjoyment is positively related to intrinsic motivation and interest (Pekrun, 2006; Schukajlow & Rakoczy, 2016). An intrinsic motivation is closely related to the concept of interest, as an intrinsic motivation is based on the ability to realize individual interests (Krapp, 2005). As motivation-related factors enhance strategy use (Uesaka & Manalo, 2017), students' enjoyment of strategies can be expected to be positively related to their use of these strategies for learning and problem solving. Thus, we hypothesized that students who enjoy generating drawings will be more interested in using this strategy to solve a modelling problem than students who do not enjoy generating drawings or enjoy it less. The reason for this assumption is that interested students are willing to engage with the object of their interest and apply sophisticated and deep-level strategies during learning (Grigg, Perera, McIlveen, & Svetleff, 2018; Hidi & Renninger, 2006). Empirical evidence for the positive relation between the enjoyment and the use of learner-generated drawings comes from studies on emotions and self-reported learning strategies. As enjoyment was found to be related to deep-level cognitive processing such as the elaboration and organization of learning content (Ahmed et al., 2013; Obergriesser & Stoeger, 2020; Pekrun, Goetz, Titz, & Perry, 2002a), and the drawing strategy can be classified as a deep-level processing strategy (Van Meter & Garner, 2005; Weinstein & Mayer, 1986), we expected positive effects of the enjoyment of the drawing strategy on the use of the drawing strategy to solve geometrical modelling problems.

Anxiety about using strategies is another emotion that is likely to influence strategy use such as by preventing students from applying the drawing strategy while solving a problem. If a student fears failure from generating a drawing, he or she is less likely to use this strategy and more likely to solve the problem by using another less effective strategy or might even give up on solving the problem from the very beginning. This expected negative relation between anxiety and students' strategy use has rarely been investigated. Initial evidence for the negative impact of anxiety on generating drawings has come from research on anxiety. Researchers see the use of inappropriate strategies to solve problems as one outcome of mathematical anxiety (Ramirez, Chang, Maloney, Levine, & Beilock, 2016) and have emphasized the close relation between anxiety and strategy use (Johnson, Clohessy, & Chakravarthy, 2020). Students' negative emotions, including anxiety, are expected to impede their use of deep-level processing strategies (Pekrun, 2006) such as the drawing strategy. Thus, anxiety about making self-generated drawings is likely to negatively affect the use of the drawing strategy.

In sum, in the context of solving modelling problems, students who enjoy the activity of drawing will more often generate drawings, construct an appropriate mathematical model, and solve the modelling problem. By contrast, students who are anxious about drawing will less often generate drawings but will instead try to combine the numbers from the tasks by applying arithmetic operations. We therefore hypothesized that we would find a positive indirect effect of the enjoyment of generating drawings to solve problems on modelling performance, whereas we expected to find a negative indirect effect of anxiety about generating drawings to solve problems on modelling performance with the use of the drawing strategy as an intervening variable.

Moreover, we expected overall effects of emotions about the drawing strategy on modelling performance. Prior research has demonstrated effects of the enjoyment of mathematics and anxiety about mathematics on students' performance (Ahmed et al., 2013; Ramirez et al., 2016, 2018; Schukajlow & Rakoczy, 2016). Because students' enjoyment of mathematics and their use of deep-level processing strategies are positively related (Goetz et al., 2006), these findings can be used as an indication of the effect of the enjoyment of using drawings to solve problems on modelling performance. Similar arguments can be used to ground the effects of anxiety about generating drawings to solve problems on modelling performance. However, the relation between anxiety and performance is less clear. On the one hand, anxiety might reduce cognitive resources and hinder problem solving, but on the other hand, it might inspire students to put more effort into problem solving (Pekrun, Lichtenfeld, Marsh, Murayama, & Goetz, 2017) and induce extrinsic motivation to avoid failure (Muis et al., 2015).

2.4. *Intramathematical performance as a moderator of the relations between emotions, use of the drawing strategy, and modelling*

The magnitudes of the effects of enjoyment and anxiety with respect to certain strategies on the use of these strategies can depend on students' prior mathematical performance. Students who have delivered strong prior mathematical performances can more easily identify the need to generate a drawing as an appropriate strategy, embrace their enjoyment of the drawing strategy in order to use it, and are thus likely to have better performances in the future. Further, students with strong mathematical performances are better able to focus on tasks and compensate for the negative effects of anxiety about using the drawing strategy on the use of the drawing strategy and their future mathematical performances (Gumora & Arsenio, 2002). Consequently, the positive relation between the enjoyment and the use of the drawing strategy might be stronger for students with strong intramathematical performances, and the negative relation between anxiety about using and actually using the drawing strategy might be weaker for these students. Initial empirical indications for these considerations have come from research on anxiety, as frustration was observed to have less of an effect on the drawing strategy in high-ability engineering students (Mohler, 2007). Negative effects of frustration on the use of the drawing strategy were consequently stronger in low-ability students. As there have been only a few studies on the interaction between emotions and prior performance, we could not derive specific hypotheses about whether there would be negative or positive moderation effects of enjoyment and anxiety about using the drawing strategy on the use of the drawing strategy and modelling performance. Therefore, we used an exploratory approach to test whether the relationship between drawing and emotions would depend on students' prior mathematical performance.

2.5. *Path model and hypotheses*

On the basis of theoretical considerations, we hypothesized a path model (Fig. 2) that linked students' prerequisites (emotions about strategy use and intramathematical performance) with the outcome (modelling performance). The use of the drawing strategy served as the intervening variable between emotions and modelling performance.

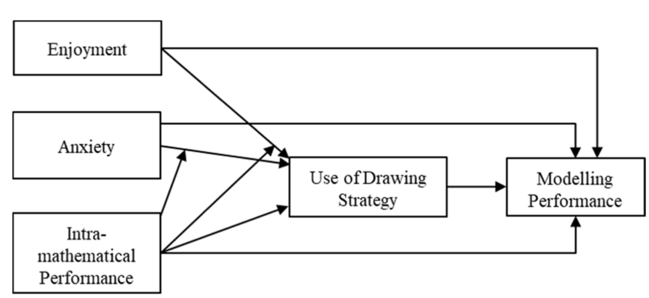


Fig. 2. Path-Analytic Model. Note. Paths illustrate the direct effect of one construct (e.g., enjoyment) on the other construct (e.g., use of the drawing strategy) or the moderating effect of a construct (e.g., intramathematical performance) on the effect of one construct (e.g., enjoyment) on the other construct (e.g., use of the drawing strategy).

Intramathematical performance was further included as a moderator of the effects of emotions on the use of the drawing strategy and modelling performance.

Hypotheses

1. (Direct effects; enjoyment/anxiety/intramathematical performance -> use of the drawing strategy; intramathematical performance -> effects of enjoyment/anxiety on the use of the drawing strategy) Enjoyment and intramathematical performance will positively (Hypothesis 1a) and anxiety will negatively (Hypothesis 1b) affect students' use of the drawing strategy. Intramathematical performance will moderate the effects of enjoyment (Hypothesis 1c) and the effects of anxiety (Hypothesis 1d) on the use of the drawing strategy. In short, the primary goal of this study is to test the ideas that the enjoyment of using the drawing strategy, anxiety about using the drawing strategy, and intramathematical performance will affect the use of the drawing strategy and that the effects of these emotions will differ in students with low and high intramathematical performance.
2. (Direct effects; use of the drawing strategy -> modelling; indirect effects; enjoyment/anxiety/intramathematical performance -> (via the use of the drawing strategy) -> modelling; intramathematical performance -> effects of enjoyment/anxiety on modelling) The use of the drawing strategy will positively affect modelling performance (Hypothesis 2a). Enjoyment and intramathematical performance will positively affect (Hypothesis 2b) and anxiety will negatively affect (Hypothesis 2c) modelling performance indirectly via the use of the drawing strategy as an intervening variable. Intramathematical performance will moderate the effects of anxiety and enjoyment on modelling performance (Hypothesis 2d). In short, the second goal of this study is to test the ideas that the use of the drawing strategy will affect modelling performance, that emotions about using the drawing strategy and intramathematical performance will indirectly affect modelling performance with the use of the drawing strategy as an intervening variable, and that the indirect effects of emotions on modelling will differ in students with low and high intramathematical performance.
3. (Total effects; enjoyment/anxiety/intramathematical performance -> modelling) Enjoyment and intramathematical performance will positively affect (Hypothesis 3a) and anxiety will negatively affect (Hypothesis 3b) students' modelling performance. In short, the third goal of this study is to examine the effects of the enjoyment of using the drawing strategy, anxiety about using the drawing strategy, and intramathematical performance on modelling.

3. Method

3.1. Participants and procedure

A total of 220 ninth and 10th graders (109 female adolescents; mean age 14.9 years, SD = 0.63) from 10 German comprehensive and academic-track secondary schools (Gesamtschulen and Gymnasium schools in the western part of Germany) took part in this study. Most of the students in our sample were familiar with the concept of using drawings to solve mathematical problems. They achieved a mean of 3.41 on a Likert scale ranging from 1 (completely agree) to 5 (completely disagree) for measuring familiarity with drawing by responding to the statement: "While solving demanding word problems, our teacher often makes a drawing." To the best of our knowledge, they did not receive any explicit training in the use of the drawing strategy during mathematics lessons. Questionnaires and performance tests were administered in two different test sessions in class by trained research assistants. There were at least 14 days between the first and second sessions.

In the first session, students filled out various questionnaires that asked about demographic information, enjoyment when using the drawing strategy, and anxiety about using the drawing strategy, among others. In the second session, they completed a test on modelling and after that a test on intramathematical performance (40 min). While taking the modelling test, students were not explicitly asked to make a drawing. Because we administered questionnaires about using the drawing strategy and because we aimed to assess the spontaneous use of the drawing strategy, there were at least 14 days between the first and second sessions so that we could avoid prompting students to use drawings to solve the modelling problems. Students participated voluntarily with no financial reward.

3.2. Measures

3.2.1. Enjoyment, anxiety, and the intramathematical performance test

Emotions about generating drawings to solve problems were assessed with Likert scales, which we adapted from the enjoyment and anxiety scales from the Achievement Emotions Questionnaire (AEQ; Pekrun, Goetz, Frenzel, Barchfeld, & Perry, 2011) by focusing the items on self-generated drawings. Pekrun et al. (2007) originally developed the AEQ-M to assess emotions about mathematics in school. In the original version, the items assessed class-related emotions, test-related emotions, and homework-related emotions in mathematics, whereas in our study, the items focused on applying a drawing strategy while solving problems (e.g., "When I do homework, I am in a good mood" was changed to "When I make a drawing to solve a complicated word problem, I am in a good mood"). In order to keep the questionnaire short, we assessed the affective component of enjoyment and the affective and cognitive components of anxiety. The enjoyment scale consisted of three items, and the anxiety scale consisted of five items (see Table 1) that ranged from 1 (*not at all true*) to 5 (*completely true*). The reliabilities (Cronbach's α) for enjoyment and anxiety were 0.766 and 0.860, respectively.

In order to address the part of intramathematical performance that was specifically relevant for solving the modelling problems that were presented to students in this study, we constructed an intramathematical test with nine items. We took five items from a prior study (Schukajlow et al., 2015) that assessed the application of the Pythagorean theorem and constructed five new items that assessed the solving of quadratic equations (see Fig. 3). Students received 1 point for the correct solution and 0 points for an incorrect or missing solution. The reliability was 0.760.

3.2.2. Modelling performance test and test on the use of the drawing strategy

The modelling performance test included eight problems that could be solved by applying the Pythagorean theorem (Fig. 1). Two raters gave scores to students' solutions of 2 (correct problem solution), 1 (partly correct solution resulting from errors in calculations), or 0 (incorrect solution resulting from an incorrect mathematical model or a missing

Table 1
Items used in the study to assess enjoyment and anxiety about using the drawing strategy.

Scale	Item
Enjoyment	When I make a drawing to solve a complicated word problem, I am in a good mood.
	I enjoy making a drawing to solve a difficult word problem.
	It is not fun to make a drawing to solve a difficult word problem (reversed).
Anxiety	I get scared when I make a drawing to solve a complicated word problem.
	If I make a drawing to solve a complicated word problem, I worry that others can do it better than me.
	I feel nervous when I make a drawing while working on a complicated word problem.
	I often worry that making a drawing to solve a demanding word problem is too complicated for me.
	When I make a drawing to solve a complicated word problem, I feel helpless.

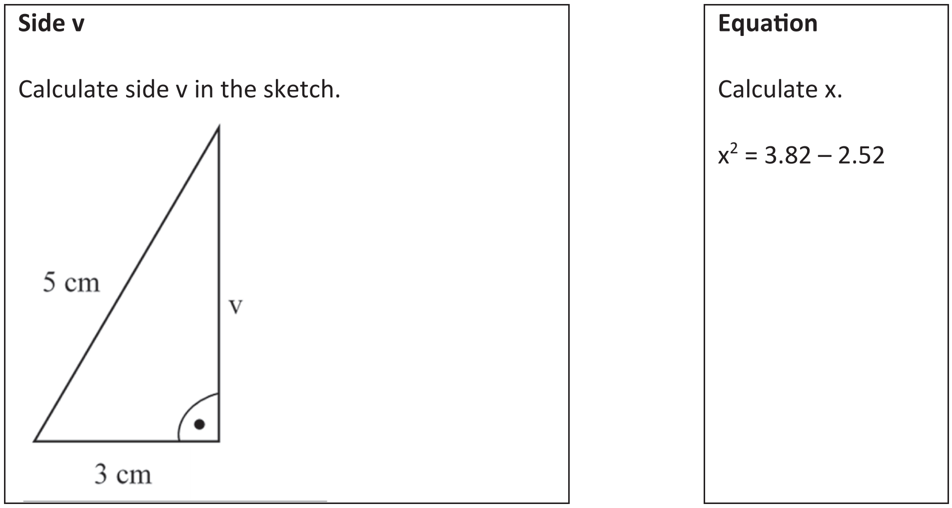


Fig. 3. Two Sample Problems from the Intramathematical Performance Test.

solution). The interrater reliability Cohen's κ was at least 0.82 for all modelling problems. The test reliability was 0.772.

Students' use of the drawing strategy was assessed via the number of drawings they constructed while solving eight problems on the modelling test. If students made a drawing for a problem, they received a score of 1; if they did not make a drawing, they received a score of 0.

3.3. Data analysis

We tested the hypothesized model with regard to modelling performance using enjoyment, anxiety, and intramathematical performance as independent variables and the use of a drawing as an intervening variable. To test the hypotheses, we used a path model with 33 free parameters and 220 participants. The ratio of participants to parameters was 6.7 (220/33), which was above the critical value of 5 for obtaining solid results (Kline, 2005). We used manifest variables instead of latent variables because of the large number of parameters included in the model. Latent modelling of emotions has revealed very similar results in terms of estimates and significance. A power analysis from a Monte Carlo simulation study revealed that the sample size was sufficient for the analysis of the hypothesized effects (power > 0.80).

3.4. Clustering of the data

Because school students learn mathematics in classes rather than individually, their perceptions of emotions, use of strategies, and performance can depend on the respective class. To examine the degree of dependence within the class (n = 10) for all measures, we calculated the intraclass correlation coefficient (*ICC*) using the statistical program Mplus (Muthén & Muthén, 1998–2017). The *ICC* was very low for enjoyment (0.04) and was low for anxiety (0.12), intramathematical performance (0.14), modelling performance (0.20), and use of the drawing strategy (0.21). Because we were interested in effects on the individual level, we calculated fit statistics and assessed the effects using maximum likelihood estimations with adjusted standard errors (*MLR*) using the type = complex analysis in Mplus.[1] This statistical method takes into account the nonnormality of the indicator variables and the dependence of observations for parameter estimates and goodness-of-fit model testing (Muthén & Muthén, 1998–2017).

3.5. Missing values

The percentage of missing values ranged from 4.1% for enjoyment and anxiety to 7.7% for modelling performance. We estimated the parameters with the maximum likelihood algorithm (*FIML*) implemented in Mplus, which uses all of the information from the covariance matrices.

4. Results

4.1. Analysis of model fit for the path model

We calculated model parameter estimates using the correlation matrix presented in Table 2. The analysis of the correlations showed that all the values were in the expected directions (e.g., the use of the drawing strategy was positively correlated with enjoyment and negatively correlated with anxiety).

Two goodness-of-fit statistics—the comparative fit index (*CFI*) and the standardized root mean square residual (*SRMR*)—were used to examine whether the hypothesized path model fit the data, and the estimated parameters were used to test the hypotheses (Hu & Bentler, 1999). The *CFI* was 1.00, which was above the critical value of 0.95. The *SRMR* was 0.009, which was below the critical value of 0.05. The Chi-Square test was not significant, $\chi^2(2) = 131.656$, $p = .72$. Thus, the data fit the hypothesized path model well according to all fit indices. The predictors explained 29.4% of the variance (R^2) in modelling performance.

4.2. Tests of hypotheses

In this section, we present the results of the estimates we calculated for the hypothesized path model (see the graphical representation in Fig. 4).

4.2.1. Effects of enjoyment, anxiety, and intramathematical performance on the use of the drawing strategy

Positive effects of the enjoyment of using the drawing strategy and intramathematical performance on the use of the drawing strategy were confirmed in our study (Hypothesis 1a). Further, the effects of anxiety about using the drawing strategy on the use of the drawing strategy missed the cutoff for significance but were in the expected direction (Hypothesis 1b). We did not find effects of intramathematical performance on the relation between the enjoyment of the drawing strategy

[1] In addition, we applied a bootstrapping procedure (5000 bootstrapped samples) to analyze the significance of paths representing indirect and moderating effects because the distribution of product terms might be only asymptotically normal. The *p*-values for the effects under investigation were identical or even lower than when we used the type = complex analysis.

Table 2
Means, standard deviations, and bivariate correlations between all variables.

Variable	1	2	3	4	5	*M*	*SD*
1. Enjoyment	–					2.35	0.81
2. Anxiety	−0.07	–				1.89	0.84
3. Intramathematical performance	−0.02	−0.14	–			0.49	0.25
4. Use of the drawing strategy	0.15*	−0.17*	0.13	–		0.61	0.36
5. Modelling performance	0.04	−0.34***	0.47***	0.35***	–	0.64	0.67

***$p < .001$, **$p < .01$, and *$p < .05$, two-tailed.

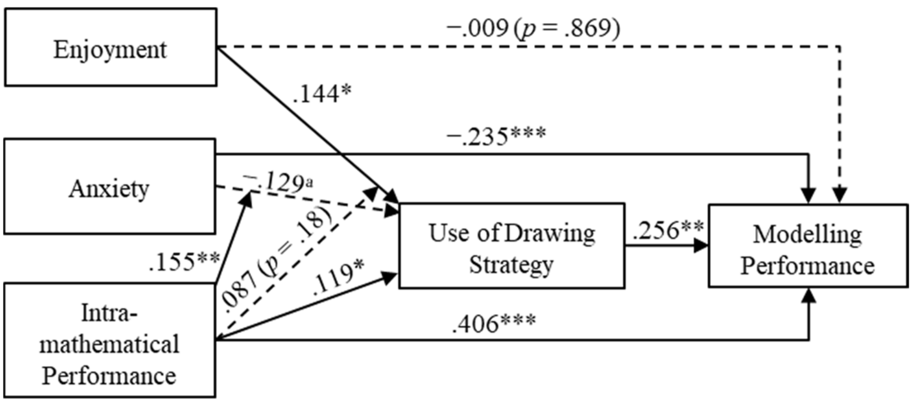


Fig. 4. The Hypothesized Path Model for Testing the Direct, Indirect, and Total Effects of Enjoyment, Anxiety, and Intramathematical Performance on the Use of the Drawing Strategy and Modelling Performance. Note. p = one-tailed. Significant paths ($p < .05$) are illustrated with solid lines and nonsignificant paths with broken lines; ***$p < .001$, **$p < .01$, *$p < .05$, and [a]$p < .10$. Enjoyment, anxiety, and intramathematical performance were centered. We report standardized regressions. Thus, the β coefficients can be interpreted as the predicted change in (residualized) criterion measures (in standard deviation units) when the independent measure changes by one standard deviation. According to the hypothesized mediation model, if enjoyment increases by one standard deviation (*SD*enj), the use of the drawing strategy directly changes by β * *SD*ud = 0.144 * *SD*ud.

and the use of the drawing strategy (Hypothesis 1c), but we found positive effects of intramathematical performance on the relation between anxiety about using the strategy and the use of the strategy (Hypothesis 1d). We performed an additional analysis in students with low intramathematical performance (one standard deviation below the mean of intramathematical performance) and high intramathematical performance (one standard deviation above the mean of intramathematical performance) for exploratory reasons. Whereas anxiety negatively affected the use of the drawing strategy in students with low intramathematical performance (β = −0.476, $p < .001$), this negative emotion positively affected the use of the drawing strategy in students with high intramathematical performance (β = 0.219, $p < .05$). Hence, our findings indicate the importance of students' emotions about strategies and intramathematical performance for strategy use, and they reveal that anxiety has different effects on the use of the drawing strategy in students with different levels of prior intramathematical performance.

4.2.2. *Direct effects of the use of the drawing strategy on modelling and indirect effects of enjoyment, anxiety, and intramathematical performance on modelling via the use of the drawing strategy*

As expected (Hypothesis 2a), the use of the drawing strategy positively affected modelling performance. In line with our expectations (Hypothesis 2b), the enjoyment of using the drawing strategy affected modelling indirectly via the use of the drawing strategy (β = 0.037, $p < .05$), and the indirect effects of intramathematical performance on modelling performance just missed the cutoff for significance (β = 0.031, $p = .07$). The indirect effects of anxiety about using the drawing strategy on modelling performance were not significant (Hypothesis 2c), but the effects were in the hypothesized direction (β = −0.033, $p = .097$). Moreover, we found a moderating effect of intramathematical performance on the indirect effect of anxiety on modelling via the use of the drawing strategy and confirmed Hypothesis 2d. Our exploratory analysis revealed that the indirect negative effect of anxiety about using the drawing strategy on modelling was significant in students with low intramathematical performance (β = −0.476, $p < .001$), and, although not quite significant, this effect went in the opposite direction and was even positive in students with high intramathematical performance (β = 0.056, $p = .087$). In summary, emotional and cognitive factors might have an impact on modeling via the use of strategies.

4.2.3. *Effects of enjoyment, anxiety, and intramathematical performance on modelling performance*

The enjoyment of drawing did not have a total effect on modelling performance (β = 0.027, $p = .336$), but intramathematical performance affected modelling performance (β = 0.442, $p < .001$), partly confirming Hypothesis 3a. Consistent with Hypothesis 3b, anxiety about using the drawing strategy negatively affected modelling performance (β = −0.259, $p < .001$). These results indicate the importance of anxiety about strategies and prior intramathematical performance for solving modelling problems.

5. Discussion

5.1. *Empirical contributions*

A main finding of the present study is that experiencing activating emotions (i.e., enjoyment or anxiety) with respect to the use of a strategy and prior performance are important predictors of the use of the strategy. More specifically, enjoyment of the drawing strategy and intramathematical performance positively affected the use of the drawing strategy. However, the effects of anxiety about using the drawing strategy on the use of the drawing strategy were more complicated. Anxiety about using the drawing strategy was a strong negative predictor of the use of the drawing strategy in low-achieving students. In students with high intramathematical performance, anxiety about using the drawing strategy went in the opposite direction. Higher anxiety about the drawing strategy was associated with more frequent use of the drawing strategy. This is a new contribution to research on the use of strategies because previous studies did not investigate the role of enjoyment and anxiety regarding the drawing strategy, prior intramathematical performance, and the interaction of these factors on the use of the drawing strategy and more generally on the use of deep-level processing strategies.

The second finding is that the use of the drawing strategy was positively associated with performance in solving modelling problems and that activating emotions (i.e., enjoyment or anxiety) about the drawing strategy and prior intramathematical performance affected students' performance when they solved problems with a connection to reality by using the drawing strategy. More specifically, students' enjoyment of the drawing strategy and their intramathematical performance positively affected modelling with the use of the drawing strategy as an intervening variable. Anxiety about using the drawing strategy was negatively associated with modelling performance via the use of the drawing strategy as an intervening variable in low-achieving students, whereas the effects of anxiety in students with high intramathematical performance went in the opposite direction but were not quite significant. This is a new contribution because prior studies did not investigate the effects of enjoyment and anxiety with respect to the drawing strategy and prior performance on modelling and more generally on achievement-related measures. Further, our work investigated for the first time the role of the use of the drawing strategy as an intervening variable between emotions about using the drawing strategy and performance.

The third result is that experiencing activating emotions (i.e., enjoyment or anxiety) about the drawing strategy and prior intramathematical performance are important predictors of modelling performance. More specifically, anxiety about using the drawing strategy negatively affected and intramathematical performance positively affected students' modelling performance.

The results of the present study add to previous research about the effects of enjoyment, anxiety, and prior performance on the use of the drawing strategy (Fiorella & Mayer, 2016; Fiorella & Zhang, 2018; Goetz et al., 2006; Uesaka & Manalo, 2017; Van Meter & Garner, 2005; Wu & Rau, 2019), on factors that affect the use of deep-level processing strategies (Ahmed et al., 2013; Muis et al., 2015; Obergriesser & Stoeger, 2020; Pekrun, Goetz, Titz, & Perry, 2002b; Taub et al., 2014; Winne & Hadwin, 1998), and on achievement-related variables such as modelling performance (Krawitz, Schukajlow, & Van Dooren, 2018; Lai, Zhu, Chen, & Li, 2015; Mohler, 2007; Ramirez et al., 2016; Schukajlow & Rakoczy, 2016; Verschaffel & De Corte, 1997).

5.2. Theoretical contributions

Learner-generated drawing is an important strategy for learning and problem solving. On the basis of the theory of learner-generated drawing (Arcavi, 2003; Rellensmann et al., 2017; Uesaka & Manalo, 2017; Van Meter & Firetto, 2013; Van Meter & Garner, 2005), the domain-specific theory of solving intramathematical problems with a connection to reality (Niss et al., 2007; Schukajlow, Kaiser, & Stillman, 2018; Verschaffel et al., 2000), and the control-value theory of achievement emotions (Pekrun, 2006), we hypothesized and tested a model linking activating emotions about using the drawing strategy, intramathematical performance, the use of the drawing strategy, and modelling performance.

Our findings are consistent with Hypothesis 1 concerning the effects of the activating emotions of enjoyment and anxiety and prior intramathematical performance on the use of the drawing strategy. First, we confirmed the positive effects of enjoyment when using the drawing strategy on the use of the drawing strategy, and we found negative effects of anxiety about using the drawing strategy in students with low prior performance on the use of the drawing strategy. Thus, our findings support the control-value theory of achievement emotions from which we drew this hypothesis. Second, we found positive effects of prior performance on the use of the drawing strategy, as suggested in the theory of learner-generated drawing. We did not expect to find positive effects of anxiety about using the drawing strategy on the use of the drawing strategy in students with high intramathematical performance, and this finding might reflect the activating nature of anxiety. Anxiety can induce extrinsic motivation to avoid failure (Pekrun, 2006) and this in turn can increase the use of more sophisticated deep-level processing strategies such as critical thinking in a prior study (Muis et al., 2015) or the drawing strategy in this work. These effects occur specifically in students with high intramathematical performance, as these students are more likely to identify the importance of the drawing strategy for solving modelling problems (Schukajlow, 2011). We know from prior research that when activation results from anxiety, it can have different effects on students' performance depending on prior performance or the initial level of anxiety. Students with low math anxiety do not suffer from reduced working memory and might demonstrate task-relevant activation and achieve higher scores in mathematics (Pletzer, Kronbichler, Nuerk, & Kerschbaum, 2015; Ramirez et al., 2018).

The findings are consistent with Hypothesis 2 concerning the positive effects of using the drawing strategy on performance that was suggested in the theory of learner-generated drawing. Further, Hypothesis 2, which concerned the indirect effects of the activating emotions of enjoyment and anxiety about using the drawing strategy and prior performance on modelling via the use of the drawing strategy, was partly confirmed in our study. As predicted in the control-value theory of emotions and in the theory of self-generated drawing, indirect effects of emotions on modelling were found for enjoyment in students with low and high intramathematical performance and for anxiety in students with low intramathematical performance. However, we did not confirm the indirect negative effects of anxiety about using the drawing strategy on modelling via the use of the drawing strategy in high-performing students. Again, anxiety about using the drawing strategy might not have negative effects on performance under specific conditions such as high prior performance in our study. Consistent with the theory of self-generated drawing, students' prior intramathematical performance affected their modelling performance indirectly via the use of the drawing strategy.

Further, the findings were partly consistent with Hypothesis 3, which concerned the effects of the activating emotions of enjoyment and anxiety about using the drawing strategy and prior intramathematical performance on modelling performance. As proposed in the theory of emotions, anxiety about using the drawing strategy negatively affected modelling performance. This finding confirms the theoretical considerations about the negative effects of mathematical anxiety, specifically for anxiety about learner-generated drawing and more generally for anxiety about strategy use. However, positive effects of enjoying the use of the drawing strategy on modelling could not be confirmed. One explanation for the zero relation between enjoying the use of the drawing strategy and modelling is that we assessed the enjoyment of one specific strategy and not whether the student enjoyed modelling in general. Solving modelling problems requires many more activities than just drawing. Students might focus more on applying mathematical procedures while solving modelling problems than on the use of the drawing strategy. Consequently, we suggest that researchers pay more attention to the objects of emotions in theories of emotions. Further, drawing upon the processes of solving modelling problems, we demonstrated that prior intramathematical performance affects performance in solving real-world problems. This result is consistent with the domain-specific theory of solving problems with a connection to reality that underlines the importance of prior intramathematical performance for modelling.

This study extended the theory of emotions (Pekrun, 2006) to emotions about the drawing strategy, and it is consistent with research on self-generated drawing (Fiorella & Zhang, 2018; Van Meter & Firetto, 2013; Van Meter & Garner, 2005). This study is consistent with the theory of modelling (Niss et al., 2007; Schukajlow et al., 2018; Verschaffel, Schukajlow, Star, & Van Dooren, 2020) as it demonstrated the importance of prior mathematical knowledge (assessed via prior intramathematical performance) for solving real-world problems.

5.3. Practical contributions

Along with other evidence from research on learner-generated drawing, the current study suggests that the use of the drawing strategy is beneficial for problem solving in the domain of mathematics and more specifically for geometrical modelling problems. However, it identified important emotional and cognitive factors that affect the use of the drawing strategy while solving mathematical problems. When a person enjoys using drawings to solve problems, this fosters the use of the drawing strategy, whereas anxiety about using the drawing strategy can increase or hinder the use of the drawing strategy. This work suggests that modifications should be applied to the boundary conditions for learning by drawing (Fiorella & Zhang, 2018) that underline the importance of guidance for successful learning but that do not include emotional factors. The guidance for applying the drawing strategy can indeed be helpful for students with poor prior intramathematical performance, as not only might it increase the use of the drawing strategy and its quality, but it might also compensate for the negative effects of anxiety about using the drawing strategy on the use of the drawing strategy and performance. An important practical implication is that it might make sense to consider increasing students' enjoyment of using the drawing strategy in strategy training programs. This can be done by emphasizing the value of this strategy and improving students' control appraisals as suggested in the control-value theory of emotions. Prior research indicated that utility value interventions (e.g., reflection on utility value of mathematics) successfully improved motivational variables (Eccles & Wigfield, 2020) and can be promising for increasing enjoyment and decreasing anxiety about using of the drawing strategy (for the detailed analysis of the conditions for the effects of reflection on utility value in mathematics see Gaspard et al. (2019) and Liebendörfer and Schukajlow (2020)). Other promising interventions might be to improve student transformative experience (Pugh, 2011) or prompting students to develop multiple solutions for modelling problems (Schukajlow & Rakoczy, 2016) which were demonstrated to improve emotions, performance and interest in the domains of biology and mathematics. Our findings about the effects of the use of the drawing strategy for geometrical modelling problems adds to a large body of research on the positive effects of using the drawing strategy on students' performance in mathematics and science.

5.4. Strengths and limitations

The roles of enjoyment, anxiety, and intramathematical performance as predictors of strategy use and modelling performance were tested with path analyses. Caution should be applied when interpreting the paths in the proposed model as causal. The validity of the analysis of path models strongly depends on evidence from previous research on the possibility that there are directed effects such as the influence of anxiety on students' performance. Our assumptions about the implied causal structure came from the theory of learner-generated drawing, the theory of solving modelling problems, and the control-value theory of achievement emotions and were based on empirical results from prior studies. However, experimental studies are needed to collect stronger evidence for the direction and power of the tested effects. Moreover, our path model might be incomplete because other strategies such as taking notes or highlighting might also transmit the effects of emotional and cognitive prerequisites on performance. In future studies, researchers should increase the number of strategies they assess for problem solving in order to collect more support for generalizing their findings to other strategies and more generally to the roles of emotional and cognitive factors for applying deep- and surface-level strategies in problem solving. Our findings are limited to two activating emotions enjoyment and anxiety. In future studies, including a wider range of emotions, assessment of emotions via varies components (affective, cognitive, motivational and physiological) and addressing emotions about mathematics are essential to obtain a generalized finding about the role of emotions about drawings for strategy use and performance. Further investigating the reciprocal effects (e.g., the effects of the use of the drawing strategy on emotions) is important in the future.

For the assessment of emotions, we decided to use questionnaires because the validity of questionnaires has been confirmed and is widely accepted in the field. Because visual representations can be used for different purposes in mathematics, we asked students about their emotions concerning the drawing strategy while solving complicated word problems. However, although the questionnaires focused on the application of the drawing strategy, some students may have reported on the emotions they experience when solving complicated word problems. This is a limitation of the present study that should be addressed in future research. Students' performance was assessed with tests that have been used in prior studies. Strategy use was assessed with analyses of students' solutions and not by self-reports so that we could obtain a more valid measure of students' strategies (Rovers, Clarebout, Savelberg, de Bruin, & van Merriënboer, 2019).

6. Summary

In our study, we answered several calls for studies to link emotional, strategic, and cognitive variables (e.g., Carden & Cline, 2015; Pekrun, 2006), and we investigated interaction effects of individual differences in problem solving behavior with a focus on strategy use. In summarizing our findings, we would like to emphasize that enjoyment, anxiety, and prior performance are powerful prerequisites of learning outcomes that facilitate strategy use and intramathematical performance. Our finding concerning the link between emotions about using the drawing strategy and prior performance on the one hand and the use of a deep-level processing drawing strategy and modelling performance on the other hand is the main contribution of this project. This work has theoretical implications for the theory of learner-generated drawing, control-value theory, and the theory of solving modelling problems as well as practical implications for modifications that can be applied to drawing strategy training programs.

Declaration of Competing Interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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