Ranking game on networks: The evolution of hierarchical society

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Interacting with each other, individuals in a population form various social network topologies. Models of evolutionary games on networks provide insight into how the collective behaviors of structured populations are influenced by individual decision making and network topologies. In a hierarchical society, many social resources are allocated according to certain social rankings, such as class, social status, and social hierarchy. In this context, to climb the social ladder, individuals will try to improve their social ranking among the population. It is essential to understand the impact that changes in social ranking have on decision making, which very few literature discuss. To capture this social nature, a ranking game model on networks was introduced in this study. Three decision-making strategies – random, follow, and centrality-based – are introduced. Systematic numerical simulations of the different strategies are conducted on three social network topologies: random, small-world, and scale-free networks. The results reveal that the rankings of the whole population evolve differently with various dynamics in network topologies and social liquidities. The centrality strategy leads to relatively larger than average centrality, while the follow strategy tends to form networks with significantly larger edge density, indicating overall improvements for the whole population. Notably, the centrality strategy results in the least similarity, lowest survival rate, and highest liquidity, showing that this strategy allows larger social-structure changes with relatively better social mobility. In contrast, for the random and follow strategies, the social network becomes more rigid. However, in all cases, individuals are observed to appear in different ranking positions. This ranking game model could serve as a basis for further sophisticated ranking-related evolutionary games on social networks, with implications for policymaking in ranked social scenarios.

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1. Introduction

In a structured social system or organization, individual players dynamically interact with each other to form an evolving social network [1,2] in order to exchange knowledge and diffuse innovations [3–6]. By establishing new connections or canceling existing ones between players, the network topology changes constantly to demonstrate collective dynamics [7–10]. In turn, the evolving environment drives individuals to adapt to the surroundings. In the context of a network environment, evolutionary game theory is married with social network theory, which has recently provided insight into *agent–action–network* evolutions [11–17]. In this series of studies on spatial evolutionary games, a variety of games have been tested on social networks, including prisoner's dilemma (PD) [18–23], snowdrift [24], public goods [25], and minority games [26–28].

Cooperation is an important concept in social network studies, and is usually explored as a property of a given social network [29,30]. However, in studies of evolutionary games on networks, the cooperation among players is not only a result of evolution, but also a driving force of it. These studies shed light on understanding behaviors in networked populations or organizations, particularly on cooperation evolution [2,12–14,19,31–36]. In most models, players are placed in a network and play iterative rounds of games against each other to maximize their utilities by accumulating game rewards. In other cases, the utilities are represented by network properties like centrality [37,38]. Centrality is a measurement of network importance in the structured population [38–42]. In certain cases, players try to reach better social positions with larger centralities. While some models keep the networks unchanged [32], in realistic dynamic network evolution, it is possible for a corresponding edge to be established or canceled when bilateral cooperation is reached or discharged [13]. Though existing models provide insight into the underlying behaviors, the aims or motivations for game players are the values of rewards, which are independent of the rankings. In other words, the utilities are absolute continuous values. However, the motivations or incentives behind individual decision making might vary in different scenarios. In many real-life situations, individuals are only concerned with the relative discrete rankings rather than the utility values. In these scenarios, the overall aim of an individual is to secure a relative position in the population, regardless of the distance between themselves and the individuals in the adjacent rankings. For instance, a candidate only needs to take the lead to win an election, but the leading margin does not alter the result. In a ranked evaluation system, the ranking is more important than anything else to the players.

To investigate the dynamics of networks in which the game players are concerned with their ranking, we introduce a *ranking game* (RG) on social networks. In this model, an individual establishes or cancels edges based on the topology with the deliberate aim of increasing their ranking position among the population. The ranking is determined by the topology of the networked society, indicated by network centrality. Centralities of both local degree and global betweenness are considered. In each round, players make decisions to climb the social rankings. Each of them chooses the best candidate with whom to establish a new edge or cancel an existing edge to deliver the best outcome, in the hope of achieving the highest possible rank. The ranking game model is simulated on three networks: a random network (ER), a small-world network (SW), and a scale-free network (SF), generated from corresponding models [7,8]. Three decision making strategies of *random* S_r, *follow* S_f, and *centrality* S_c are considered for comparison. We observe the dynamics of network evolution as well as changes in the social rankings.

The main contributions of this work are threefold. First, we formalize social ranking dynamics as a ranking game. Proposed as a new evolutionary game on networks, the model is simple and vividly captures the competition dynamics in a hierarchical population, whereas social status is quantified as network centralities. Second, the results show that the centrality strategy introduces more mobility to a structured population, while the random and follow strategies result in lower mobilities, with possibilities for individuals to change ranking positions still remaining. Identical evolutionary patterns are observed in all three types of network. This study provides new insight into the evolutionary dynamics concerning the mobility of a structured population. Third, this work contributes to the collection of literature on evolutionary games on networks by expanding the studies into the domain of social rankings in network-structured populations. This work also provides new research directions to related objectives from the widely used absolute payoffs in existing evolutionary game studies. We hope this ranking game model can stimulate studies on ranking behaviors of social systems, and provide new evidence and implications to policymakers on issues of social hierarchy, mobility, inequality, and governance.

The rest of this paper is organized as follows: Section 2 presents a discussion on evolutionary games played on networks, with literature reviews. In Section 3, the ranking game model is introduced. Subsequently, detailed numerical simulations and results are presented in Section 4. Finally, Section 5 concludes the study and provides discussion.

2. Background

Human society and the business world are typical hierarchical systems in which players are ranked in classes according to certain measurements like social status, wealth, resources, influences, and authorities in the population. Those in higher-ranked positions usually enjoy social advantages compared to those with poor rankings. The social competition drives low-ranked players to climb the social ladder [37] in the hopes of reaching a better rank, while the top-ranked try to maintain their leading positions.

In ranked situations, a relative competitive advantage, rather than an absolute advantage, matters the most for players to reach better ranking positions. For example, in the case of an oligopoly, firms try to be ranked high enough to be accepted into the dominating group and enjoy the market shares and pricing. In a tiered education system, schools are ranked in groups according to their performance and are granted different levels of financing. In these cases, capability drives the competition for higher ranking.

Without loss of generality, we shall indicate the top-ranked players as *the rich* and the low-ranked players as *the poor*. Here, players might be ranked according to the wealth for individuals, competitiveness for firms, trading power for states, etc. In a society, the interactions among players involve cooperation and defection. Cooperation usually benefits the two sides bilaterally, while defection harms both. This makes the ranking competition complicated. The rich might tend to be reluctant to establish cooperation with the poor because helping the poor might threaten their high positions. In contrast, the poor are not willing to defect from existing partnerships for fear of losing social status. It would be interesting to investigate how a hierarchical population might evolve in ranking competitions. Since ranked hierarchical structures are common in human society, the understanding of social phenomena like inequity, mobility, and class solidification are critical for social regulation and governance [43]. Will the rich stay in top-ranking positions? Do the poor have a chance to become rich? Will the social ranking maintain rigidity or show mobility? Overall, how does the social ranking change in ranking games?

Emerging studies on evolutionary games have provided insight into how social systems evolve [15]. Without considering a game played by a structured population, the hierarchy and network structures have been compared in the context of organization coordination [44], and the cost factor of network structures on organizational level change has been investigated [45]. More recently, there has been a new series of studies on spatial games played on networks that appeared as extensions to traditional games [12,46–48], which greatly advanced our understanding on the origin of cooperation and defection [11]. In these evolutionary games on networks, topologies were taken into consideration to study how the decision making of structured individuals emerged as collective behaviors and in turn influenced network topologies [19]. In a network, players have different levels of influence due to their heterogeneous topological properties. The dynamics of influence was investigated in hierarchical populations [5]. In another study, influential invaders were identified in a structured evolutionary population [17]. Most models for evolutionary games on networks simply consider a single network. However, some efforts have been made to understand how cooperation is determined in multiplex networks, and how evolutionary games are played on multilayer networks [6,14,16].

Prisoner's dilemma (PD) games are well explored in game theories. With the development of network science, PD games have been incorporated into social networks to study cooperation behaviors [19,20,22,23,32,49]. The payoff is awarded to two game players according to their choices to either cooperate or defect. For a networked population, cooperation is achievable through PD games [13]. Unlike the regularities of periodic lattice networks, heterogeneous networks are more suitable for modeling social systems in which local connectivities vary throughout the population [50,51]. The topologies of networks have significant influence on the results of games.

Understanding the cooperation among players is the central objective of evolutionary games on networks [33–36]. PD games in heterogeneous networks do not necessarily reach cooperation [18], nor do they reach direct reciprocity [52]. However, cooperation is promoted on networks [53,54]. Neighborhood diversity in local interactions has been found to promote cooperation [54]. Moreover, a structured population tends to favor cooperation if the benefit-to-cost ratio is large enough [55]. The SF network structure is found to be helpful in enhancing cooperation [56], while for PD games played on SW networks, a certain degree of heterogeneity plays an influential role in promoting cooperation [22]. In a similar study, at a certain average degree, the cooperation level reaches a peak [34]. The influence of network properties like the average degree, variance in degree distribution, clustering coefficient, and assortativity on the cooperation level is investigated in a comparative study [33]. Meanwhile, for snowdrift games, cooperation is reduced if the cost of cooperative behavior is high [24,57]. From the perspective of information, another study investigates the emergence of cooperation within groups of individuals in the evolutionary dynamics of public goods games played on structured networks. The results show that if the mesoscopic information about the structure of the real groups is available for players, cooperation is enhanced [25]. Nevertheless, maintaining cooperation in a structured population is difficult [58]. When the fitness of a player is determined by their own payoff and the average payoff of all direct neighbors, the player adopts the strategy from one neighbor with probability in proportion to the difference in fitness [59]. This model captures asymmetric finesses and enhances cooperation levels. If the player has a heterogeneous strategy adoption between the best performer and a random candidate, it has been found that following the best performer can promote the cooperation level [53]. Most networks considered in the study of evolutionary games on networks are single-layer networks. However, it is worth exploring interdependent multilayer networks. In a study, players played a PD game on one layer, while other players played a snowdrift game on another layer. The strategy information sharing between layers enhanced the cooperation [6].

Some experiments are performed with human players. The games played by humans on networks promote cooperation [60]. In a mixed setup of bots and humans, the noises introduced by bots contributed to the improvement of coordination [61]. Unlike most models where game players have no memory, a model considers the neighbor's past actions [62]. This reputation information has effects on high cooperation, while social knowledge does not. Another human experiment, in which subjects were arranged on networks, reveals that strategy updating impacts strategy-imitation dynamics [63]. Statistical physics provides valuable approaches to modeling social inequality [43]. The visibility of subjects is found to influence the outcome of wealth distribution in a network and leads to severe inequality [46]. By extending the resolution of standard PD games from a binary strategy space to a continuous space, the cooperation level can be amplified with strategy resolutions [64]. However, if players have multiple strategy choices rather than binary choices in standard PD games, the population tends to reach a lower cooperation level [65].

The study of networked evolutionary games has rich implications for management. Recently, a series of studies on alliance networks demonstrated that topologies have fundamental influences over the dynamics of alliance portfolio formation in structured social networks [66,67]. In the context of innovation networks [3], complementary knowledge is considered the driving force in forming an alliance [68]. In the formation of inter-firm alliances in fields like nanotechnology, alliance-making is not only influenced by technological uncertainty, but also by the given network positions [69]. Investigations of the alliance portfolios of software firms reveal that micro dynamics have an influence on decisions regarding alliance formation [70], and that the performances and value creation of software firms are closely related to alliance portfolios [71]. Environmental change impacts steel industry alliances, in which entrepreneurial firms benefit while the prominent firms suffer [72]. For the network topologies to play the games, most literature adopts homogeneous ER networks while some adopt heterogeneous SF networks [18,56,73].

Though there are studies on how actors optimize network structure and centrality measures [42,74,75], among these studies of emerging network structures and evolutionary games on networks, many discuss games with a focus on the dynamics of cooperation formation and player motivations. The games are played pair-wisely, and a binary strategy is chosen either completely randomly or by following the neighbors with best performances. In this context, the social hierarchy has not been thoroughly investigated and a link has not been established between game payoff and player ranking, thereby limiting information about the players' motivations to change position. Moreover, the payoffs are continuous values rather than discrete ranking positions. Furthermore, the topological centrality of a player is considered in decision making.

Structural centrality is a measure indicating the importance of a player in the population, and is reliant on the network topology [38,41]. Degree centrality indicates the number of connections between one player and their neighbors, and is therefore a local property. To indicate the importance of a player among the structured population, the betweenness centrality is introduced as the number of all shortest paths passing through the player, indicating that betweenness centrality is a global property. Players with high centralities are considered important players. In the existing literature, there is still a lack of discussion on the centrality properties in games on networks. In the investigation of social hierarchy, a social climbing game is proposed, as agents try to become more central in social networks [37]. They find that the hierarchy can be reinforced with reduced social mobility [40], and report that the nestedness property emerges when players try to maximize their centrality values, though this is not dependent on the types of centralities. To overcome the difficulty of missing data, a confidence-level measurement is introduced to describe the central player in a network [39], which is a good supplement to the above centrality statistics. While degree centrality describes the local importance of a node and betweenness centrality describes its global importance, eigenvector centrality treats edges differently according to the scores of connected nodes. PageRank is a variant of eigenvector centrality. Compared to the degree and betweenness centralities, eigenvector centrality is especially suitable for directed networks in which incoming and outgoing connections are treated differently. The entropy-based centrality introduced is derived from a transfer entropy matrix [76]. The transfer entropy was originally introduced to describe how stocks influence each other using historical information [76]. In other words, the entropy quantifies how the history of one node can be used to predict the future of another. Since it is a nonlinear version of the Granger causality [76], it is suitable for describing directed networks, and the importance of nodes is evaluated as a prediction power to other nodes. Other entropy-based centralities might focus on the path-transfer flow [77,78]. In the former [78], the entropy concept is built upon the probability or likelihood that a path through which information flows passes a given vertex. Based on this [78], the entropy concept is further developed [77]; however, it still concerns the centrality based on a discrete Markovian transfer process. However, our present ranking game concerns the centrality solely based on the topologies of the networks and not a Markovian transfer process. Furthermore, we consider undirected networks in this work, and introduce a hybrid centrality combining degree and betweenness centralities to consider the local and global importance simultaneously.

To study how the hierarchical social structure changes from individual behaviors, investigates the social mobility in structured population settings in networks, and captures the network evolution of players with upward ranking mobility, we examine a ranking game on networks in this study. The ranking is based on centralities determined by topological position. In this game, a player seeks a counterpart from the structured population, attempting to reach a better ranking position by establishing a new edge or canceling an existing edge. The ranking game on networks serves as a contribution to existing literature on evolutionary games on networks by expanding the scope from existing non-ranking games to ranking games, with focus on social hierarchy, social mobility, and centrality-based social advantages.

3. Ranking game model

In a ranking game, we consider a structured population with N players on a network $G(V, E)$, where V is the vertices set and E is the edges set. If two players v_i and v_j are directly connected, then the edge $e_{ij} = 1$, otherwise $e_{ij} = 0$. The connected players form a neighbor set, $\Gamma_i^t = \left\{ v_j \left| e_{ij} = 1 \right. \right\}$. The non-neighbors are denoted as $\mathrm{H}_i^t = \left\{ v_j \left| e_{ij} = 0 \right. \right\}$. Three different network models are considered, i.e. *Erdős–Rényi network* G_{er} with edge probability p_{er}, *small-world network* G_{sw} generated from initial Γ_{sw} neighbors with rewiring probability p_{sw}, and *scale-free network* G_{sf} with l_{sf} preferential attached edges. Using these generative models and parameters, we can set up the initial networks for the games. SW and SF networks are two widely observed topological structures in social systems [79,80], while the random Erdős–Rényi network is included for comparison. We conducted multiple rounds of iterative ranking games on each of the three networks to investigate how the initial network structure evolves and influences the dynamics. For a given network, players are allocated to the vertices V, and the network size $|V|$ is equal to the population size N. In each round, each player adopts the same strategy to establish a new edge to one non-neighbor player or cancel an existing edge with one neighbor player. We introduced three strategies, *random strategy* S_r, *follow strategy* S_f, and *centrality strategy* S_c. In S_r, the players all randomly choose whether to establish or cancel an edge with a random counterpart player in the population. This random strategy mimics purely irrational decision making. In the opposite extreme case, a follow strategy is adopted by the population. In S_f, a player chooses to establish or cancel according to their ranking position. If the player chooses to establish, he/she establishes an edge to the player with the highest centrality among all non-neighbors. If the player chooses to cancel, he/she cancels the edge to the player with the lowest centrality value. In other words, players are purely rational in following the best and avoiding the worst choices in the population.

In *centrality strategy* S_c, all players are ranked in a ranking R^t in descending order of centrality value. The socially leading players with larger centrality values occupy the top-ranking positions. In contrast, the socially disadvantaged players with small centrality values stay in the bottom-ranking positions. Considering that the centralities can be based on degree and betweenness, we use a hybrid form:

$$c_i^t = \alpha \hat{k}_i^t + (1-\alpha)\hat{b}_i^t, \tag{1}$$

where $0 \leq \alpha \leq 1$ is the weight parameter to adjust contributions of normalized degree centrality $\hat{k}_i^t$ and normalized betweenness centrality $\hat{b}_i^t$. For v_i, degree centrality k_i^t indicates the local importance, while betweenness centrality b_i^t indicates the global importance. We normalize the original centralities to fit them into the same ranges between 0 and 1, i.e.:

$$\begin{cases} \hat{k}_i^t = \left(k_i^t - k_{\min}^t\right)/\left(k_{\max}^t - k_{\min}^t\right), \\ \hat{b}_i^t = \left(b_i^t - b_{\min}^t\right)/\left(b_{\max}^t - b_{\min}^t\right). \end{cases} \tag{2}$$

In a ranking R^t, larger r_i^t indicates larger centrality c_i^t, and vice versa. R^t is available to all players as public information for decision making. A ranking game is like the process of climbing a social ladder. The mission of a player is to climb the ladder to achieve a higher ranking position; in other words, to be relatively better than the rest of the population. Normally, a player with a better ranking position has less motivation to establish an edge and more motivation to cancel an established edge with another player, while a player in a poor ranking position has a strong motivation to establish an edge and a weak motivation to cancel an established edge. At time t, a player v_i first chooses to establish or cancel based on their ranking position r_i^t on the ranking R^t of the whole population. A bottom-ranked player tends to establish a new edge rather than cancel an existing one in an attempt to improve their ranking position, while a selfish top-ranked player is more likely to cancel rather than establish an edge because connecting with others will benefit others and threaten their own position. Thus, starting from the bottom part of the social ladder, a player v_i has a bigger p_i^t to establish an edge, and when v_i reaches the upper part of the ladder, it has a smaller p_i^t and is more likely to cancel. Disadvantaged players want to benefit from establishing edges with other players to gain larger centrality from climbing up the ladder, while advantaged players are more likely to cancel with other players to discourage the latter from climbing the ladder, and in this way maintain their relatively better ranking positions. To capture this, player v_i chooses to establish with probability p_i^t defined as

$$p_i^t = \frac{N - r_i^t}{N-1}, \tag{3}$$

where r_i^t is the centrality rank of v_i among the population of size $N = |V|$. Meanwhile, v_i chooses to cancel with a probability $1 - p_i^t$. If v_i chooses to establish, then v_i considers each non-neighbor player $v_j \in \mathrm{H}_i^t$, and calculates the possible rank $\hat{r}_i$. Then, all possible ranks are evaluated to choose the best candidate player v_j, allowing the best ranking position for v_i, that is,

$$\operatorname*{arg\,max}_{v_j \in \mathrm{H}_i^t} \left(\hat{r}_i \left| G^t + e_{ij}\right.\right). \tag{4}$$

Afterwards, an edge is established between v_i and v_j, making $e_{ij} = 1$. Similarly, when v_i chooses to cancel, v_i evaluates all candidates among neighbors $\varGamma_i^t$ and chooses the v_j of

$$\operatorname*{arg\,max}_{v_j \in \varGamma_i^t} \left(\hat{r}_i \left| G^t - e_{ij}\right.\right). \tag{5}$$

Then, the edge e_{ij} is canceled. Each time, all players conduct decision making and edges are established or canceled, thus the network is updated from $G^t \to G^{t+1}$.

To investigate how the ranked social hierarchy changes, we introduce three properties to quantify the dynamics of rankings. For two given times t_1 and t_2, the ranking evolves from R^{t_1} to R^{t_2}. *Spearman correlation* ρ_{t_a,t_b} is used to describe the ranking similarity between R^{t_a} and R^{t_b}. If the two rankings are similar, ρ_{t_a,t_b} tends towards 1; conversely, dissimilarity produces a negative value tending towards –1. $\rho_{t-1,t}$ is calculated for two sequential times $t-1$ and t, while $\rho_{t_0,t}$ indicates how the ranking at time t is different to the initial ranking at $t = 0$. *Survival rate* ς_{t_1,t_2} is introduced to indicate how players' ranking positions remain unchanged. If $r_i^{t-1} = r_i^t$, then v_i survives from $t-1$ to t, i.e., $\varsigma_i^{t_1,t_2} = 1$. Again, survival rates between the last time and the initial time are calculated. For the whole population, the survival rate is normalized as

$$\varsigma_{t_1,t_2} = \frac{\sum_{v_i \in V} \varsigma_i^{t_1,t_2}}{N}. \tag{6}$$

Similarly, both $\rho_{t-1,t}$ and $\rho_{t_0,t}$ are considered. *Rank liquidity* μ_{t_1,t_2} is used to quantify the ranking position changes. To normalize the changes, we divide the total position changes by the maximum possible changes as follows:

$$\mu_{t_1,t_2} = \begin{cases} \frac{2\sum_i \left|r_i^{t_1} - r_i^{t_2}\right|}{N^2} & \text{if } N \text{ is even,} \\ \frac{2\sum_i \left|r_i^{t_1} - r_i^{t_2}\right|}{(N-1)(N+1)} & \text{if } N \text{ is odd.} \end{cases} \tag{7}$$

Thus, μ_{t_1,t_2} is kept as a value between 0 and 1. Again, both $\mu_{t-1,t}$ and μ_{t_0,t_2} are calculated.

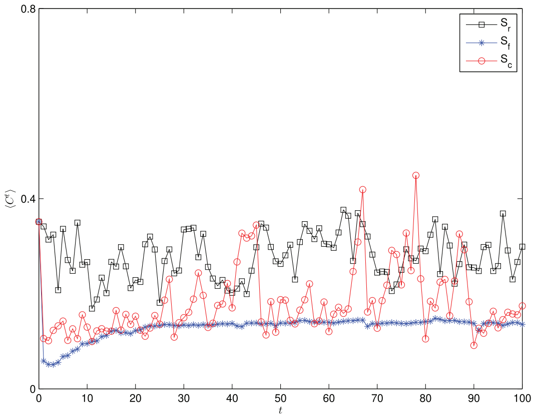
For a given network, average centrality $\langle C^t \rangle = \sum_i c_i^t / N$ measures the averaged importance of the whole population. Network density is the ratio of existing edges to possible edges, i.e. $D^t = 2\,|E|/(N\,(N-1))$. The average clustering coefficient $\langle CC^t \rangle = \sum_i cc_i^t / N$ measures the tendency of the players to connect in clusters. Average assortativity $\langle A^t \rangle = \sum_i a_i^t / N$ is introduced to measure the likelihood of players connecting to other players with similar degrees. A larger $\langle A^t \rangle$ indicates that top-ranked players tend to connect with other top-ranked players, while a smaller $\langle A^t \rangle$ indicates that low-ranked players try to connect with top-ranked players. To investigate the properties of the networks, these coefficients are calculated.

Our ranking model is significantly different from other models [37,40] in several aspects. First, in the social climbing model [37], a player tries to maximize their utility solely based on the degrees of direct neighbors in a distance of one, and neighbors of neighbors in a distance of two, which is a local degree measurement. The considered strategy for a player is to rewire a randomly selected edge from a direct neighbor to a neighbor's neighbor with a probability based on the possible increase in utility. Thus, the player tries to maximize the local degree value. Obviously, in this model, the edge numbers and edge density remain unchanged. However, in the ranking game, the ranking of the combined centrality of both degree and betweenness matters. In other words, both local and global importance are taken into consideration. Meanwhile, in ranking games, three strategies are considered. In the centrality strategy S_c and follow strategy S_f, the decision to establish or cancel is dependent on one's ranking position. The target for choosing counterpart players is also to achieve a better ranking position. Moreover, in a ranking game, the edge number changes throughout the game instead of remaining fixed. Similar to the follow strategy S_f in our ranking game, a player chooses to establish or cancel an edge in a probabilistic approach [40]. However, the probability is kept unchanged and has nothing to do with the network topology. In our ranking game, the probability is directly based on the centrality and ranking position. In this model, the decision making in choosing the counterpart player is solely based on the centrality value, in that it will simply establish with the player with the highest centrality value or cancel with the one with the lowest centrality value [40]. This approach resembles the follow strategy S_f in our ranking game. However, the initial network is kept as an empty network, and the model focuses on the growth [40]. In our ranking game, we start with different topological networks with different edge distributions to see how the initial network topology evolves in the game. Furthermore, we introduce measurements of ranking dynamics such as ranking similarity ρ_{t_1,t_2}, ranking survival rate ς_{t_1,t_2}, and ranking liquidity μ_{t_1,t_2}, and we also systematically investigate the network properties to see how the network topology changes. Based on this ranking game model framework, extensions to other relatively complex centralities like eigenvector, closeness, and PageRank are also possible and straightforward.

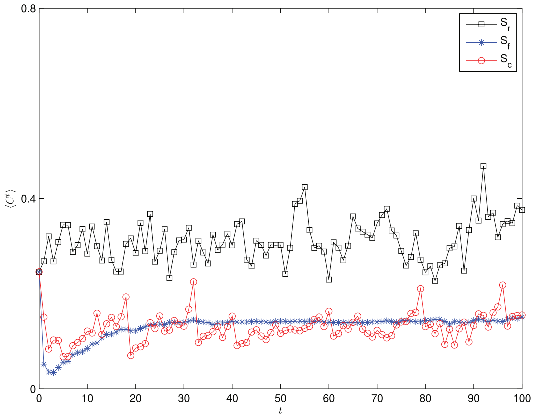
4. Results

We conducted numerical simulations to study how the ranking game evolves on networks with various configurations. *Erdős–Rényi* networks, G_{er}, are generated with different p_{er}. *Small-world* networks, G_{sw}, are generated with different rewiring probabilities p_{sw} and initial number of neighbors n_k [8]. *Scale-free* networks, G_{sf}, are generated with different preferential attached edges l_{sf} [7]. Thus, we consider homogeneous ER networks G_{er} as well as heterogeneous G_{sw} and G_{sf} in our simulations. For each generated network, all three strategies, *random strategy* S_r, *follow strategy* S_f, and *centrality strategy* S_c, are played independently. Different values of α are used for different combinations of degree and betweenness centralities. In a single simulation round, we let the agents play 100 sequential games. For each game, the network properties are calculated to see how the network topologies change throughout the games. Meanwhile, the rankings and corresponding *Spearman correlation* ρ, *survival rate* ς, and *rank liquidity* μ are calculated to investigate how the rankings evolve. We have conducted simulations in a large number of initial configurations, and we find that patterns remain largely unchanged. In the following discussion, we present the results of networks with a population of size $N = 50$, $p_{er} = 0.1$, $p_{sw} = 0.1$, $\left|\Gamma_{sw}^0\right| = 4$, $l_{sf} = 4$, $0 \le \alpha \le 1$, and 100 rounds.

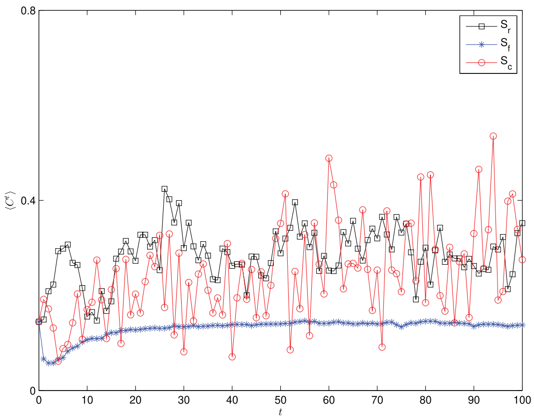
Fig. 1 shows that the average centralities $\langle C^t \rangle$ for all three strategies. In the ER network and SF network, S_r and S_c evolve similarly with significantly larger $\langle C^t \rangle$ and fluctuations compared to S_f, which remains largely stable in all rounds. In the SW network, S_r remains outstanding compared to the other two strategies, and S_c drops to a similar level to S_f.



(a) ER



(b) SW



(c) SF

Fig. 1. Average centrality $\langle C^t \rangle$ for the ER network (a), SW network (b), and SF network (c). In each network, three strategies, S_r, S_f, and S_c, are plotted. S_r and S_c evolve similarly with significantly larger $\langle C^t \rangle$ and fluctuations compared to S_f.

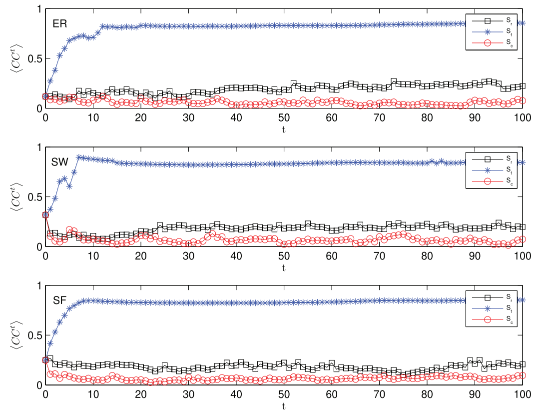
For the SF network, $\langle C^t \rangle$ increases for S_r and S_c, indicating that the networked population as a whole is improving with growing $\langle C^t \rangle$ in these cases.

Normally, connected groups form an ecosystem with better collective social welfare, such as shared resources and knowledge, than an isolated society. It will be interesting to investigate the tendency of players in a structured population to cluster together. A larger clustering coefficient $\langle CC^t \rangle$ indicates the existences of highly connected subgroups in which participants are socially connected. Fig. 2(a) shows that $\langle CC^t \rangle$ behaves similarly in different networks. Strategy S_f significantly outperforms both S_r and S_c in all networks with growing $\langle CC^t \rangle$, while it remains largely stable around the initial value for the other two strategies. This shows that participants tend to form clusters when adopting strategy S_f and the random strategy S_r, but the centrality strategy S_c does not encourage the formation of clusters. For a structured population, the edge density D^t can be seen as a measure that indicates the cooperation level [13]. A highly connected network has more edges and normally enjoys better global benefits such as resilience to risks, smaller network diameter, lower synchronization costs, and more efficient information spreading. The results show that all strategies improve D^t for all networks. As shown in Fig. 2(b), the follow strategy S_f increases the density by more than the other two strategies in all networks. In the ER network, the centrality strategy S_c outperforms the random strategy S_r, while in the SW and SF networks, the two strategies perform similarly. Furthermore, the density grows at first but becomes stable at relatively small values in the SW and SF networks. This result demonstrates that a simple follow strategy can better enhance the formation of edges to increase the density D^t and encourage cooperation. However, the other two strategies poorly encourage cooperation with random and competitive establishing or canceling decision making. In the centrality strategy S_c in particular, players in better ranking positions are less likely to establish edges to others and tend to cancel existing edges. The results hint that if the aims for players are to achieve better ranking positions rather than promote collective global cooperation, the population improves poorly. From another perspective, social networks usually exhibit a tendency in which participants with similar centrality are more likely to be connected [81]. In a society, the rich are more likely to connect with other rich people, leaving the poor to only be connected with the poor. In a rigidly stratified society, the assortativity coefficient $\langle A^t \rangle$ is large and positive. As illustrated in Fig. 2(c), for all networks, starting from neutral situations, i.e., $\langle A^t \rangle = 0$, the follow strategy S_f and random strategy S_r demonstrate slight fluctuations around the initial value. In other words, the network neither becomes assortative nor disassortative. It is interesting to observe that the centrality strategy S_f decreases $\langle A^t \rangle$ dramatically from the beginning in all networks to larger, negative values. This means the networks become much more disassortative and exhibit more opportunities for connection between the rich and the poor. In S_c, to climb the social ranking ladder, the poor have relatively better chances to connect with the rich than in S_f.

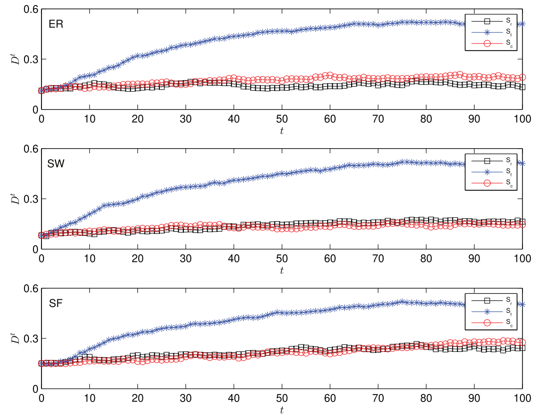
To quantify the changes in rankings with time, we introduce and calculate the values of similarity ρ, survival rate ς, and liquidity μ. As defined previously in Section 3, for two rankings at different times, similarity ρ describes how similar the rankings are to each other. In Fig. 3(a), S_f and S_r show large values of similarity ρ, while S_c has relatively small values. S_f and S_r gradually reach stable situations with ρ close to 1. The consequences of high similarities imply that stable rankings are reached. For S_c in the SF network, ρ fluctuates around zero. However, the high similarity for the overall ranking still allows individuals to change their ranking positions. To further evaluate the stability of the ranking, the ratio of unchanged players in two sets of rankings is defined as survival ratio ς. A small value of ς indicates that most players experience ranking position changes. As shown in Fig. 3(b), the values of ς fluctuate dramatically for S_f while S_r and S_c remain largely unchanged in a small range just above zero for all networks. Similar patterns are observed in all networks. This shows that the follow strategy S_f has a larger survival rate compared to the other two strategies, indicating less mobility for individuals in S_f. To investigate the mobility, we plot the curves of liquidity μ for all networks in Fig. 3(c). As shown, the centrality strategy S_c has the highest liquidity in all networks, followed by the random strategy S_r, while S_f has the lowest liquidity. The liquidity μ demonstrates larger fluctuations in the centrality strategy S_c, but remains stable in the other two strategies S_r and S_f. In the SF network, the liquidity μ increases gradually for S_c, while it drops to near zero for S_f. This again verifies that S_f leads to a less liquid ranking regarding position changes.

In a comparison of the changes of the above coefficients, the centrality strategy S_c has the least similarity ρ, smallest survival rate ς, and highest liquidity μ. Conversely, S_f has the largest similarity ρ and survival rate ς, and almost zero liquidity μ. S_r generally lies between the two. The results also indicate that the ER and SW networks show similar dynamics for all strategies, while the SF network has particular effects on S_c where the similarity ρ significantly declines to a low level and the liquidity μ increases with time. In other words, in S_f, a population in which individuals are trying to climb the social ranking ladder by blindly following the best results reinforces the social hierarchy with growing similarity ρ and survival rate ς, as well as declining liquidity μ. In this case, the overall chance for social mobility decays and the ranking becomes even more rigid. In contrast, for S_c, rational decision making to optimize one's social ranking allows the population to demonstrate better mobility, in which individuals enjoy more chances. This implies that if individuals are motivated to change their own positions in a structured population instead of following the best results, it results in better social mobility and higher chances for social-structure changes.

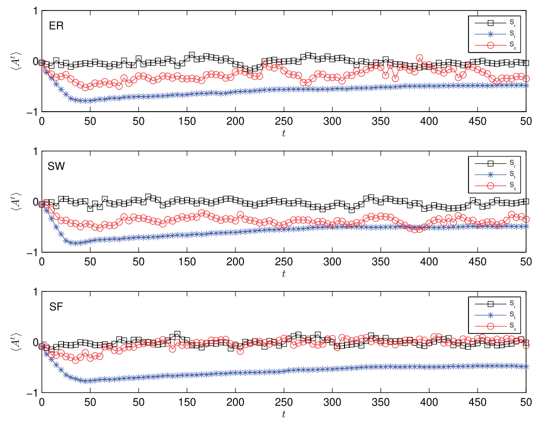
We have presented the average results for the whole population. It is also interesting to look into how individual players might change ranking positions. In Fig. 4, we present the results for both top-ranked and bottom-ranked players for the three strategies. As plotted, the rankings for the players appear more scattered for S_c and S_r, demonstrating higher mobility and randomness. In other words, the top-ranked players might quickly drop from the top-ranked positions, while the bottom-ranked players might quickly escape from the bottom positions. In both cases, regardless of their initial positions, players can appear in all possible positions in the rankings. It is surely beneficial for certain disadvantaged



(a) $\langle CC^t \rangle$

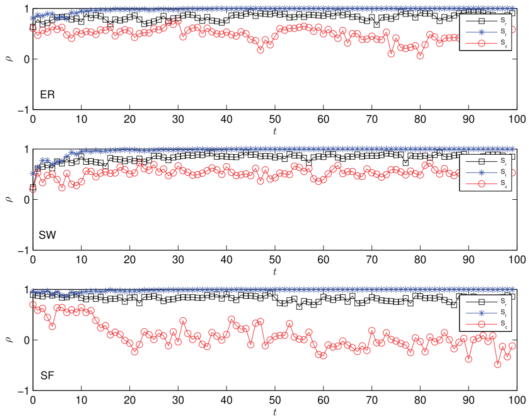


(b) D^t

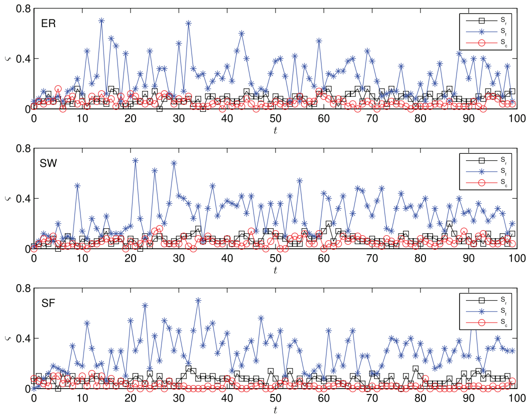


(c) $\langle A^t \rangle$

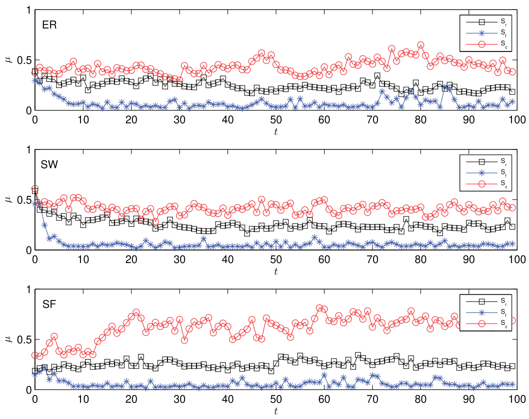
Fig. 2. Topological properties of clustering coefficient $\langle CC^t \rangle$ (a), edge density D^t (b), and degree assortativity $\langle A^t \rangle$ (c) for all strategies played on different networks. This shows that participants tend to tie in clusters when adopting strategy S_f and the random strategy S_r, but the centrality strategy S_c does not encourage the formation of clusters. The follow strategy S_f can better enhance the formation of edges to increase the density D^t and encourage cooperation. In S_r and S_c, to climb the social ranking ladder, the poor have relatively better chances to connect with the rich than in S_f.



(a) ρ

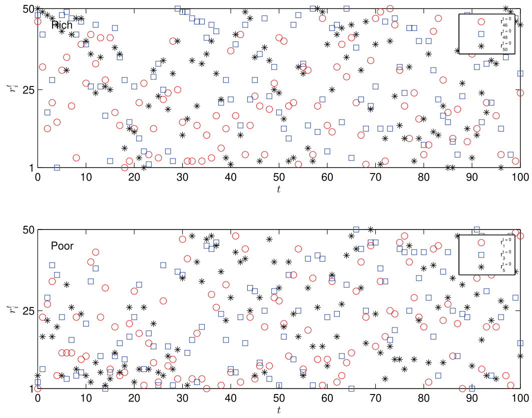


(b) ς

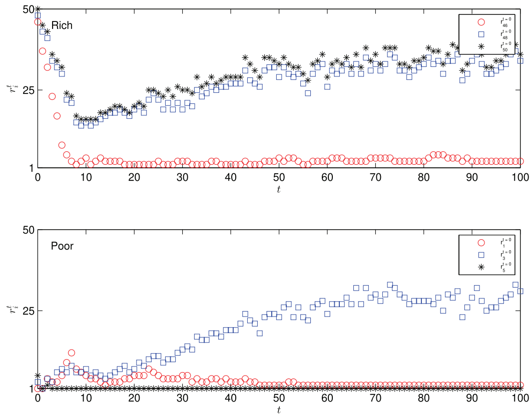


(c) μ

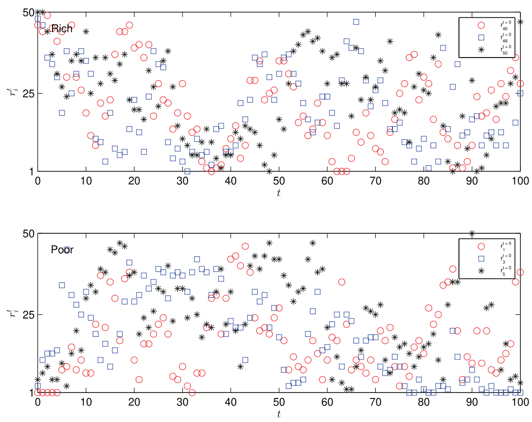
Fig. 3. Evolutions of similarity ρ (a), survival ratio ς (b), and liquidity μ (c) for all strategies in ER, SW, and SF networks. The consequences of high similarities imply that stable rankings are reached. The follow strategy S_f has a larger survival rate compared to the other two strategies, indicating poor mobility for individuals in S_f. S_c has the largest liquidity, while S_f leads to a less liquid ranking regarding position changes.



(a) S_c



(b) S_f



(c) S_r

Fig. 4. Ranking changes for top-ranked (top) and bottom-ranked (bottom) players in S_c (a), S_f (b), and S_r (c). Again, the results show that S_f makes the population less mobile than S_c and S_r.

players to have the possibility of reaching better ranking positions, and vice versa, as some top-ranked players do not always remain unchanged but slide down the social ladder. However, for the follow strategy S_f, the randomness is very limited. Top-ranked players are observed to lose their positions after the beginning and regain them slowly, or drop to the bottom permanently. For bottom-ranked players, they might remain unchanged at the bottom, but also have the chance to climb slowly. Due to the limited mobility brought by S_f, players tend to stay in a relatively stable state and either change slowly with less randomness or remain trapped in certain positions. This again shows that unlike the strategies S_r and S_c, strategy S_f makes the population less mobile, with the social hierarchy solidified with less opportunity for players to move.

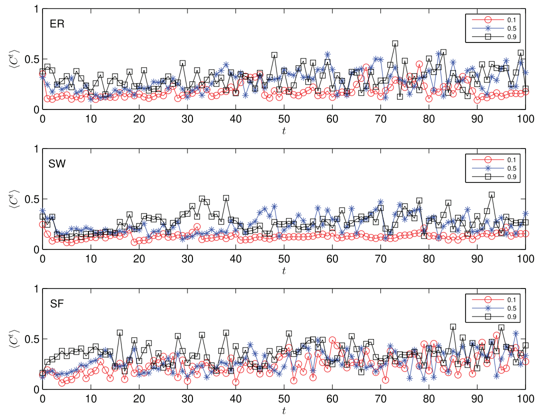
The average centrality $\langle C^t \rangle$ curves for all strategies (S_c, S_f, and S_r) are plotted in Figs. 5(a), 5(b), and 5(c), respectively. Different α values of 0.1, 0.5, and 0.9 are used. Since $c_i^t = \alpha \hat{k}_i^t + (1-\alpha)\hat{b}_i^t$, the value of α adjusts the contributions from local degree $\hat{k}_i^t$ and global betweenness $\hat{b}_i^t$. As shown in the results, curves with larger α stay above others with smaller α. This indicates that the larger the weight for local centrality $\hat{k}_i^t$, the higher centrality it has. In other words, the local degree $\hat{k}_i^t$ is the major factor in the centrality. This is more obvious for the centrality strategy S_c shown in Fig. 5(a). Additionally, as shown in the figures, $\langle C^t \rangle$ for all three strategies on all networks remains predominantly at small values.

In Fig. 6, we further investigate the influence of α values on the ranking properties of similarity ρ, survival rate ς, and liquidity μ for all situations. For combinations of networks and strategies, as α increases, all the properties of ρ also increase to relatively high values, indicating the networks are becoming jammed with higher ρ due to increasing weights on degree centrality. With small α, the ranking is less influenced by local degree centralities and is more reliant on global betweenness centralities. The results show that focusing on local centrality tends to enhance the ranking similarity. Additionally, the survival rates ς fluctuate at small values, indicating that changes happen for the individuals with only a small portion of players remaining exactly unchanged. Meanwhile, liquidity μ declines significantly in S_f, showing that the overall averaged abilities for players to change decays slowly, leading to fixed rankings, while liquidity μ remains largely unchanged for S_r and S_c in all situations.

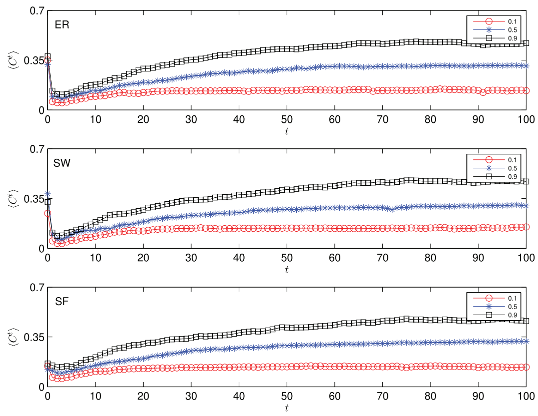
5. Conclusions

The study of spatial evolutionary games played on networks gives insight into understanding the collective behaviors of a structured population. Most literature focuses on using traditional games to explore how network structures influence the outcomes of iterative games. In those models, players aim to optimize their payoffs from games played with neighbors. The ranking information of the hierarchical population is not taken into consideration in decision making. However, in many realistic social situations, individuals are at times motivated to climb a certain social ladder with the aim of achieving a better ranking position in the population. In these scenarios, individuals optimize relative advantages among the population to occupy relatively better ranking positions. To capture this ranking nature in a structured population, we proposed a simple stylized ranking game on networks to study the relationships of behaviors and network topologies. We consider three different ranking decision making strategies for players to adopt: random, follow, and centrality strategies. In addition, random ER networks, heterogeneous SW networks, and SF networks are considered. A systematic numerical simulation is performed in various combinations. The network and ranking properties are calculated. The results indicate that for the follow strategy, networks tend to be less liquid, while the random and centrality strategies remain at a higher mobility, allowing players to change ranking positions. In summary, this research contributes to the literature of evolutionary games on networks by introducing a ranking game model. Our numerical simulations introduce new thoughts on the ranked social phenomenon and implications for regulators to reconsider the dynamics behind the ranked social ladder for a structured population.

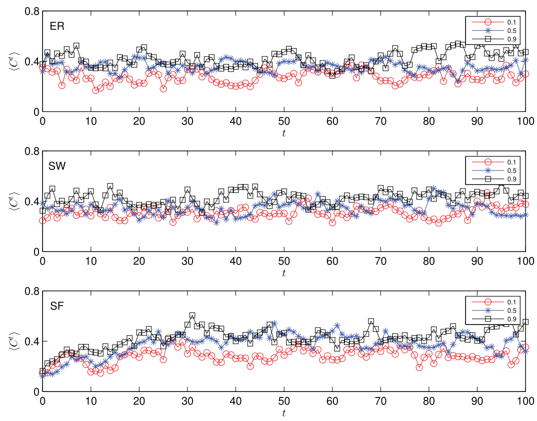
The proposed ranking game model is oversimplified in focusing on the essentials, but it serves as a starting point. Further extensions would not only be of interest, but also quite straightforward and intuitive with the introduction of more complicated considerations, such as social wealth distribution, cooperation promotion, and alliance formation. Furthermore, it is worth investigating ranking games on networks with other models of statistical physics. Spin-glasses [82] are classical models with adoptions in social network studies. For example, a player on a network with two action options or states can be treated as a spin. The dynamics of the whole network might be modeled as a spin network. This allows spin-glass models to be adopted to study certain social phenomena. One aspect we might consider introducing in further research is the phase transition [32]. For our case in particular, it is worth investigating whether a phase transition is possible for the rankings, network topologies, and critical conditions. The percolation theory has been applied in social network studies of network robustness and fragility [83], and epidemic spreading over networks [84]. Percolation theory is mainly used to study how signals, opinions, or diseases spread over a social network. With percolation theory, it might be possible to further investigate how a ranking might evolve in a social network in which dynamics are modeled as percolation. For example, given certain opinions spreading over a network, one might be interested to study how the rankings of the opinion leaders in the network might evolve. In this study, we focus on the degree and betweenness centralities. It is possible to consider other centrality measurements like eigenvector and entropy-based centralities. However, this might require a change from undirected networks to directed networks and bringing the predictions among players into consideration. This work is a theoretical evolutionary game model and is not meant to provide predictability of social norms. However, it is worth exploring the possibilities for these kinds of models to be utilized to predict certain social norms in future studies.



(a) S_c

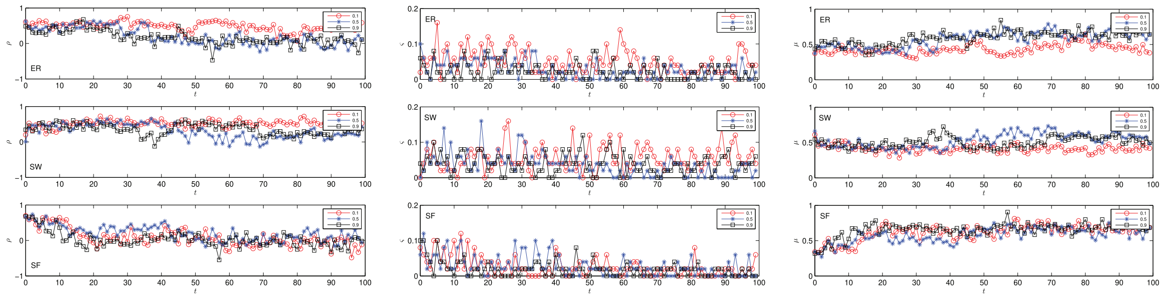


(b) S_f

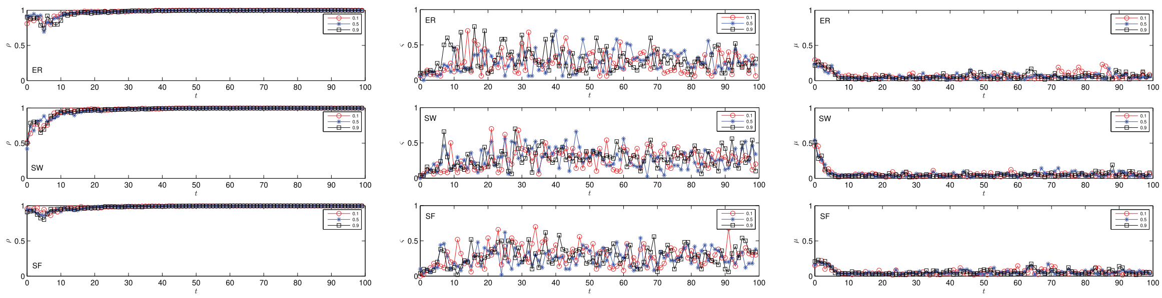


(c) S_r

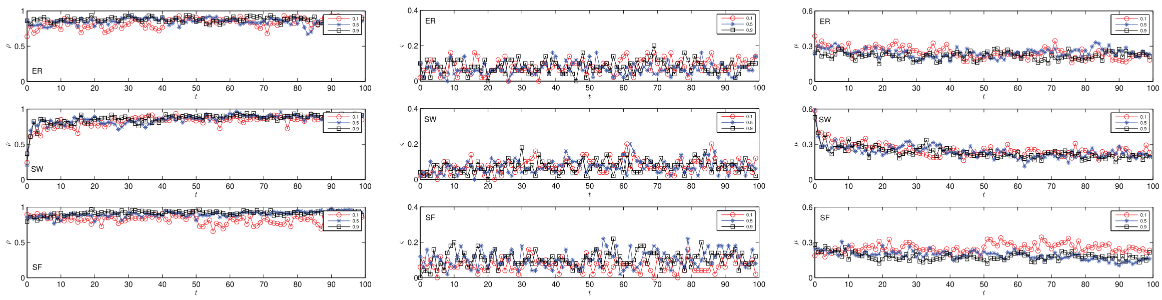
Fig. 5. Three strategies played on ER, SW, and SF networks in different α. Larger local-degree centrality weights contribute to the higher average centralities, especially for S_f.



(a) S_c (b) S_c (c) S_c



(d) S_f (e) S_f (f) S_f



(g) S_r (h) S_r (i) S_r

Fig. 6. Similarity ρ, survival rate ς, and liquidity μ for S_c (a, b, c), S_f (d, e, f), and S_r (g, h, i) played on ER, SW, and SF networks with different α. S_f again leads to poor mobility.

We hope this work can stimulate and inspire further studies in evolutionary games on networks to understand how a population and individuals behave in ranked social structures. After all, in-depth understanding of evolutionary dynamics is essential to regulate a strictly hierarchical and ranked society hoping to achieve social equality with opportunities for all.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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References

[1] V. Sekara, A. Stopczynski, S. Lehmann, Fundamental structures of dynamic social networks, Proc. Natl. Acad. Sci. 113 (36) (2016) 9977–9982, http://dx.doi.org/10.1073/pnas.1602803113.

[2] D. Banks, Dynamic network models: introduction to new and interdisciplinary approaches, Comput. Math. Organ. Theor. 15 (4) (2009) 259, http://dx.doi.org/10.1007/s10588-009-9062-6.

[3] T.W. Valente, Network models of the diffusion of innovations, Comput. Math. Organ. Theor. 2 (2) (1996) 163–164, http://dx.doi.org/10.1007/BF00240425.

[4] A. Hyatt, N. Contractor, P. Jones, Computational organizational network modeling: Strategies and an example, Comput. Math. Organ. Theor. 2 (4) (1996) 285–300, http://dx.doi.org/10.1007/BF00132313.

[5] B. Fotouhi, M.G. Rabbat, Dynamics of influence on hierarchical structures, Phys. Rev. E 88 (2013) 022105, http://dx.doi.org/10.1103/PhysRevE.88.022105.

[6] C. Xia, X. Li, Z. Wang, M. Perc, Doubly effects of information sharing on interdependent network reciprocity, New J. Phys. 20 (7) (2018) 075005, http://dx.doi.org/10.1088/1367-2630/aad140.

[7] A.-L. Barabási, R. Albert, Emergence of scaling in random networks, Science 286 (5439) (1999) 509–512, http://dx.doi.org/10.1126/science.286.5439.509.

[8] D.J. Watts, S.H. Strogatz, Collective dynamics of 'small-world' networks, Nature 393 (6684) (1998) 440–442, http://dx.doi.org/10.1038/30918.

[9] P. Doreian, Equivalence in a social network, J. Math. Sociol. 13 (3) (1988) 243–281, http://dx.doi.org/10.1080/0022250X.1988.9990034.

[10] T.L. Frantz, Garry robins: Doing social network research: network-based research design for social scientists, Comput. Math. Organ. Theor. 23 (4) (2017) 587–589, http://dx.doi.org/10.1007/s10588-016-9236-y.

[11] M. Doebeli, The evolutionary origin of cooperators and defectors, Science 306 (5697) (2004) 859–862, http://dx.doi.org/10.1126/science.1101456.

[12] P.D. Taylor, T. Day, G. Wild, Evolution of cooperation in a finite homogeneous graph, Nature 447 (7143) (2007) 469–472, http://dx.doi.org/10.1038/nature05784.

[13] N. Hanaki, A. Peterhansl, P.S. Dodds, D.J. Watts, Cooperation in evolving, Sci. Rep. Manag. Sci. 53 (7) (2007) 1036–1050, http://dx.doi.org/10.1287/mnsc.1060.0625.

[14] F. Battiston, M. Perc, V. Latora, Determinants of public cooperation in multiplex networks, New J. Phys. 19 (7) (2017) 073017, http://dx.doi.org/10.1088/1367-2630/aa6ea1.

[15] M. Perc, A. Szolnoki, Coevolutionary games—a mini review, Biosystems 99 (2) (2010) 109–125, http://dx.doi.org/10.1016/j.Biosyst.2009.10.003.

[16] Z. Wang, L. Wang, A. Szolnoki, M. Perc, Evolutionary games on multilayer networks: a colloquium, Eur. Phys. J. B 88 (5) (2015) 124, http://dx.doi.org/10.1140/epjb/e2015-60270-7.

[17] G. Yang, T.P. Benko, M. Cavaliere, J. Huang, M. Perc, Identification of influential invaders in evolutionary populations, Sci. Rep. 9 (1) (2019) 7305, http://dx.doi.org/10.1038/s41598-019-43853-9.

[18] C. Gracia-Lázaro, A. Ferrer, G. Ruiz, A. Tarancón, J.A. Cuesta, A. Sánchez, Y. Moreno, Heterogeneous networks do not promote cooperation when humans play a Prisoner's Dilemma, Proc. Natl. Acad. Sci. 109 (32) (2012) 12922–12926, http://dx.doi.org/10.1073/pnas.1206681109.

[19] B. Ranjbar-Sahraei, Theory of cooperation in complex social networks, Proc. AAAI (2014) 1471–1477.

[20] G. Szabo, C. Toke, Evolutionary prisoner's dilemma game on a square lattice, Phys. Rev. E 58 (1) (1997) 6, http://dx.doi.org/10.1103/PhysRevE.58.69.

[21] C. Luo, X. Zhang, Y. Zheng, Chaotic evolution of prisoner's dilemma game with volunteering on interdependent networks, Commun. Nonlinear Sci. Numer. Simul. 47 (2017) 407–415, http://dx.doi.org/10.1016/j.cnsns.2016.12.004.

[22] F. Fu, L.-H. Liu, L. Wang, Evolutionary prisoner's dilemma on heterogeneous newman-watts small-world network, Eur. Phys. J. B 56 (4) (2007) 367–372, http://dx.doi.org/10.1140/epjb/e2007-00124-5.

[23] A. Szolnoki, M. Perc, Z. Danku, Towards effective payoffs in the prisoner's dilemma game on scale-free networks, Physica A 387 (8) (2008) 2075–2082, http://dx.doi.org/10.1016/j.physa.2007.11.021.

[24] C. Hauert, M. Doebeli, Spatial structure often inhibits the evolution of cooperation in the snowdrift game, Nature 428 (6983) (2004) 643–646, http://dx.doi.org/10.1038/nature02360.

[25] J. Gómez-Gardeñes, M. Romance, R. Criado, D. Vilone, A. Sánchez, Evolutionary games defined at the network mesoscale: The public goods game, Chaos 21 (1) (2011) 016113, http://dx.doi.org/10.1063/1.3535579.

[26] M. Anghel, Z. Toroczkai, K.E. Bassler, G. Korniss, Competition-driven network dynamics: Emergence of a scale-free leadership structure and collective efficiency, Phys. Rev. Lett. 92 (2004) 058701, http://dx.doi.org/10.1103/PhysRevLett.92.058701.

[27] L. Shang, X.F. Wang, Evolutionary minority game on complex networks, Physica A 377 (2) (2007) 616–624, http://dx.doi.org/10.1016/j.physa.2006.11.080.

[28] X.-J. Zhang, Y. Tang, J. Xiong, W.-J. Wang, Y.-C. Zhang, Dynamics of cooperation in minority games in alliance networks, Sustainability 10 (12) http://dx.doi.org/10.3390/su10124746.

[29] M. Li, Y. Fan, J. Chen, L. Gao, Z. Di, J. Wu, Weighted networks of scientific communication: the measurement and topological role of weight, Physica A 350 (2) (2005) 643–656, http://dx.doi.org/10.1016/j.physa.2004.11.039.

[30] M. Li, J. Wu, D. Wang, T. Zhou, Z. Di, Y. Fan, Evolving model of weighted networks inspired by scientific collaboration networks, Physica A 375 (1) (2007) 355–364, http://dx.doi.org/10.1016/j.physa.2006.08.023.

[31] C. Luo, X. Zhang, Effect of self-organized interdependence between populations on the evolution of cooperation, Commun. Nonlinear Sci. Numer. Simul. 42 (2017) 73–82, http://dx.doi.org/10.1016/j.cnsns.2016.05.009.

[32] L.M. Floría, C. Gracia-Lázaro, J. Gómez-Gardeñes, Y. Moreno, Social network reciprocity as a phase transition in evolutionary cooperation, Phys. Rev. E 79 (2009) 026106, http://dx.doi.org/10.1103/PhysRevE.79.026106.

[33] S. Iyer, T. Killingback, Evolution of cooperation in social dilemmas on complex networks, PLoS Comput. Biol. 12 (2) (2016) 1–25, http://dx.doi.org/10.1371/journal.pcbi.1004779.

[34] C.-L. Tang, W.-X. Wang, X. Wu, B.-H. Wang, Effects of average degree on cooperation in networked evolutionary game, Eur. Phys. J. B 53 (3) (2006) 411–415, http://dx.doi.org/10.1140/epjb/e2006-00395-2.

[35] H.-X. Yang, Z. Wang, Promoting cooperation by reputation-driven group formation, J. Stat. Mech. Theory Exp. 2017 (2) (2017) 023403, http://dx.doi.org/10.1088/1742-5468/aa569f.

[36] F. Fu, X. Chen, L. Liu, L. Wang, Social dilemmas in an online social network: The structure and evolution of cooperation, Phys. Lett. A 371 (1) (2007) 58–64, http://dx.doi.org/10.1016/j.physleta.2007.05.116.

[37] M. Bardoscia, G. De Luca, G. Livan, M. Marsili, C.J. Tessone, The social climbing game, J. Stat. Phys. 151 (3–4) (2013) 440–457, http://dx.doi.org/10.1007/s10955-013-0693-0.

[38] T.L. Frantz, M. Cataldo, K.M. Carley, Robustness of centrality measures under uncertainty: Examining the role of network topology, Comput. Math. Organ. Theor. 15 (4) (2009) 303, http://dx.doi.org/10.1007/s10588-009-9063-5.

[39] T.L. Frantz, K.M. Carley, Reporting a network's most-central actor with a confidence level, Comput. Math. Organ. Theor. 23 (2) (2017) 301–312, http://dx.doi.org/10.1007/s10588-016-9229-x.
[40] M.D. Koenig, C.J. Tessone, Network evolution based on centrality, Phys. Rev. E 84 (5) (2011) 056108, http://dx.doi.org/10.1103/PhysRevE.84.056108.
[41] L.C. Freeman, Centrality in social networks conceptual clarification, Soc. Netw. 1 (3) (1978) 215–239, http://dx.doi.org/10.1016/0378-8733(78)90021-7.
[42] B. Buechel, V. Buskens, The dynamics of closeness and betweenness, J. Math. Sociol. 37 (3) (2013) 159–191, http://dx.doi.org/10.1080/0022250X.2011.597011.
[43] A. Chatterjee, A. Ghosh, J. ichi Inoue, B.K. Chakrabarti, Social inequality: from data to statistical physics modeling, J. Phys.: Conf. Ser. 638 (2015) 012014, http://dx.doi.org/10.1088/1742-6596/638/1/012014.
[44] K. Ishida, T. Ohta, On a mathematical comparison between hierarchy and network with a classification of coordination structures, Comput. Math. Organ. Theor. 7 (4) (2001) 311–330, http://dx.doi.org/10.1023/A:1013454200524.
[45] A. Fortune, The other side of the network coin: Cost considerations of network structure, Comput. Math. Organ. Theor. 9 (2) (2003) 109–125, http://dx.doi.org/10.1023/B:CMOT.0000022751.56620.99.
[46] A. Nishi, H. Shirado, D.G. Rand, N.A. Christakis, Inequality and visibility of wealth in experimental social networks, Nature 526 (7573) (2015) 426–429, http://dx.doi.org/10.1038/nature15392.
[47] H. Ebel, S. Bornholdt, Coevolutionary games on networks, Phys. Rev. E 66 (5) (2002) 056118, http://dx.doi.org/10.1103/PhysRevE.66.056118.
[48] J. Poncela, J. Gómez-Gardeñes, L.M. Floría, A. Sánchez, Y. Moreno, Complex cooperative networks from evolutionary preferential attachment, PLoS One 3 (6) (2008) e2449, http://dx.doi.org/10.1371/journal.pone.0002449.
[49] M. Nakamaru, H. Matsuda, Y. Iwasa, The evolution of cooperation in a lattice-structured population, J. Theoret. Biol. 184 (1) (1997) 65–81, http://dx.doi.org/10.1006/jtbi.1996.0243.
[50] P. Doreian, On the connectivity of social networks, J. Math. Sociol. 3 (2) (1974) 245–258, http://dx.doi.org/10.1080/0022250X.1974.9989837.
[51] D. Brenner, A. Dellnitz, F. Kulmann, W. Rödder, Compressing strongly connected subgroups in social networks: An entropy-based approach, J. Math. Sociol. 41 (2) (2017) 84–103, http://dx.doi.org/10.1080/0022250X.2017.1284070.
[52] M. van Veelen, J. Garcia, D.G. Rand, M.A. Nowak, Direct reciprocity in structured populations, Proc. Natl. Acad. Sci. 109 (25) (2012) 9929–9934, http://dx.doi.org/10.1073/pnas.1206694109.
[53] Z. Deng, S. Wang, Z. Gu, J. Xu, Q. Song, Heterogeneous preference selection promotes cooperation in spatial prisoners' dilemma game, Chaos Solitons Fractals 100 (Supplement C) (2017) 20–23, http://dx.doi.org/10.1016/j.chaos.2017.04.031.
[54] Y. Ma, J. Lu, L. Shi, Diversity of neighborhoods promotes cooperation in evolutionary social dilemmas, Physica A 468 (2017) 212–218, http://dx.doi.org/10.1016/j.physa.2016.11.010.
[55] H. Ohtsuki, C. Hauert, E. Lieberman, M.A. Nowak, A simple rule for the evolution of cooperation on graphs and social networks, Nature 441 (7092) (2006) 502–505, http://dx.doi.org/10.1038/nature04605.
[56] J. Gómez-Gardeñes, M. Campillo, L.M. Floría, Y. Moreno, Dynamical organization of cooperation in complex topologies, Phys. Rev. Lett. 98 (10) (2007) 108103, http://dx.doi.org/10.1103/PhysRevLett.98.108103.
[57] C.P. Roca, A. Sánchez, J.A. Cuesta, Individual strategy update and emergence of cooperation in social networks, J. Math. Sociol. 36 (1) (2012) 1–21, http://dx.doi.org/10.1080/0022250X.2010.520830.
[58] A.J. Stewart, J.B. Plotkin, Collapse of cooperation in evolving games, Proc. Natl. Acad. Sci. 111 (49) (2014) 17558–17563, http://dx.doi.org/10.1073/pnas.1408618111.
[59] C. Shen, X. Li, Z. Deng, L. Shi, Asymmetric evaluation promotes cooperation in network population, Physica A 474 (2017) 391–397, http://dx.doi.org/10.1016/j.physa.2017.02.005.
[60] D.G. Rand, S. Arbesman, N.A. Christakis, Dynamic social networks promote cooperation in experiments with humans, Proc. Natl. Acad. Sci. 108 (48) (2011) 19193–19198, http://dx.doi.org/10.1073/pnas.1108243108.
[61] H. Shirado, N.A. Christakis, Locally noisy autonomous agents improve global human coordination in network experiments, Nature 545 (7654) (2017) 370–374, http://dx.doi.org/10.1038/nature22332.
[62] E. Gallo, C. Yan, The effects of reputational and social knowledge on cooperation, Proc. Natl. Acad. Sci. 112 (12) (2015) 3647–3652, http://dx.doi.org/10.1073/pnas.1415883112.
[63] A. Traulsen, D. Semmann, R.D. Sommerfeld, H.-J. Krambeck, M. Milinski, Human strategy updating in evolutionary games, Proc. Natl. Acad. Sci. 107 (7) (2010) 2962–2966, http://dx.doi.org/10.1073/pnas.0912515107.
[64] J. Tanimoto, How does resolution of strategy affect network reciprocity in spatial prisoner's dilemma games? Appl. Math. Comput. 301 (Supplement C) (2017) 36–42, http://dx.doi.org/10.1016/j.amc.2016.11.036.
[65] A.J. Stewart, T.L. Parsons, J.B. Plotkin, Evolutionary consequences of behavioral diversity, Proc. Natl. Acad. Sci. 113 (45) (2016) E7003–E7009, http://dx.doi.org/10.1073/pnas.1608990113.
[66] S.J. Cranmer, B.A. Desmarais, J.H. Kirkland, Toward a network theory of alliance formation, Int. Interact. 38 (3) (2012) 295–324, http://dx.doi.org/10.1080/03050629.2012.677741.
[67] L. Rosenkopf, G. Padula, Investigating the microstructure of network evolution: Alliance formation in the mobile communications industry, Organ. Sci. 19 (5) (2008) 669–687, http://dx.doi.org/10.1287/orsc.1070.0339.
[68] J.A.C. Baum, R. Cowan, N. Jonard, Network-independent partner selection and the evolution of innovation networks, Manag. Sci. 56 (11) (2010) 2094–2110, http://dx.doi.org/10.1287/mnsc.1100.1229.
[69] C.H. Wang, C.C. Lo, H.M. Liang, P.Y. Chien, H.M. Chen, S.Y. Lo, Inter-firm alliances networks in nanotechnology: The roles of network position and technological uncertainty, in: 2013 Proceedings of PICMET '13: Technology Management in the IT-Driven Services, PICMET, 2013, pp. 259–269.
[70] I. Castro, C. Casanueva, J.L. Galán, Dynamic evolution of alliance portfolios, Eur. Manage. J. 32 (3) (2014) 423–433, http://dx.doi.org/10.1016/j.emj.2013.06.006.
[71] D. Lavie, Alliance portfolios and firm performance: A study of value creation and appropriation in the U.S. software industry, Strateg. Manage. J. 28 (12) (2007) 1187–1212, http://dx.doi.org/10.1002/smj.637.
[72] B.R. Koka, J.E. Prescott, Designing alliance networks: the influence of network position, environmental change, and strategy on firm performance, Strateg. Manage. J. 29 (6) (2008) 639–661, http://dx.doi.org/10.1002/smj.679.
[73] M. Tomassini, E. Pestelacci, L. Luthi, Social dilemmas and cooperation in complex networks, Internat. J. Modern Phys. C 18 (07) (2006) 1–13, http://dx.doi.org/10.1142/S0129183107011212.
[74] V. Buskens, A. van de Rijt, Dynamics of networks if everyone strives for structural holes, Am. J. Sociol. 114 (2) (2008) 371–407, http://dx.doi.org/10.1086/590674.
[75] M.O. Jackson, A. Wolinsky, A strategic model of social and economic networks, J. Econ. Theory 71 (1) (1996) 44–74, http://dx.doi.org/10.1006/jeth.1996.0108.
[76] S. Bekiros, D.K. Nguyen, L.S. Junior, G.S. Uddin, Information diffusion cluster formation and entropy-based network dynamics in equity and commodity markets, European J. Oper. Res. 256 (3) (2017) 945–961, http://dx.doi.org/10.1016/j.ejor.2016.06.052.

[77] A.G. Nikolaev, R. Razib, A. Kucheriya, On efficient use of entropy centrality for social network analysis and community detection, Soc. Netw. 40 (2015) 154–162, http://dx.doi.org/10.1016/j.socnet.2014.10.002.
[78] F. Tutzauer, Entropy as a measure of centrality in networks characterized by path-transfer flow, Soc. Netw. 29 (2) (2007) 249–265, http://dx.doi.org/10.1016/j.socnet.2006.10.001, special Section: Advances in Exponential Random Graph (p*) Models.
[79] A. Flache, M.W. Macy, Small worlds and cultural polarization, J. Math. Sociol. 35 (1–3) (2011) 146–176, http://dx.doi.org/10.1080/0022250X.2010.532261.
[80] B. Albert-László, Linked: The New Science of Networks, Perseus, 2002.
[81] M.E.J. Newman, Assortative mixing in networks, Phys. Rev. Lett. 89 (2002) 208701, http://dx.doi.org/10.1103/PhysRevLett.89.208701.
[82] M. Mézard, G. Parisi, M. Virasoro, Spin Glass Theory and beyond: An Introduction to the Replica Method and Its Applications, Vol. 9, World Scientific, Publishing Company, 1987.
[83] D.S. Callaway, M.E.J. Newman, S.H. Strogatz, D.J. Watts, Network robustness and fragility: Percolation on random graphs, Phys. Rev. Lett. 85 (2000) 5468–5471, http://dx.doi.org/10.1103/PhysRevLett.85.5468.
[84] S.-W. Son, G. Bizhani, C. Christensen, P. Grassberger, M. Paczuski, Percolation theory on interdependent networks based on epidemic spreading, Europhys. Lett. 97 (1) (2012) 16006, http://dx.doi.org/10.1209/0295-5075/97/16006.