Topological Effects of a Vorticity Filament on the Coherent Backscattering Cone
Supplemental Material

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The Supplemental Material contains information on the fitting procedure of the phase shifts of Fig. 2, and on the numerical simulations justifying the use of the Poisson distribution in Eq. (6).

CONTENTS

I. PHASE SHIFT FITS OF FIG. 2

In order to fit the phase shifts of Fig. 2, we first convert the positions x on the transducer array into the polar coordinates (r, θ) used in Eq. (1), as defined in the inset of Fig. 1,

$$\theta(x) = \pi - \arctan \frac{x}{d_0}, \tag{S1}$$

$$\rho(x) = \frac{x}{\sin\left(\pi - \theta(x)\right)}. \tag{S2}$$

The experimental data are then fitted by the following formula

$$\phi(x) = \psi_\alpha\left(\rho(x + x_0), \theta(x + x_0)\right) + \phi_0 \tag{S3}$$

where ψ_α is defined by Eq. (1) with the Aharonov-Bohm parameter α, and two other fit parameters, x_0 that takes into account that the vorticity filament is not directly facing the center of the transducer array, and ϕ_0 that account for a small phase shift adjustment between the two arrays. In Eq. (1), $k = \frac{2\pi f}{c}$ with $c = 1500\,\mathrm{m/s}$ in water, and the sum was performed between $m = -2000$ and $m = 2000$. The Bessel function of the first kind $J_{|m-\alpha|}$ in Eq. (1) was calculated using the SciPy wrapper [1] of the AMOS `zbesj` routine [2]. On the curves of Fig. 2, the two other fit parameters ranges are $-0.8\,\mathrm{mm} < x_0 < 0.1\,\mathrm{mm}$ and $0.01\,\mathrm{rad} < \phi_0 < 0.04\,\mathrm{rad}$. Let us mention that fixing these two additional parameters to zero do not change quantitatively the values of the extracted α. Moreover, we also checked that taking into account the scattering by a finite size vorticity filament [Eq. (2.126) of ref. [3]] provide very similar α values, and very small vorticity filament core size compared to the ultrasonic wavelength.

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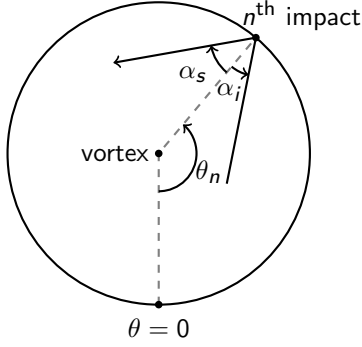† philippe.roux@univ-grenoble-alpes.fr

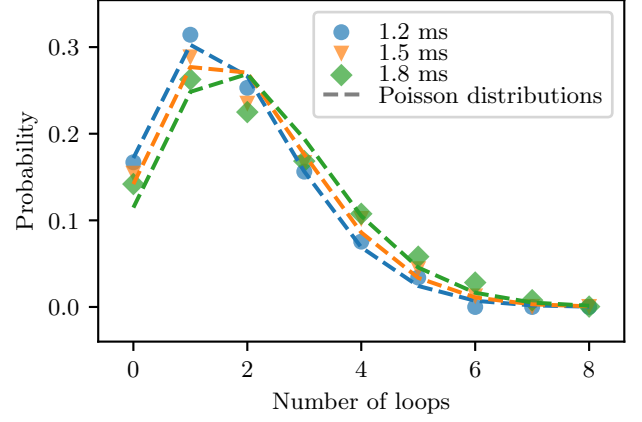Figure S1. Geometry of the numerical simulations

Figure S2. Statistics of the number of loops done by the random walks described in the text for different random walk lengths (symbols) and the corresponding Poisson distributions (dashed lines, the colors encode the path lengths).

II. NUMERICAL SIMULATIONS OF THE NUMBER OF LOOPS IN THE CAVITY

In order to get an estimation of the number of loops done by each closed paths in the cavity, we perform an angular random walk in polar coordinates. The geometry is shown in Fig. S1. The random walks starts at the bottom point $(\theta = 0)$, with a random angle, and travels on straight lines between successive scattering events. A scattering event happens each time the random walk crosses the circle. Let us call α_i the angle with respect to the normal at which the path impacts the circle, the path then continues with an angle θ_s which is chosen on a normal law centered on $\alpha_s = \alpha_i$ (specular reflection) with a certain width (standard deviation σ) accounting for the roughness of the surface. We continue the random walk until the total travel time of the walk is equal to the measured ones (1.2 ms, 1.5 ms or 1.8 ms; radius of the cavity, 7.25 cm; sound velocity, 1500 m/s). We are only interested in the statistics of the backscattered paths, which mean that we only keep the paths which end at $\theta = 0$ after the total travel time. During the random walks, we log the number of loops done by the path around the vortex. Fig. S2 shows an example of the statistics of the number of loops done by 50,000 random walks ending at $\theta = 0$, when $\sigma = 40°$. The dashed lines correspond to the Poisson distributions calculated using the average number of loops extracted from the statistics. The good agreement between the simulated statistics and the corresponding Poisson distributions justifies Eq. (6).

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