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On star and biclique edge-colorings

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Abstract

A biclique of G is a maximal set of vertices that induces a complete bipartite subgraph $K_{p,q}$ of G with at least one edge, and a star of a graph G is a maximal set of vertices that induces a complete bipartite graph $K_{1,q}$. A biclique (resp. star) edge-coloring is a coloring of the edges of a graph with no monochromatic bicliques (resp. stars). We prove that the problem of determining whether a graph G has a biclique (resp. star) edge-coloring using two colors is NP-hard. Furthermore, we describe polynomial time algorithms for the problem in restricted classes: K_3-free graphs, chordal bipartite graphs, powers of paths, and powers of cycles.

Keywords: star edge-coloring; biclique edge-coloring; NP-hard

1. Introduction

Bicliques have been studied in several contexts such as telecommunications (Gualandi et al., 2013; Faure et al., 2014) and bioinformatics (Zhang et al., 2014). In the graph theory, the biclique vertex-coloring problem was proposed by Groshaus et al. (2014) and it is attracting much attention in recent works (Macêdo Filho et al., 2012, 2015). In the present work, we address the edge-coloring version of the biclique coloring problem, that is, we investigate how to color the edges of a graph in such a way that no biclique is monochromatic. We also address a variant of the problem in which the stars cannot be monochromatic, the so-called star edge-coloring problem. We prove that the problem of determining whether a graph has a biclique (resp. star) edge-coloring using two colors is NP-hard. Observe that the NP-hardness of biclique (resp. star) edge-coloring using two colors does not imply that the problem remains NP-hard if three or more colors are allowed. Indeed, for the analogous problem of clique vertex-coloring, the classical NP-hardness result of Kratochvíl and Tuza (2002) refers to clique vertex-coloring using precisely two colors; only almost a decade later, Marx (2011) proved the $\Sigma_2 P$-completeness of clique k-vertex-coloring for any $k \geq 2$. The NP-hardness of biclique (resp. star) edge-coloring using two colors motivates the study of these two problems in restricted graph classes. The present work proposes some techniques that allow us to obtain biclique edge-colorings and star edge-colorings of graphs in the following classes: K_3-free graphs, chordal bipartite graphs, powers of paths, and powers of cycles. Coloring problems have been largely studied in these classes, see, for example, Cerioli and Posner (2012), Dabrowski et al. (2012), Campos and de Mello (2007), Luiz et al. (2015), and Macêdo Filho et al. (2015).

Let $G = (V, E)$ be a simple graph with order $n = |V|$ vertices and $m = |E|$ edges. A *biclique* of G is a maximal set of vertices that induces a complete bipartite subgraph $K_{p,q}$ of G with at least one edge; and a *star* of a graph G is a maximal set of vertices that induces a complete bipartite graph $K_{1,q}$ of G. A *biclique edge-coloring* of G is a function C'_b that associates a color to each edge of G such that no biclique with at least two edges is monochromatic. If the function C'_b uses at most c colors, we say that C'_b is a *biclique c-edge-coloring*. The *biclique chromatic index* of G is the least c for which G has a biclique c-edge-coloring. Similarly, a *star edge-coloring* of G is a function C'_s that associates a color to each edge of G such that no star with at least two edges is monochromatic. If the function C'_s uses at most c colors, we say that C'_s is a *star c-edge-coloring*. The *star chromatic index* of G is the least c for which G has a star c-edge-coloring.

The paper is organized as follows. In Section 2, we prove that the BICLIQUE 2-EDGE-COLORING and the STAR 2-EDGE-COLORING problems are NP-hard. We observe that the construction of the particular instance in this last proof has a polynomial amount of bicliques, so the STAR 2-EDGE-COLORING problem is NP-complete for the class of graphs in which each vertex belongs to at most one K_3, have degree at most 3, and are C_4-free, case in which we can define the precise complexity of the problems. In Section 3, we investigate the problems in the class of K_3-free graphs and determine its star chromatic index in polynomial time. Finally, in Section 4, we construct biclique edge-colorings and star edge-colorings for chordal bipartite graphs, powers of cycles, and powers of paths.

2. Star and biclique 2-edge-colorings are NP-hard

In this section, we prove that both BICLIQUE 2-EDGE-COLORING (2-BEC) and STAR 2-EDGE-COLORING (2-SEC) problems are NP-hard by reducing the NP-hard problem NOT-ALL-EQUAL 3-SATISFIABILITY (Schaefer, 1978) to 2-BEC problem (Fig. 1).

These two decision problems are defined as follows:

NOT-ALL-EQUAL 3-SATISFIABILITY (NAE 3-SAT). Instance: Set $X = \{x_1, \ldots, x_n\}$ of Boolean variables, collection $C = \{c_1, \ldots, c_m\}$ of clauses over X such that each clause $c_i \in C$ has $|c_i| = 3$.
Question: Is there a truth assignment for X such that each clause in C has at least one true literal and at least one false literal?

BICLIQUE 2-EDGE-COLORING (2-BEC). Instance: Graph $G = (V, E)$.
Question: Does G admits a 2-BEC?

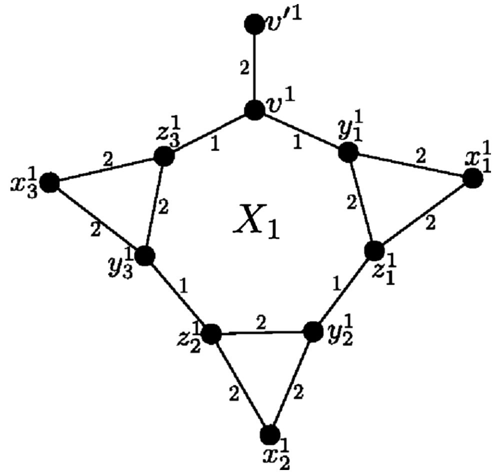

Fig. 1. Gadget X_1 corresponding to a variable x_1 that occurs in exactly three clauses of C with a unique BICLIQUE 2-EDGE-COLORING determined by color 2 in $v^1v'^1$.

Theorem 1. *The 2-BEC problem is NP-hard.*

Proof. In order to reduce the NOT-ALL-EQUAL 3-SATISFIABILITY to the 2-BEC problem, we need to construct, in polynomial time, a particular instance $G = (V, E)$ of 2-BEC problem from a generic instance (X, C) of NOT-ALL-EQUAL 3-SATISFIABILITY, such that C is satisfiable if, and only if, $G = (V, E)$ admits a biclique 2-edge-coloring. First, we construct a particular instance $G = (V, E)$ of 2-BEC described next; second, we prove that every graph G that admits a biclique 2-edge-coloring defines a not-all-equal truth assignment for (X, C) (Lemma 1); third, we prove that every not-all-equal truth assignment for (X, C) defines biclique 2-edge-coloring for graph G (Lemma 2). □

Construction of particular instance. Let (X, C) be a generic instance of NAE 3-SAT such that $X = \{x_1, \dots, x_n\}$ is the variable set and $C = \{c_1, \dots, c_m\}$ is a collection of clauses, where $c^j = (l_1^j, l_2^j, l_3^j)$ and $|c_j| = 3$.

For each variable x_i, we have a gadget X_i such that $V(X_i) = \bigcup_{t=1}^{k}\{y_t^i, z_t^i, x_t^i\} \cup \{v^i, v'^i\}$ and $E(X_i) = \bigcup_{t=1}^{k-1}\{y_t^i z_t^i, z_t^i x_t^i, x_t^i y_t^i, z_t^i y_{t+1}^i\} \cup \{y_k^i z_k^i, z_k^i x_k^i, x_k^i y_k^i, z_k^i v^i, v^i v'^i, v^i y_1^1\}$, where k is the number of occurrences of literal corresponding to x_i or $\overline{x_i}$ in C. We note that for each $1 \le i \le n$, the number of vertices of X_i is $3k + 2$ (e.g., see Fig. 2).

For each clause $c^j = (l_1^j, l_2^j, l_3^j)$ we have one *clause vertex* c_j. For each $1 \le j \le m$ and $d \in \{1, 2, 3\}$, if l_d^j is equal to variable x_i then we have edge $c_j x_t^i$, where t is one of the k vertices x_t^i in X_i with no edge to some clause vertex c. Otherwise, if l_d^j is equal to $\bar{x}_i$ then we add vertex $x_t'^i$ and edges $\{x_t^i x_t'^i, c_j x_t'^i\}$, again t is one of the k vertices x_t^i in X_i with no edge to some clause vertex c. Note that the constructed graph is C_4-free, so that any biclique edge-coloring is a star edge-coloring and vice versa.

Lemma 1. *If the particular instance $G = (V, E)$ of 2-BEC admits a biclique 2-edge-coloring, then there exists an NAE truth assignment that satisfies (X, C).*

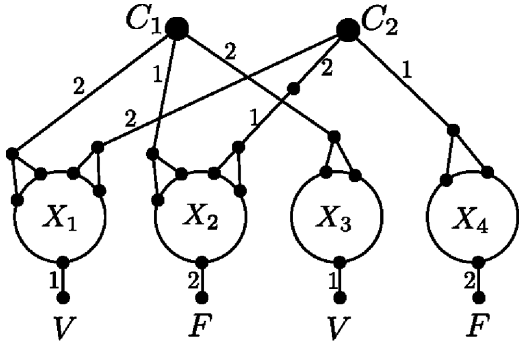

Fig. 2. Instance $G = (V, E)$ of 2-BEC obtained from the satisfiable instance of 3-SAT: $I = (X; C) = (\{x_1, x_2, x_3, x_4\}, \{(x_1 \vee x_2 \vee x_3), (x_1 \vee \bar{x}_2 \vee x_4)\})$.

Proof. The truth assignment is defined based on the following property (we refer to Fig. 2): Each gadget X_i contains a unique hole H_i, which is odd. Moreover, X_i has the property that all P_3s are bicliques, except for $\{v^i, y_1^i, z_k^i\}$, which is included in the biclique $\{v^i, v'^i, y_1^i, z_k^i\}$. It follows that if in some biclique 2-edge-coloring (with colors 1,2) the edge $v^i y_1^i$ has color $\lambda \in \{1, 2\}$, then every second edge along the hole H_i must have color λ too, up to edge $v^i z_k^i$, and consequently $v^i v'^i$ must have color $3 - \lambda$; moreover, all other edges of X_i must have color $3 - \lambda$.

Assume that G admits a biclique 2-edge-coloring. Define a truth assignment as follows: the value of variable x_i is `True` if the edge $v^i v'^i$ is colored 1 and is `False` if the edge $v^i v'^i$ is colored 2. The truth assignment is valid because each clause is the center of a star that is a biclique, hence not all incident edges have the same color and, equivalently, not all literals have the same truth value. □

The converse of Lemma 1 is given next by Lemma 2.

Lemma 2. *If there exists a not-all-equal truth assignment that satisfies (X, C), then the particular instance $G = (V, E)$ of 2-*BEC *admits a biclique 2-edge-coloring.*

Proof. Assume that (X, C) has a not-all-equal truth assignment. We color each pendant edge incident to X_i with color 1 if x_i is `True`; and with color 2 if x_i is `False`. Thus, we extend the coloring to each X_i as in Fig. 2. Finally, we color each edge incident to x_i^j with the color distinct from the color of the pendant edge incident to X_j, and each remaining edge—corresponding to negative literals—receives the unique available color. The coloring is valid because it corresponds to a not-all-equal truth assignment, so that not all colors incident to a clause are equal, and the stars centered in the clause vertices are bicolored. □

Since the particular instance of 2-BEC does not contain C_4s, we obtain the following result.

Corollary 1. *The 2-*SEC *problem is NP-hard.*

Furthermore, since a graph G resulting from the construction above is such that each vertex belongs to at most one K_3, G is C_4-free and has degree at most 3; we obtain one more result.

Corollary 2. *The 2-*BEC *and 2-*SEC *problems are NP-complete for the class of graphs G such that each vertex belongs to at most one K_3, G is C_4-free and has degree at most 3.*

3. On the star chromatic index of K_3-free graphs

In this section, we determine the star chromatic index of odd cycles and of all connected K_3-free graphs in polynomial time. The first result is an immediate consequence of the well-known result of edge-coloring of cycles.

Theorem 2. *If G is a chordless cycle C_n with odd $n \geq 5$ vertices, then the star chromatic index of G is equal to 3. Furthermore, if G is a connected K_3-free graph that is not isomorphic to C_n, for odd $n \geq 5$, then the star chromatic index of G is equal to 2.*

Proof. Since G is K_3-free, every vertex with all its neighbors form an induced star. If G has a vertex with degree 1 (leaf), construct a depth-first search tree starting on a leaf of G. If G has no leaf, then G has a cycle, and let $C = (v_1, v_2, \ldots, v_k, v_1)$ be an induced cycle of G such that v_k has a neighbor not in C (note that such a cycle always exists). Construct a depth-first search tree on G with v_1 as a root and choosing vertices $v_2, v_3, \ldots, v_k$ in that order.

Note that if there are return edges to the root (there is no leaf in G to be used as a root), then there is at least one return edge that is not from a leaf—the return edge $v_k v_1$.

Color the tree edges from level i to level $i+1$ with color $(i \bmod 2) + 1$. If the root vertex is a leaf, then its star has just one edge. If the root vertex has degree greater than 1, then choose a return edge leaving from a vertex that is not a leaf and color it with color 2.

For each leaf f with return edges, choose a return edge to color with a color different from the tree edge arriving in f. If f has no return edge, the star of f has just one edge. Therefore, every star has two colors or has only one edge. □

4. On the biclique chromatic index of some graphs

In this section, we determine the biclique chromatic index of all chordal bipartite graphs and an upper bound for the biclique chromatic index of powers of cycles and powers of paths.

A graph is *chordal bipartite* if it is bipartite and each cycle of length at least 6 has a chord. The proof of the next result is based on the property that these graphs have a bisimplicial elimination ordering. Note that if an edge uv belongs to two distinct bicliques, then $N(u) \cup N(v)$ cannot be a biclique. Hence, a bisimplicial edge of the graph belongs to precisely one biclique. The coloring is obtained by an algorithm based on induction of the number of edges.

Theorem 3. *Every chordal bipartite graph has biclique chromatic index 2.*

Proof. Let $G = (V, E)$ be a chordal bipartite graph. We may assume G connected. If G has two edges, color one edge with color 1 and the other edge with color 2. Now, assume that G has more than two edges and let uv be a bisimplicial edge, such that $N(u) \cup N(v)$ induces a complete bipartite graph.

Note that the bicliques of $G \setminus uv$ are the same as the bicliques of G, except for $G[N(u) \cup N(v)]$, which is a biclique of G that is not a biclique of $G \setminus uv$. By induction, graph $G \setminus uv$ has a biclique 2-edge-coloring, that is, an association of colors to the edges is in such a way that every biclique is 2-colored. We construct a biclique 2-edge-coloring of G as follows. First, color the edges of $E \setminus \{uv\}$ as in a biclique 2-edge-coloring of $G \setminus uv$ using colors 1 and 2. If this coloring results in

$G[N(u) \cup N(v)]$ monochromatic with color 1, then color uv with color 2; otherwise color uv with color 1. $\square$

A *power of a cycle* C_n^k, for $n, k \geq 1$ is a simple graph on n vertices with $V(G) = \{v_0, \ldots, v_{n-1}\}$ and $\{v_i, v_j\} \in E(G)$ if, and only if, $\min\{(j-i) \bmod n, (i-j) \bmod n\} \leq k$. Note that C_n^1 is the induced cycle C_n, and C_n^k with $n \leq 2k+1$ is the complete graph K_n. In a power of a cycle C_n^k, we take $(v_0, \ldots, v_{n-1})$ to be a cyclic order on the vertex set and we always perform arithmetic modulo n on vertex indices. A *power of a path* P_n^k, for $k \geq 1$, is a simple graph on n vertices with $V(G) = \{v_0, \ldots, v_{n-1}\}$ and $\{v_i, v_j\} \in E(G)$ if, and only if, $|i-j| \leq k$. Note that P_n^1 is the induced path P_n, and P_n^k with $n \leq k+1$ is the complete graph K_n. In a power of a path P_n^k, we take $(v_0, \ldots, v_{n-1})$ to be a linear order on the vertex set.

In the following, we obtain an upper bound for the biclique chromatic index of both classes by analyzing the cases according to the number of vertices of the graph, as described next.

The bicliques of a power of a path P_n^k, $n > k+1$, are precisely (Macêdo Filho et al., 2015):

- K_2 and P_3 bicliques, if $k+2 \leq n \leq 2k$; and
- P_3 bicliques, if $n \geq 2k+1$.

The bicliques of a power of a cycle C_n^k, $n > 2k+1$, are precisely (Macêdo Filho et al., 2015):

- C_4 bicliques, if $2k+2 \leq n \leq 3k+1$;
- P_3 and C_4 bicliques, if $3k+2 \leq n \leq 4k$; and
- P_3 bicliques, if $n \geq 4k+1$.

Theorem 4. *Every noncomplete power of a cycle has a biclique edge-coloring using at most four colors.*

Proof. Let $G = C_n^k$ be a power of a cycle with $n \geq 2k+2$. We show how to color G in such a way that no induced P_3 is monochromatic, hence, no biclique of G is monochromatic.

First define $\lceil n/k \rceil$ sets of vertices, each set having k consecutive vertices of G, as follows:

- $B_1 = \{v_1, v_2, \ldots, v_k\}$
- $B_2 = \{v_{k+1}, v_{k+2}, \ldots, v_{k+k}\}$
- $\ldots$
- $B_i = \{v_{(i-1)k+1}, v_{(i-1)k+2}, \ldots, v_{(i-1)k+k}\}$
- $\ldots$
- $B_{\lfloor n/k \rfloor} = \{v_{(\lfloor n/k \rfloor-1)k+1}, v_{(\lfloor n/k \rfloor-1)k+2}, \ldots, v_{(\lfloor n/k \rfloor-1)k+k}\}$
- if $n \neq 0 \bmod k$, that is, if $\lceil n/k \rceil = \lfloor n/k \rfloor + 1$, then $B_{\lfloor n/k \rfloor+1} = V(G) \setminus (B_1 \cup B_2 \cup \ldots \cup B_{\lfloor n/k \rfloor})$ (note that this vertex set has size less than k).

Consider an auxiliary graph G_B with vertices $b_1, \ldots, b_{\lceil n/k \rceil}$ corresponding, respectively, to the blocks $B_1, \ldots, B_{\lceil n/k \rceil}$ of G, in a such way that two vertices, b_i and b_j, are adjacent in G_B, if there exists an edge in G from a vertex of B_i to a vertex of B_j. If vertex $b_{\lfloor n/k \rfloor+1}$ exists, then G_B is composed by a cycle $b_1, \ldots, b_{\lfloor n/k \rfloor+1}$ having vertex $b_{\lfloor n/k \rfloor}$ adjacent to both $b_{\lfloor n/k \rfloor+1}$ and b_1. Note that if $n = 0 \bmod k$, then G_B is a cycle, and so it has maximum degree 2. If $n \neq 0 \bmod k$, then G_B has maximum degree 3. In what follows, we present how to construct a biclique 4-edge-coloring of G from a 4-total-coloring of G_B. A k-total-coloring of a graph G is an assignment of k colors to the elements (vertices and edges) of a graph, such that adjacent or incident elements have different colors, and a k-total-coloring of a graph with maximum degree Δ, uses at least $\Delta + 1$ colors.

First, we prove that G_B has a 4-total-coloring. If $n \leq 3k$ then G_B is a C_3, which has a 4-total-coloring by coloring elements b_1, b_1b_2, b_2, b_2b_3, b_3, b_3b_1 with colors 1, 2, 3, 1, 4, 3, respectively. Hence, we consider the case $n > 3k$. If $n = 0 \bmod k$, then G_B is a cycle $b_1, b_2, \ldots, b_{\lfloor n/k \rfloor}$ and so it is easily 4-total-colorable. If $n \neq 0 \bmod k$, then we color elements $b_{\lfloor n/k \rfloor+1}b_{\lfloor n/k \rfloor}$, $b_{\lfloor n/k \rfloor}$, $b_{\lfloor n/k \rfloor}b_1$, b_1, b_1b_2 with colors 1, 2, 3, 4, 1, and extend the total coloring to the remaining elements using four colors.

We need some additional notations. An edge of G whose endvertices belong to the same block B_i is called an *internal-B_i* edge. An edge of G whose endvertices belong to distinct blocks B_i and B_j is called an *external-B_iB_j* edge.

Now, we construct a biclique 4-edge-coloring of G from a 4-total-coloring of G_B: For each block B_i of G, each internal-B_i edge of G receives the same color as b_i received in the 4-total-coloring of G_B; for each block B_i of G, each external-B_iB_j edge of G receives the same color as b_ib_j received in the 4-total-coloring of G_B.

It remains to prove that there is no edge-monochromatic P_3 in G. Consider a set $\{v_1, v_2, v_3\}$ of vertices that induces a P_3 such that v_1v_2 and v_2v_3 are edges of G and v_1v_3 is a nonedge of G. Note that v_1 and v_3 are not in the same block, for otherwise, they would be adjacent. There are three possible cases.

1. Vertices v_1, v_2, and v_3 belong to distinct blocks B_i, B_j, and B_k, respectively, of G—which implies that b_i and b_j, resp. b_j and b_k, are adjacent in G_B. In this case, edges v_1v_2 and v_2v_3 receive, resp., the color of b_ib_j and b_jb_k in G_B, which are distinct because b_ib_j and b_jb_k are adjacent edges of G_B.
2. Vertices v_1 and v_2 belong to the same block B_i—which implies that v_3 belongs to a block B_j that is adjacent to B_i. In this case, edges v_1v_2 and v_2v_3 receive, resp., the color of b_i and b_ib_j in G_B, which are distinct because b_ib_j is incident to b_i in G_B.
3. Vertices v_2 and v_3 belong to the same block B_i. This case is analogous to the previous case. □

The above result provides an upper bound for the biclique chromatic index of powers of cycles. It is important to note that this upper bound is tight. Indeed, we could find a power of a cycle[1] whose biclique chromatic index is 4, namely, C_{56}^{10}. In addition, there exist powers of cycles with biclique chromatic index equal to 2 and 3. An example of a power of a cycle with biclique chromatic index equal to 2 is any graph C_n^k with $2k + 2 \leq n \leq 3k + 1$. A valid coloring is constructed by defining set B_1 with $n - k$ consecutive vertices and set B_2 with the remaining k consecutive vertices. In B_1, we color the edges, having both endvertices, with color 1 and the remaining edges with color 2 (we invite the reader to check that each C_4 contains at least one vertex from each color class and all bicliques are C_4s).

An example of a power of a cycle with biclique chromatic index equal to 3 is the graph C_9^2 with vertices $\{v_0, v_1, v_2 \ldots, v_8\}$. In fact, suppose that there exists a biclique 2-edge-coloring of C_9^2. Without loss of generality, start by assigning color 1 to edge v_0v_1, and so both edges v_1v_3 and v_0v_7 must be colored with color 2. This implies that edges v_5v_7 and v_3v_5 must have color 1, but

[1]We run the `smallk` vertex coloring software (Culberson, 2000) over the graph H whose vertices are the edges of C_{56}^{10}, such that two vertices are adjacent if the corresponding edges of C_{56}^{10} are the edges of an induced P_3. The conversion is valid because all bicliques of C_{56}^{10} are P_3. The software could not find any valid 3-coloring of H.

in this case, the biclique $v_3v_5v_7$ would be monochromatic, which is a contradiction. A biclique 3-edge-coloring is obtained by coloring edges v_0v_1, v_5v_6, v_1v_8, v_0v_2, v_4v_6, v_5v_7 with color 1, edges v_2v_3, v_7v_8, v_1v_3, v_2v_4, v_6v_8, v_0v_7 with color 2, and edges v_1v_2, v_3v_4, v_4v_5, v_6v_7, v_0v_8, v_3v_5 with color 3.

Theorem 5. *Every noncomplete power of path has a biclique edge-coloring using at most four colors.*

Proof. Every noncomplete power of a path G is an induced subgraph of a power of a cycle H. By the proof of Theorem 4, H has a 4-coloring of its edges in such a way that no induced P_3 is monochromatic. We claim that the restriction of this coloring to the edges of G is a biclique 4-edge-coloring of G, because each induced P_3 of G is an induced P_3 of H—therefore, it cannot be monochromatic by the coloring given to H. □

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