Appendix

Table 1: Summary of Selected Literature

Research domain/topics	Representative Literature
Complex network science	Refs. [2, 75] pioneered the complex network science with evidence and modeling of small-world and scale-free networks. Ref. [53] provides a detailed survey of this emerging research which has now been applied in many domains.
Financial network analysis	Financial markets are typical complexity systems. Recent years have seen a growing trend of applying complex network science to models and studying the topological structures of financial markets. This financial network analysis approach has been applied in financial practices like portfolio management, risk management, and trading strategies design, etc. Using the network analysis, the interbank market [10, 40], investor networks in market [58], global economies [45], money market [41], market stability [55] has been studied revealing stylized properties and models.
Global stock market studies	Refs. [1, 66] invested the global sentiment impacts on the performances of global markets. The applications of investor sentiment of global markets are also discussed.
Granger, lead/lag, and PageRank centrality	The Granger causality test indicates the existence of predictive information among variables [34]. Recently, this approach has been seen in modeling financial markets to study the interrelationships among assets [72]. The lead/lag effects in high

	frequency data are investigated in [39, 44]. Refs. [12, 21] provide more evidence of lead/lag in global markets. These empirical findings reveal the stylized lead/lag effects exist in various markets and implicate possible applications in trading, for example, possible trading based on the lead/lag between index and derivatives like futures. PageRank algorithm is first proposed to quantitatively evaluate the importance of network vertices [59]. To compare with other metrics of centrality like degree, between-ness, etc., in the similar spirit, the alternative of eigenvector centrality, PageRank centrality is widely used to calculate the importance of vertices in [9, 53].

Granger Causality and Data

Granger Causality

The stocks in the market not only fluctuate correlatively but also influence with each other. This applies to the global financial market as well, where different stock markets have significant impacts with each other. Based on the correlation matrix generated from the price return series of a set of stocks, correlation of two stocks s_i and s_j, can be defined as

$$\rho_{ij} = \frac{\langle Y_i Y_j \rangle - \langle Y_i \rangle \langle Y_j \rangle}{\sqrt{(\langle Y_i^2 \rangle - \langle Y_i \rangle^2)(\langle Y_j^2 \rangle - \langle Y_j \rangle^2)}} . \ (1)$$

Eq. 1 shows how the two series Y_i and Y_j of stock S_i and S_j co-move with each other. Since $-1 \leq \rho_{ij} \leq 1$, the two series can move in the same direction or the opposite based on the sign of ρ_{ij}. It is thus possible to evaluate the collective behaviors of a pair of stocks or a set of stocks in a given portfolio.

However, the shortcomings of correlation-based approaches are apparent. The primary issue is that the correlations do not give statistical information of the causality between stock pairs. Due to this, the correlation approach lacks the ability in describing the lead/lag behaviors of stocks. Correlation information only reveals the pattern of movements of two series but fails to explain the causal relationships. In the real world, two phenomena, especially those happening along time, often have specific cause/effect relationship. For example, A has an influence over B or A contributes certain causality to the happening or effects of B.

A set of stocks in a given portfolio are not only co-moving with each other, but also have mutual influence with each other. Some stocks can cause other stocks to change. Unfortunately, the correlation method fails to explain. Thus, new measurements beyond the correlations to provide causality information between pairs with statistical meanings are needed.

To describe the aspect of causalities between events, the *Granger causality test* was introduced by Granger [34]. The hypothesis test is setup in evaluating the predictive ability between variables in a context of linear regressions. There are two time series $x_t, x_{t-1}, \dots, x_0$, and $y_t, y_{t-1}, \dots, y_0$ over a time period of $t = 0, 1, \dots, T$. A linear regression can be set up as:

$$y_t = \sum_{i=1}^{q} \alpha_i x_{t-i} + \sum_{j=1}^{q} \beta_j y_{t-j} + u_t^1 , (2)$$

and similarly,

$$x_t = \sum_{i=1}^{s} \gamma_i x_{t-i} + \sum_{j=1}^{s} \lambda_j y_{t-j} + u_t^2, (3)$$

where u_t^1 and u_t^2 are independent white noises. These regressions take the previous behaviors of both x_t and y_t into consideration to predict the present values.

The null hypothesis can be set up as:

$$H_0: \alpha_1 = \alpha_2 = \cdots = \alpha_q = 0 \text{ for Eq. 2, and } \lambda_1 = \lambda_2 = \cdots = \lambda_s = 0 \text{ for Eq. 3.}$$

The alternative hypothesis can be set up as:

$$H_a: Not\ all\ of \alpha_1 \dots\dots \alpha_q = 0 \text{ for Eq. 2, and } not\ all\ of\ \lambda_1 \dots\dots \lambda_s = 0 \text{ for Eq. 3.}$$

The statistical philosophy behind the Granger test is that if the lagged values of x together with its values of y is included, it enables us to get a better prediction of the future values of y than without the help of lagged x values, then x Granger-causes y. Otherwise, if the lagged values of x fail to contribute in the prediction of future y, then x does not Granger-cause y. It is also the same if y Granger causes x.

To conduct hypothesis testing, the F-test will be utilized. Taking the two H_0 for both Eq. 2 and Eq. 3 all together, there are four scenarios:

1. x_t unidirectional Granger causes y_t, i.e. $\alpha_i \neq 0$ and $\lambda_j = 0$;

2. y_t unidirectional Granger causes x_t, i.e. $\lambda_j \neq 0$ and $\alpha_j = 0$;

3. x_t and y_t bidirectional cause each other, i.e. $\alpha_i \neq 0$ and $\lambda_j \neq 0$;

4. x_t and y_t are independent, i.e. $\alpha_j = 0$ and $\lambda_j = 0$.

ADF and Unit Root Test

Before the Granger causality test is conducted, it is important to make sure the time series are stationary. This is done by conducting a unit root test. The *Dickey-Fuller* test [65] is usually applied to test the unit root; an extension is further developed as *Augmented Dickey-Fuller (ADF)* test. In ADF, a t-statistic can be compared with critical values on different levels of statistical significances. If the t-statistic is larger, then we do not reject the null hypothesis that a unit root exists. In this case, the time series is not stationary. Otherwise, we reject the null hypothesis and believe that the time series is stationary and fits to conduct ADF. For stock returns, an event can bring certain impact into the fluctuations to the series, but this impact will decay allowing the return back to its mean.

Granger causality test has become a standard tool in the study of the causality relationships for pairs. With these pairs of causality information, a causality network for a set of variables can be constructed. Given the time lag nature of Granger test, it has become widely used in economics and finance studies [24].

Data Setting Introduction

The causalities among financial series is discussed in this section. Based on these tests, the Granger-causality networks are built, and the properties of these networks are investigated.

The daily close data for 33 major stock market indices around the world in the period of 04/01/2007 to 06/11/2015 with 2307 total trading dates from Yahoo Finance was collected in this research. Since each stock market has its trading calendar with different holidays and closed dates, the missing dates were replaced with the next available data. There are discussions of the time-zone effect in study of global indices because of the different closing time for each market. However, we concern the closing prices in an extended period and treat all indices in the same way. This approach focuses on the closing prices without special procession of adjusting. For each index, the log return is applied as:

$$Y_i = ln\, P_i(t) - ln\, P_i(t-1), \qquad (4)$$

where $P_i(t)$ is the closing price for index i at time t. In Table 2, all 33 indices utilized in this study are listed. It represents major stock markets, including 4 from the US, 12 from Europe countries, 11 from Asian countries, 5 American countries and one Middle Eastern country. In Fig. 1, log daily returns for all indices over the study period is plotted. The returns demonstrate fluctuation but the means are around zero. ADF tests over all return time series were conducted. The average t-statistic is larger than all 6 critical values (in absolute value terms) at significance levels of 1%, 5%, 10%, 90%, 95%, and 99%. In fact, t-statistic ranges from -20.4871 to -17.5458, suggesting all return series are stationary. This aligns with the visualizations, that all return series move around the mean of zero. With this stationary background, the Granger causality test can be carried out later.

We first use all logged return data over the whole study period to construct the correlation matrix C_{ij}. This approach is widely adopted in financial network analysis to convert price or return time series data into correlation matrices [13, 53, 54]. We plot the probability density function (PDF) to show the distribution of C_{ij} of all indices calculated from the data over the whole study period in Fig. 2. The average correlation $\rho_{ij} = 0.4269$ with standard deviation $\sigma_{ij} = 0.1963$. Since the average correlation is positive, all indices co-move together. We also observe that a maximum $max(\rho_{ij}) = 0.9858$ for S&P500 and NYSE, $min(\rho_{ij}) = 0.0337$

for NASDAQ and NZ50 (New Zealand). Based on the correlation information, the complete graph of the indices network in Fig. 3 provides the backbone of the full network. Furthermore, the minimum spanning tree MST is provided in Fig. 4, where markets of the same regions are clearly identified.

Table 2: Tick names, markets, countries and regions of 33 major stock markets indices around the world.

Tick	Name	Country	Region
GSPC	S&P 500	US	America
DJI	Dow Jones Industrial Average	US	America
IXIC	NASDAQ Composite	US	America
NYA	NYSE Composite	US	America
FTSE	FTSE 100	UK	Europe
GDAXI	DAX	Germany	Europe
FCHI	CAC 40	France	Europe
ATX	ATX	Austria	Europe
GD.AT	Athen Index Compos	Greece	Europe
IBEX	IBEX 35	Spain	Europe
FTSEMIB.MI	FTSE MIB	Italy	Europe
SMI	Swiss Market Index	Switzerland	Europe
OMXC20	OMX Copenhagen 20	Denmark	Europe
AEX	Amsterdam Exchange index	Netherlands	Europe
BEL-20	EURONEXT BEL-20	Belgium	Europe
BIST 100	XU100	Turkey	Europe
N225	Nikkei 225	Japan	Asia
HSI	Hang Seng Index	Hong Kong	Asia
SSE	SSE Composite Index	China	Asia
STI	STI Index	Singapore	Asia
AORD	ALL ORDINARIES	Australian	Asia
BSESN	S&P BSE SENSEX	India	Asia
JKSE	Jakarta Composite Index	Indonesia	Asia
KLSE	FTSE Bursa Malaysia KLCI	Malaysia	Asia
NZ50	S&P/NZX 50 Index Gross	New Zealand	Asia
KS11	KOSPI Composite Index	Korea	Asia
TWII	TSEC weighted index	Taiwan	Asia
GSPTSE	S&P/TSX Composite index	Canada	Americas
BVSP	IBOVESPA	Brazil	Americas
MXX	IPC	Mexico	Americas
IPSA	IPSA Santiago de Chile	Chile	Americas
MERV	MerVal Buenos Aires	Argentina	Americas
TA100	TEL Aviv TA-100 IND	Israel	Middle East

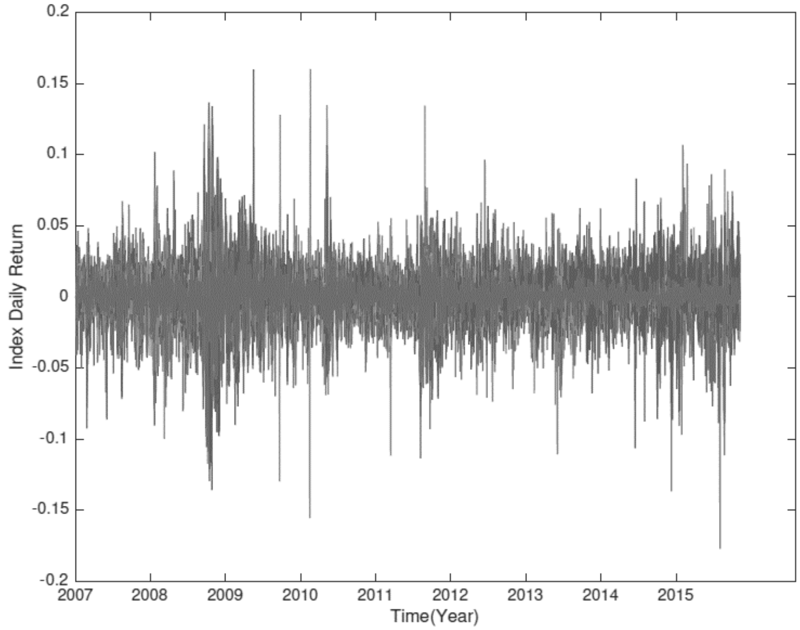

Figure 1: The daily log returns of 33 major market indices around the world. It shows the returns volatiles over our study period between 04/01/2007 and 06/11/2015 covering a total of 2307 trading dates. They are all stationary with means around zero.

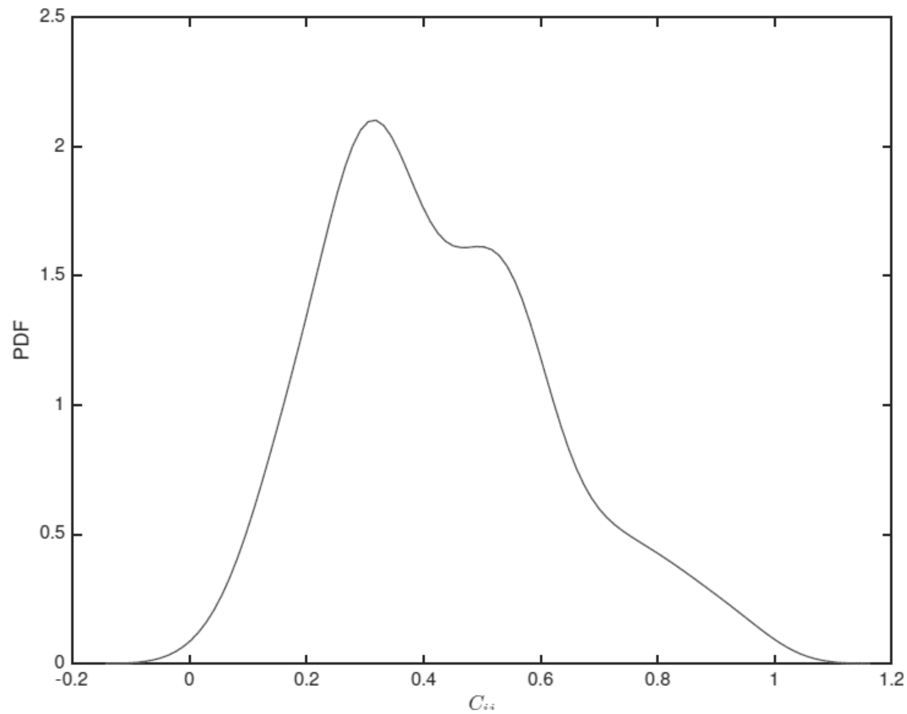

Figure 2: The probability density function (PDF) of correlations of all indices over the whole study period. The distribution falls on the right side of zero indicating that the indices are positively correlated. In other words, they are influencing each other.

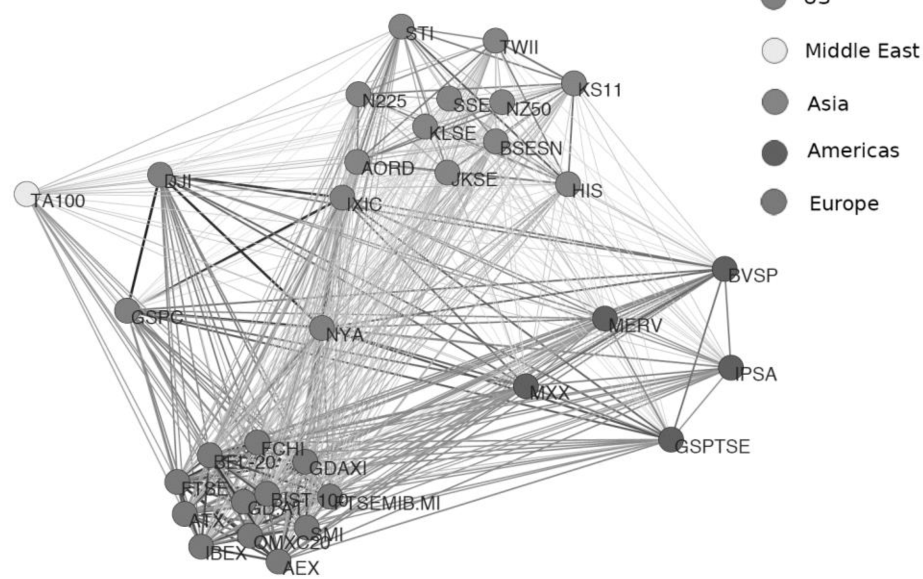

Figure 3: The correlation based network of 33 indices calculated from the daily return data over the whole study period.

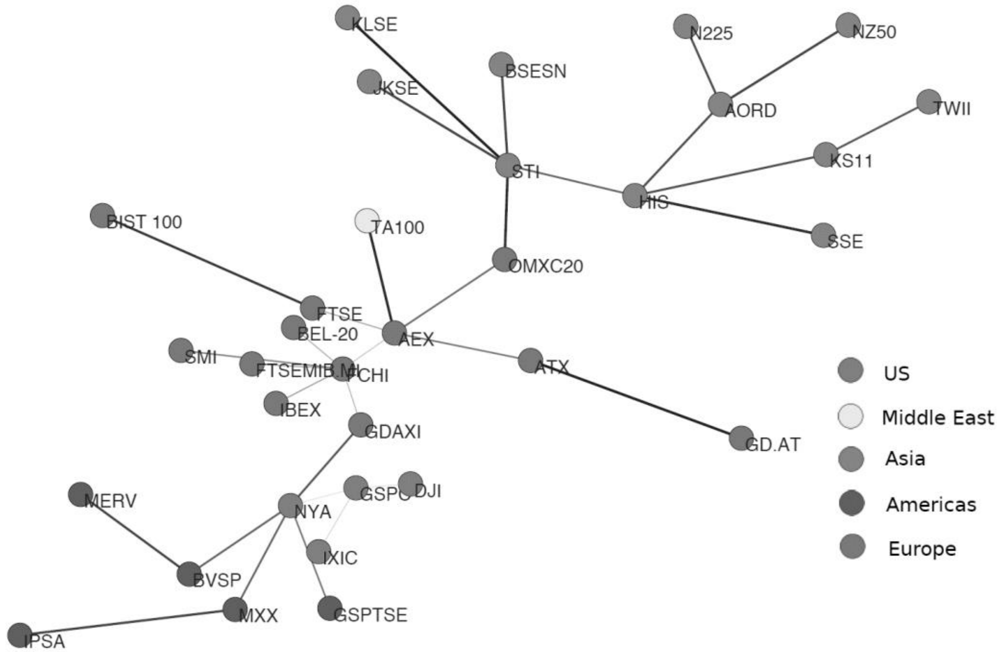

Figure 4: The minimum spanning tree *MST* extracted from the full network of 33 indices calculated from the daily return data over the whole study period. We see the indices are clustered together according to the regions.