

Figure S1: **Prediction error for partial least squares analysis (PLS)**. The root mean squared prediction error (RMSEP) is shown for parameters N_e and s as a function of increasing number of PLS components. We performed PLS using Wright-Fisher simulations of 100 loci where we assumed uniform priors for N_e and s ($U[0.5, 4.5]$ and $U[0, 1]$, respectively).

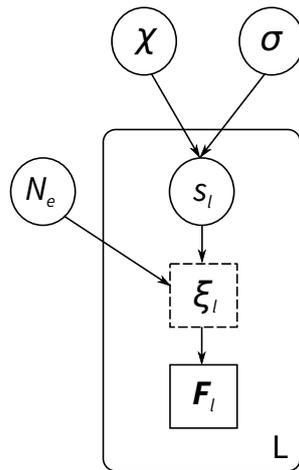


Figure S2: **Directed acyclic graph describing the Wright-Fisher model examined in this study.** Solid circles represent parameters to be estimated. The dashed square represents the full data, which is summarized here by a vector of statistics F_l , indicated by a solid square. Nodes contained in the plate are repeated for each locus $l \in \{1, \dots, L\}$ times.

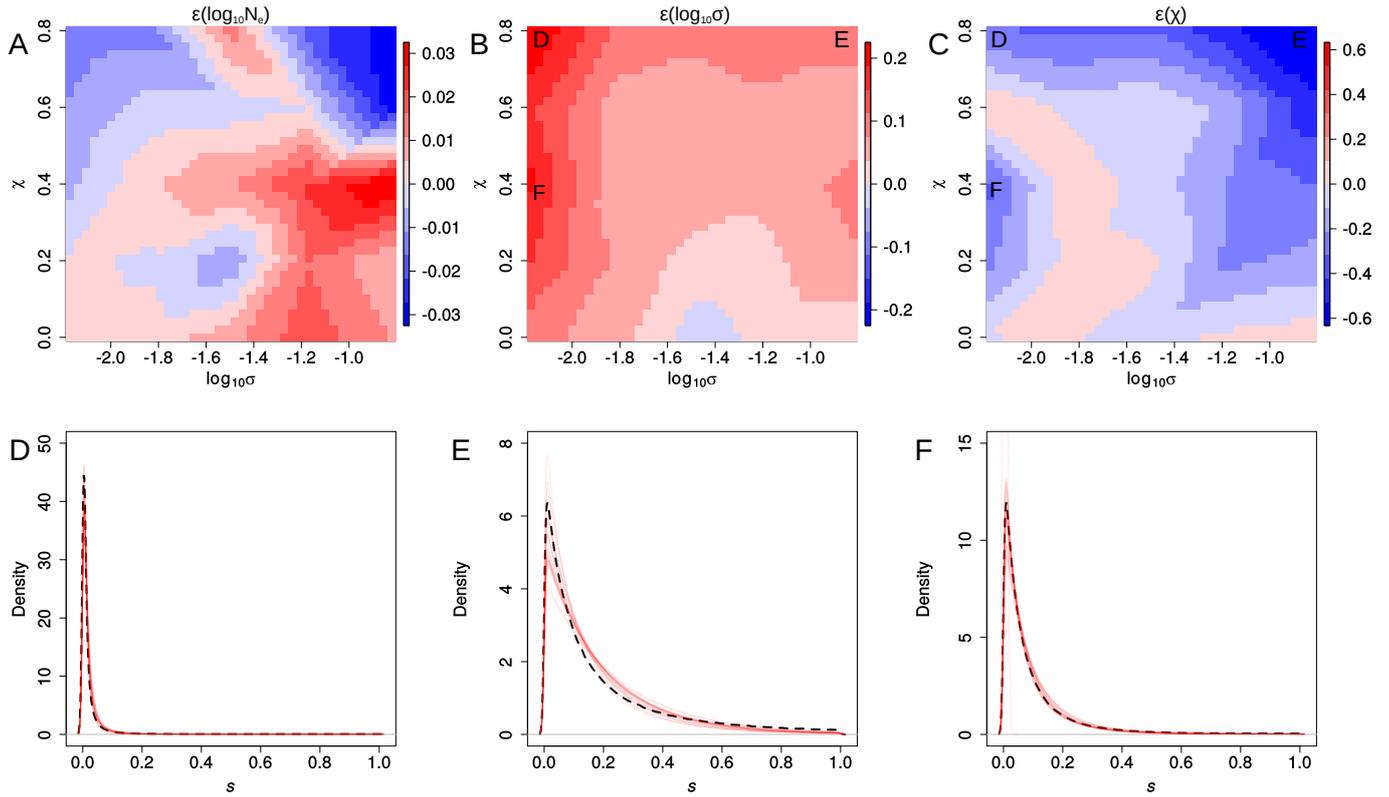


Figure S3: **Accuracy in estimating N_e and DFE parameters σ and χ jointly.** (A,B,C) Sets of simulations of 100 loci were conducted for combinations of parameters σ and χ over a grid from their prior range and we evaluated the median approximation error ($\epsilon = \text{estimate} - \text{true}$) over 25 replicates. Color gradients indicate the extent of overestimation (red) or underestimation (blue) of each parameter. These results suggest very high accuracy when estimating N_e with maximum $\epsilon \approx 0.04$ or 1% of the prior range and rather low for σ (about 10% of the prior range). In contrast, ϵ is rather large for χ , spanning up to 75% of the prior range. This is due to several combinations of χ and σ leading to very similar shapes of the truncated GPD. This is illustrated in panels D, E and F, where we show the true (dashed black line) versus estimated (red) DFE obtained for 50 replicates using parameter combinations of χ and σ as indicated in panels B and C.

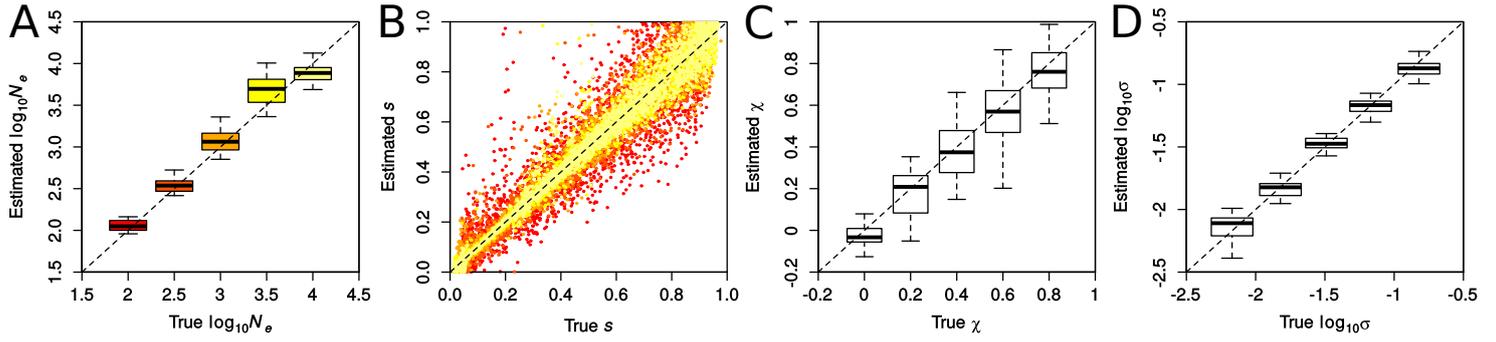


Figure S4: **Accuracy in inferring demographic and selection parameters using the PLS 5/2 set of statistics.** Results were obtained with ABC-PaSS using five and two PLS components for N_e and each s , respectively (PLS 5/2). Shown are the true versus estimated posterior medians for parameters N_e (A), s per locus (B), χ and σ of the Generalized Pareto distribution (C and D, respectively). Boxplots summarize results from 25 replicate simulations, each with 100 loci. Uniform priors over the whole ranges shown were used. (A, B): N_e assumed in the simulations is represented as a color gradient of red (low N_e) to yellow (high N_e). (C,D): Parameters μ and N_e were fixed to 0 and 10^3 , respectively, $\log_{10}\sigma$ was fixed to -1 (C) and χ was fixed to 0.5 (D).

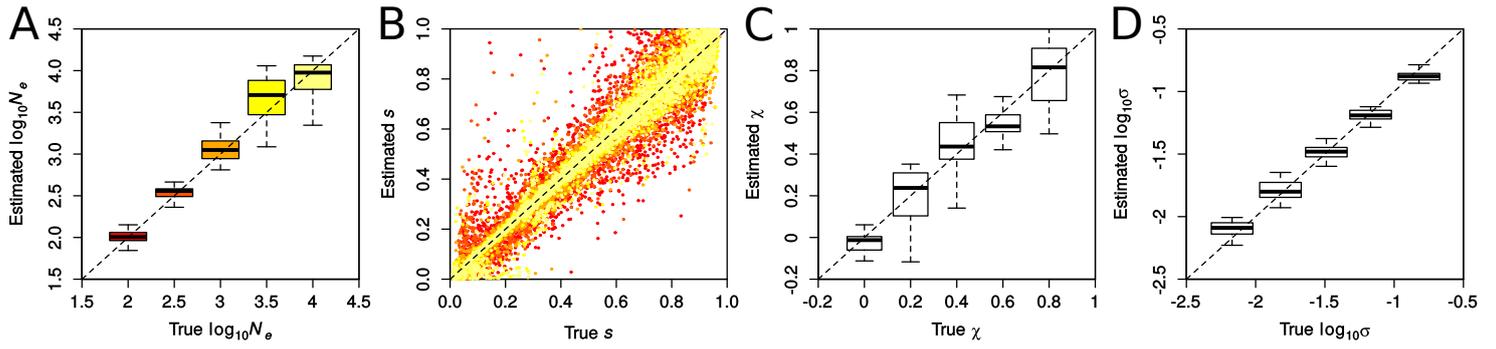


Figure S5: **Accuracy in inferring demographic and selection parameters using the PLS 3/1 set of statistics.** Results were obtained with ABC-PaSS using three and one PLS components for N_e and each s , respectively (PLS 5/2). Shown are the true versus estimated posterior medians for parameters N_e (A), s per locus (B), χ and σ of the Generalized Pareto distribution (C and D, respectively). Boxplots summarize results from 25 replicate simulations, each with 100 loci. Uniform priors over the whole ranges shown were used. (A, B): N_e assumed in the simulations is represented as a color gradient of red (low N_e) to yellow (high N_e). (C,D): Parameters μ and N_e were fixed to 0 and 10^3 , respectively, $\log_{10}\sigma$ was fixed to -1 (C) and χ was fixed to 0.5 (D).

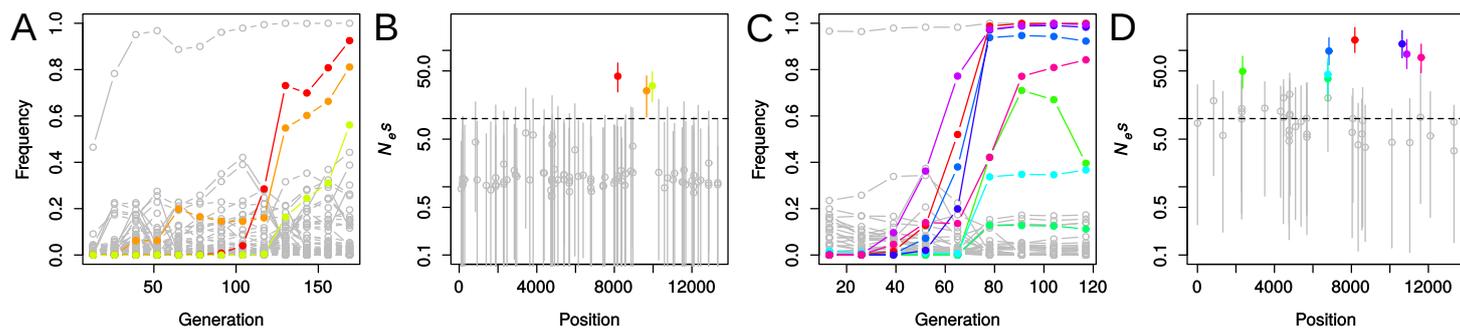


Figure S6: Allele trajectories (A, C) and posterior estimates for $N_e s$ (B, D) for control (A, B) and drug-treated (C, D) Influenza. Non-significant loci are colored grey and significant loci are colored with a unique color for each locus.

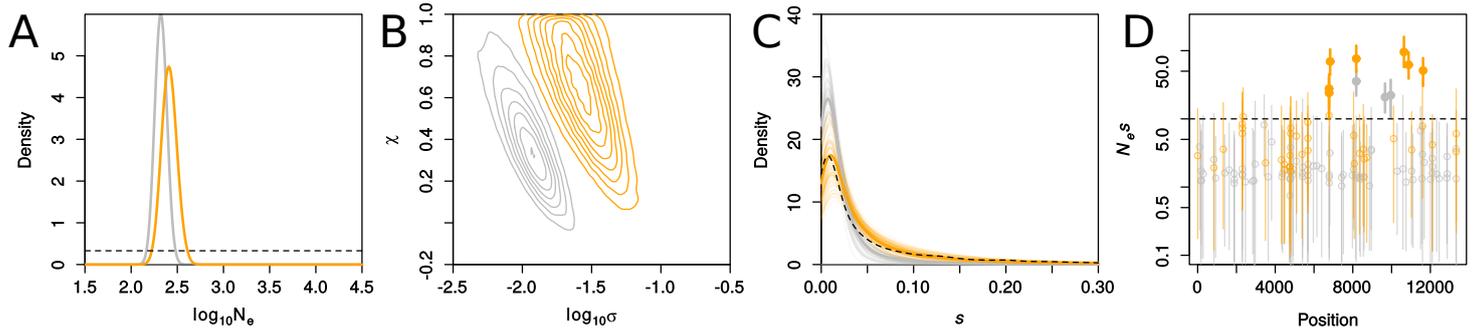


Figure S7: Inferred demography and selection for experimental evolution of Influenza using the PLS 3/1 set of statistics. We show results for the no-drug (control) and drug-treated Influenza in grey and orange, respectively. Shown are the posterior distributions for $\log_{10}N_e$ (A) and $\log_{10}\sigma$ and χ (B). In panel C, we plotted the modal distribution of fitness effects (DFE) with thick lines by integrating over the posterior of its parameters. The thin lines represent the DFEs obtained by drawing 100 samples from the posterior of σ and χ . Dashed lines in panels A and C correspond to the prior distributions. In panel D, the posterior estimates for $N_e s$ per locus versus the position of the loci in the genome are shown. Open circles indicate non-significant loci whereas closed, thick circles indicate significant loci ($P(N_e s > 10) > 0.95$, dashed line).

Simulation set	Parameter	Method	RMSE	Pearsons R^2
Set 1	s	LC 1/1	0.0700	0.970
		PLS 3/1	0.0809	0.960
		PLS 5/2	0.0743	0.966
	$\log_{10}(N_e)$	LC 1/1	0.178	0.969
		PLS 3/1	0.199	0.962
		PLS 5/2	0.171	0.972
Set 2	s	LC 1/1	0.0191	0.954
		PLS 3/1	0.0390	0.957
		PLS 5/2	0.0195	0.953
	$\log_{10}(N_e)$	LC 1/1	0.0292	-
		PLS 3/1	0.0390	-
		PLS 5/2	0.0485	-
	χ	LC 1/1	0.126	0.911
		PLS 3/1	0.127	0.910
		PLS 5/2	0.140	0.895
Set 3	s	LC 1/1	0.0226	0.987
		PLS 3/1	0.0228	0.986
		PLS 5/2	0.0227	0.986
	$\log_{10}(N_e)$	LC 1/1	0.0427	-
		PLS 3/1	0.189	-
		PLS 5/2	0.374	-
	$\log_{10}(\sigma)$	LC 1/1	0.0783	0.988
		PLS 3/1	0.0776	0.989
		PLS 5/2	0.0844	0.984
Set 4	s	LC 1/1	0.0258	0.982
		PLS 3/1	0.0259	0.981
		PLS 5/2	0.0254	0.981
	$\log_{10}(N_e)$	LC 1/1	0.0577	-
		PLS 3/1	0.0889	-
		PLS 5/2	0.351	-
	$\log_{10}(\sigma)$	LC 1/1	0.0191	0.961
		PLS 3/1	0.0180	0.968
		PLS 5/2	0.0167	0.968
χ	LC 1/1	0.306	0.667	
	PLS 3/1	0.306	0.659	
	PLS 5/2	0.291	0.666	

Table S1: **Performance of ABC-PaSS coupled with different dimension reduction techniques in Wright-Fisher simulations.** We computed the root mean square error (RMSE) and Pearson’s correlation (R^2) between true and estimated parameter values using three different dimension reduction strategies: a single linear combination per parameter calculated according to Theorem 2 (LC 1/1), three and one PLS components for parameters $\log_{10}N_e$ and s , respectively (PLS 3/1), and five and two PLS components for parameters $\log_{10}N_e$ and s , respectively (PLS 5/2). Results shown are for four sets of 25 replicate simulations: Set 1 assumed uniform priors for N_e and s ($U[0.5, 4.5]$ and $U[0, 1]$, respectively), Sets 2-4 assumed a generalised pareto distribution for s with hyperparameters σ and χ . For Set 2 we varied χ ($U[-0.2, 1]$) and kept σ fixed ($= 0.01$), for set 3 we varied $\log_{10}\sigma$ ($U[-2.5, -0.5]$) and kept χ fixed ($= 0.5$) and for Set 4 we varied both σ and χ . For Sets 2-4 we performed simulations only for $\log_{10}N_e = 3$, thus R^2 is not calculable for these. The dimension reduction strategy with the smallest RMSE for each parameter per set is highlighted in grey.