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Sebastian Böhm

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Regional Economic Integration and Factor Mobility in Unified Germany*

Sebastian Böhm†

University of Fribourg

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The massive movement of capital and labor in opposite directions is the most striking characteristic of economic integration of Eastern and Western Germany. Beyond that, wage-setting behavior during the early years of unification and massive public social transfers have affected the transition path of the Eastern economy. In this paper, I set up a two-region open economy model with capital and labor mobility, wage-setting behavior, and public social transfers to explain major empirical trends of the German integration episode. I show that the model is able to replicate aggregate migration pattern in unified Germany and that wage-setting behavior has delayed labor productivity convergence between both German regions, whereas public social transfers have reduced the effect of wage setting on East-West net migration.

Keywords: Economic Integration, German Reunification, Capital Mobility, Migration

JEL classification: F20, E60, H20, J61

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† University of Fribourg, Department of Economics, Bd de Pérolles 90, CH-1700 Fribourg, Switzerland. E-mail: sebastian.boehm@unifr.ch.

1 Introduction

German Reunification marks an example of comprehensive economic integration of two formerly separated regions into a single economic entity. What has been particularly striking is the massive movement of production factors in opposite directions. East Germany has received enormous capital inflows, whereas low wages and high unemployment rates has been responsible for large East German labor force outmigration. Beyond that, Eastern wages were set above labor productivity levels, particularly during the first decade after unification, which may have distorted private capital investment incentives and delayed productivity convergence between both German regions (e.g. Akerlof et al., 1991; Sinn and Sinn, 1992). Moreover, the high-wage policy increased Eastern unemployment rates, which in turn increased migration incentives further and initiated large public social transfers to East German households (Sinn, 2000).

The first aim of this paper is to evaluate the potential of a neoclassical growth model with capital and labor mobility to explain major empirical trends of the German integration episode between 1991 and 2012. I therefore set up a benchmark model which accounts for capital and labor movements in opposite directions and compare the model's predictions with the empirical time series for relative East-West-German labor productivity convergence, East-German labor force net outmigration, and East-German private gross capital inflows. The second aim is then to examine the impact of wage-setting behavior and public social transfers on the macroeconomic development in unified Germany. For this reason, I extend the model by both policy measures and compare the resulting predictions with the time series of the three macroeconomic variables and the benchmark model.

Economic integration of two formally separated regions into a single economic entity may be achieved by several mechanisms.[1] In this paper, I exclusively focus on capital and labor mobility representing the major driving forces behind the German East-West convergence process (see discussion in the following section). Moreover, I consider adjust-

[1] According to Burda and Hunt (2001) economic integration may be achieved by one of the following five mechanisms or their combination: (i) internal capital accumulation in the capital-poor region, (ii) capital mobility from the capital-rich to the capital-poor region, (iii) labor mobility from the capital-poor to the capital-rich region, (iv) Heckscher-Ohlin type trade, and (v) technology adoption by the backward region from the leading region.

ment costs of moving both production factors which has been associated with this process (Burda, 2006, 2008). I neglect, however, technological differences across both regions which allows me to examine the potential of one particular model which stresses capital and labor mobility to explain major empirical trends.

The key results of the paper are: *First*, the fully extended model is able to replicate aggregate migration pattern in unified Germany. It does, however, not fully match East-West labor productivity convergence and capital inflows. The former appears to be predetermined by the type of the employed model, as a neoclassical model does imply full convergence by construction. *Second*, the policy analysis shows that wage-setting behavior has delayed labor productivity convergence between both German regions and would have increased East-West net migration substantially if government interventions had not been carried out. *Third*, public social transfers have been successful in reducing the effect of wage setting on East-West net migration.

It is important to note, that the derived results are not implied by a standard neoclassical two-region economy model with two production factors and without frictions. Because then, only one of the two, labor or capital, would move instantaneously until wages and capital returns are equalized across both regions. The capital-labor ratio needs to be equalized, and if one factor moves, this equality is reached for a unique allocation of this factor over the two regions. In this paper, however, I study the case with frictions: both labor and capital move in opposite directions since each factor moves to the region where it is scarce, but neither capital nor labor can move instantaneously since factor adjustment costs are present. In the long-run, capital-labor ratios need to be equalized but this equalization can happen for multiple levels of these inputs. Hence there is hysteresis, that is the long run size of each region depends on initial conditions, factor adjustment costs as well as distortions along the transition path.

The paper complements the theoretical literature on the macroeconomic development in unified Germany. Most related to this work are Funke and Strulik (2000) as well as Burda (2006, 2008) who investigate the pattern of German East-West convergence.[2] Burda (2006, 2008) emphasizes on the importance of factor mobility and adjustment costs. He proposes

[2] Theoretical papers on the pattern of unemployment comprise Snower and Merkl (2006) and Uhlig (2006).

a centralized open economy model which accounts for factor movements in opposite directions but he does not evaluate the predictions implied by the model with respect to various empirical time series. Funke and Strulik (2000) show that government interventions and wage-setting behavior affect the transition path of the Eastern economy. Their analysis, however, does not take into account the effects of both policy measures on East-West migration rates and vice versa. The model in this paper includes migration and reveals the impact of wage-setting behavior and public social transfers with respect to labor productivity convergence, net migration rates, and private capital inflows.

There is a sizable literature on the relationship between labor mobility and private capital formation.[3] One strand of the literature emphazises the role of increasing returns to scale (see e.g. Faini, 1996; Reichlin and Rustichini, 1998; Schäfer and Steger, 2014), which I do not consider.[4] Closest to the framework in this paper is Rappaport (2005), who studies a neoclassical one-sector economy model with factor mobility and adjustment costs. He argues that an increase in labor mobility does not necessarily increase the speed of income convergence. That is, emigration raises, ceteris paribus, the wage in the source region, but it also reduces the shadow value of capital which slows down private capital investments and therewith income convergence.

The paper is organized as follows. In Section 2, I briefly discuss major empirical trends of German Reunification. In Section 3, I set up a benchmark model of economic integration and compare its prediction with the empirical series of three macroeconomic variables. In Section 4 and 5, I introduce wage-setting behavior and public social transfers, respectively, and discuss their implications with respect to the model's prediction. The last section briefly summarizes the paper and concludes.

2 Empirical stylized facts

Table 1 summarizes important macroeconomic indicators of East-West convergence from 1991 to 2012. As the first two columns reveal, East Germany's catch up in terms of in-

[3] For an extensive literature survey, see Felbermayr et al. (2015).

[4] Schäfer and Steger (2014) analyze the dynamics of comprehensive economic integration when fundamentals and expectations interact in the process of equilibrium selection. Their model is able to replicate limited income convergence between East and West Germany.

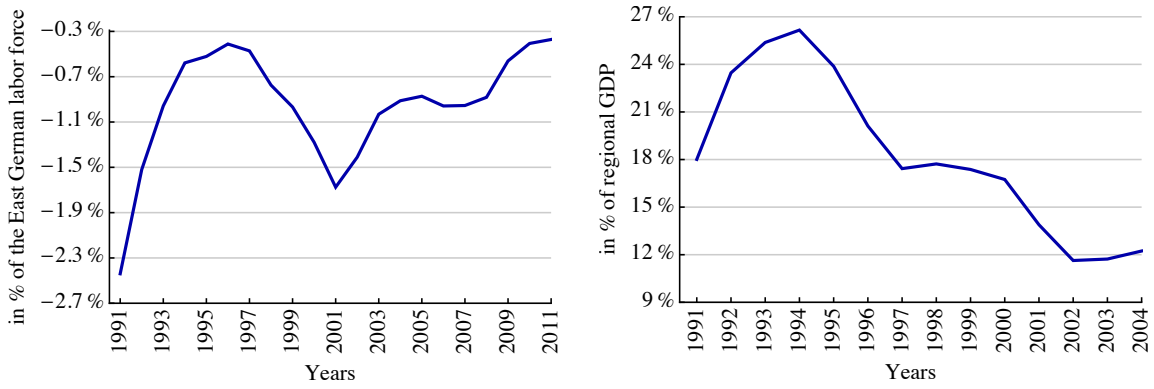
	Real GDP per Capita	Real Labor Productivity	Real Wage	Unemployment Rate
	in ratios of West German levels			
1991	0.38	0.40	0.58	1.65
1992	0.43	0.51	0.64	2.25
1993	0.51	0.60	0.68	1.93
1994	0.57	0.64	0.70	1.74
1995	0.60	0.66	0.72	1.63
1996	0.62	0.67	0.72	1.68
1997	0.62	0.69	0.72	1.77
1998	0.62	0.69	0.72	1.86
1999	0.62	0.70	0.73	1.95
2000	0.61	0.70	0.74	2.20
2001	0.61	0.71	0.74	2.35
2002	0.62	0.73	0.75	2.26
2003	0.63	0.73	0.75	2.16
2004	0.64	0.74	0.75	2.14
2005	0.65	0.74	0.76	1.87
2006	0.65	0.74	0.75	1.88
2007	0.65	0.74	0.75	2.01
2008	0.65	0.73	0.76	2.03
2009	0.66	0.75	0.77	1.88
2010	0.66	0.74	0.77	1.81
2011	0.65	0.74	0.77	1.88
2012	0.65	0.75	n.a.	1.80

Table 1: East-West convergence from 1991 to 2012. Wages are adjusted by the private consumption deflator. Berlin is excluded from calculations, except for the unemployment rate where the state belongs to East Germany. Sources: Statistisches Bundesamt (2013b,c,d), Statistik der Bundesagentur für Arbeit (2013).

come per capita and labor productivity was remarkable during the early 1990s but slowed down significantly later on. Even though average per capita convergence is still above the neoclassical prediction by Barro (1991) of 1.5-2 percent per annum, it is not clear today if East Germany will experience full income convergence at a slow pace or even limited convergence in the long run.[5] In fact, the real GDP per capita gap between East and West Germany has been closed annually by 2.7 percent on average between 1991 and 2012.

Figure 1 depicts the most striking characteristic of the German integration episode - the movement of capital and labor in opposite directions. Economic integration paved the way for the Eastern economy to achieve efficient production patterns and promoted high rates of private capital investments (panel b). However, the Eastern labor force faced on average lower wages and higher unemployment rates compared to their Western counterpart (see

[5] Uhlig (2006) was among the first to point out the possibility of limited convergence in the long run.



(a) East-West Net Migration Rates

(b) East Private Gross Capital Investment Rates

Figure 1: Migration and investment in Eastern Germany. Sources: Statistisches Bundesamt (2005, 2013a), own calculations.

Table 1), which are accountable for the large net migration outflow each year (panel a). Until 2011, total net migration amounted to nearly 1.2 million people and was especially concentrated among the higher educated youth.[6]

Moreover, German labor unions agreed on a fast wage convergence during the first decade after unification to prevent mass migration and to protect the western labor force from low-wage competition (Sinn, 2000). The increase in firms' production costs led to massive lay-offs and rising unemployment rates in the East, which in turn raised migration incentives. Hence, the German government was forced to substantially increase social security transfers to East German households in the form of unemployment and retirement benefits. Between 1991 and 2010, net public transfers amounted to 70 billion Euros, or 3 percent of Germany's GDP, on average per year, whereas about 65 percent constituted social assistances (Blum et al., 2009; Kloß et al., 2012). Even though private capital investments in East Germany were stimulated by an extensive investment subsidy law after unification (Sinn, 2002), the increase in production costs reduced investment incentives. The high-wage policy acted as a brake on the integration process as emphasized by several economists (e.g. Akerlof et al., 1991; Sinn and Sinn, 1992) and may have delayed labor productivity convergence between both regions.

[6] For a disaggregate analysis of East-West migration patterns see e.g. Hunt (2006). Note that typical migration costs as language or cultural differences do not apply.

3 The Benchmark Model

Consider a small open economy which comprises a rich region, west, and a poor region, east, in terms of the initial capital-labor ratio. Both regions are perfectly integrated into the world capital market and populated by a large number of identical individuals and firms. The interest rate, $\bar{r} > 0$, is exogenously given and domestic savings have no effect on capital accumulation, which is determined by the investment demand of firms in each region. In each period $t \in [0, \infty)$ only interregional migration between east and west is permitted. Let subscript $i \in \{E, W\}$ on a variable index region. Then, given a continuum of mass N_i of individuals in both regions and denoting $M(t)$ as net migration from east to west, total population size in region i at time t is given by initial population and migration until period t:

$$N_W(t) = N_W(0) + \int_0^t M(s)ds \tag{1}$$

$$N_E(t) = N_E(0) - \int_0^t M(s)ds. \tag{2}$$

Differentiating (1) and (2) with respect to time yields

$$\dot{N}_W(t) = -\dot{N}_E(t) = M(t), \tag{3}$$

where $M(t) > 0$ implies net migration from east to west. A dot on a variable denotes its derivative with respect to time. Overall population P in the economy is assumed to be constant, so that $P = N_W + N_E$ holds in each period.

3.1 Firms

There is mass one of identical firms in both regions operating in a perfectly competitive environment. Each firm employs a constant-returns-to-scale production technology that combines capital and labor, K and L, to produce a single homogeneous good that can be used either for consumption or investment. Firms own their capital stock. Aggregate

output $Y_i(t)$ and the evolution of capital in both regions at time t are given by

$$Y_i(t) = K_i(t)^{\alpha} L_i(t)^{1-\alpha}, \quad 0 < \alpha < 1, \tag{4}$$

$$\dot{K}_i(t) = I_i(t) - \delta K_i(t), \quad i = E, W, \tag{5}$$

where I_i denotes gross capital investment in region i and $\delta \geq 0$ is the constant capital depreciation rate.[7] According to Abel (1982) and Hayashi (1982), capital investments are assumed to be subject to convex adjustment costs. The representative firm in each region chooses paths for investment $I_i(t)$ and labor $L_i(t)$, so as to maximize the present value of its net cash flow subject to the law of motion for capital:

$$\max_{L_i, I_i} \quad \int_0^{\infty} \left\{ K_i^{\alpha} L_i^{1-\alpha} - w_i L_i - I_i \left[1 + \frac{\gamma_K}{2} \left(\frac{I_i}{K_i} \right) \right] \right\} e^{-\bar{r}t} dt \tag{6}$$

$$s.t. \quad \dot{K}_i = I_i - \delta K_i \tag{7}$$

$$K_i(0) = K_{i0} \text{ given}, \quad L_i(0) = L_{i0} \text{ given}, \quad i = E, W.$$

where $\gamma_K > 0$ measures the capital adjustment costs intensity. The first-order conditions together with (5) yield

$$w_i = (1-\alpha) \left(\frac{K_i}{L_i} \right)^{\alpha}, \tag{8}$$

$$\dot{K}_i = \left(\frac{q_i - 1}{\gamma_K} - \delta \right) K_i, \tag{9}$$

$$\dot{q}_i = (\bar{r} + \delta) q_i - \alpha \left(\frac{K_i}{L_i} \right)^{\alpha - 1} - \frac{(q_i - 1)^2}{2\gamma_K}, \tag{10}$$

where q_i denotes the shadow value of capital in region i.[8] The firms' inverse demand for labor is given by equation (8), while the evolution of the capital stock is governed by equation (9) and (10). The latter equation describes the arbitrage condition of capital which equates the return of an additional unit installed to its respective opportunity costs. Pos-

[7] McAdam and Willman (2004) estimated an elasticity of substitution for Germany of 0.7 in the pre-reunification period and of 1.1 in the post-reunification period. For both periods, their estimation does not reject the hypothesis of unit elasticity of substitution implied by the Cobb-Douglas production function.

[8] Derivations are relegated to the appendix and the time index t is suppressed whenever no ambiguity arises.

itive values of the adjustment cost parameter, γ_K, reduce the rate of capital accumulation and affect the long run capital endowment in both regions.

3.2 Individuals

Individuals in both regions supply one unit of labor inelastically to the labor market and choose the consumption path, $c_i(t)$, that maximizes their utility subject to their budget constraint. Initial conditions are assumed to imply an east-west wage differential, giving rise to migration incentives. Thus, individuals do not only choose their optimal consumption path but also their location. The optimization problem of a representative individual in both regions is summarized by

$$\max_{c_i(s)} \quad U_i(t) = \int_t^\infty \frac{c_i(s)^{1-\sigma} - 1}{1-\sigma} e^{-\rho[s-t]} ds \tag{11}$$

$$s.t. \quad \int_t^\infty e^{-\bar{r}[s-t]} c_i(s) ds = V_i(t) \tag{12}$$

$$a_i(t) = a_{it} \text{ given}, \quad i = E, W,$$

where $\rho > 0$ denotes the time-preference rate, $\sigma > 0$ the inverse of the intertemporal elasticity of substitution and $V_i(t)$ denotes individual wealth, which comprises initial financial wealth $a_i(t)$ and human wealth $\int_t^\infty e^{-\bar{r}[s-t]} w_i(s) ds$. More specifically, individual wealth of an agent of region i is given by

$$V_i(t) = \begin{cases} a_i(t) + \int_t^\infty e^{-\bar{r}[s-t]} w_i(s) ds & \text{if the agent does not migrate} \\ a_i(t) + \int_t^\infty e^{-\bar{r}[s-t]} w_j(s) ds - mc_i & \text{if the agent migrates in period } t \end{cases}, \tag{13}$$

with $i, j \in \{E, W\}$ and $i \neq j$. Following Braun (1993), interregional migration is subject to migration costs mc_i, which accrue in terms of consumable goods by the migrant at the period of leaving his region. These costs are assumed to be an increasing function in the number of migrants $M(t)$ of the same period and therefore depend on the flow of migrants and not on the stock of formerly migrated individuals.[9] The relation reflects transaction

[9] In comparison to international migration pattern, there is no evidence for migration-cost reducing network effects in the case of German East-West migration.

costs in the process of moving, for instance, increasing transportation costs or agent's commission for housing in the host region.[10] Migration costs are explicitly given by

$$mc_i \quad = \quad \gamma_L M(t), \quad i = E, W, \tag{14}$$

where $\gamma_L > 0$ denotes the migration cost intensity parameter.

Now, the important point to notice is that, in equilibrium, individual wealth does not change in response to migration. The reason is that, in equilibrium, the gain in terms of human wealth, due to prevailing east-west wage differentials, equals the migration costs, as explained below. This feature of the model allows to separate consumption decisions from migration decisions, which makes the analysis tractable.

3.3 Migration

A well know property of the individuals optimization problem stated above is that, the individual consumption path is proportional to individual wealth according to[11]

$$c_i(t) = \begin{cases} \rho \left(a_i(t) + \int_t^\infty e^{-\bar{r}[s-t]} w_i(s) ds \right) & \text{if the agent does not migrate} \\ \rho \left(a_i(t) + \int_t^\infty e^{-\bar{r}[s-t]} w_j(s) ds - mc_i \right) & \text{if the agent migrates in period } t \end{cases}, \tag{15}$$

with $i, j \in \{E, W\}$ and $i \neq j$. Substituting both expressions into the utility function in (11), one obtains the indirect utilities of a migrant and non-migrant, respectively. However, note that the only difference is given by the present value labor income differential. Then, the analysis can be simplified in the following way. Let $\lambda_j(t)$ denote the migration benefit of a migrant from the source region i at time t, such that

$$\lambda_j(t) \quad = \quad \int_t^\infty \left[w_j(s) - w_i(s) \right] e^{-\bar{r}[s-t]} ds, \quad i, j \in \{E, W\}, \quad i \neq j. \tag{16}$$

An individual of region i is willing to migrate in period t if and only if his migration benefit is positive and exceeds migration cost. Moreover, (16) implies $\lambda_i(t) = -\lambda_j(t)$, which means

[10] The proposed type of cost function may also be interpret as social costs in form of public opposition in the host region against massive immigration.

[11] See, for example, Barro and Sala-i Martin (2004).

that migration takes place only in one direction. In a competitive equilibrium cost must equal benefit, $\lambda_j(t) = mc_i$, so that individual wealth $V_i(t)$ does not change in response to migration and both, migrant and non-migrant, choose an identical consumption path. The optimal consumption path that solves the optimization problem of the representative individual above implies a consumption growth rate of:

$$\frac{\dot{c}_i(t)}{c_i(t)} = \frac{1}{\sigma}(\bar{r} - \rho), \quad i = E, W. \tag{17}$$

However, the knife-edge condition of the model requires $\bar{r} = \rho$, so that consumption does not grow.[12]

Wages are assumed to be initially lower in the eastern region, implying a positive migration benefit for eastern individuals:[13]

$$\lambda_W(t) = \int_t^\infty [w_W(s) - w_E(s)]\, e^{-\bar{r}[s-t]} ds > 0. \tag{18}$$

Using (3) and (14), the eastern population size evolves according to

$$\dot{N}_E(t) = -\gamma_L^{-1} \lambda_W(t), \tag{19}$$

Along the transition path, the migration benefit decreases due to migration and capital accumulation until capital-labor ratios are equalized across regions. In a steady state equilibrium it holds that $\lambda_W = 0$, so that population size is constant in both regions as $\dot{N}_E = 0$. For a given positive migration benefit, higher migration costs imply lower migration rates during the transition and a higher eastern population size in the steady state equilibrium.

3.4 Equilibrium and Stability Analysis

An equilibrium of the dynamic small open economy is defined as follows:

Definition 3.1. Suppose the given interest rate $\bar{r}$ is stationary and equals individual's time

[12] The knife-edge condition of the Open-Economy Ramsey model is extensively discussed e.g. in Barro and Sala-i Martin (2004, pp. 161).

[13] Equation (18) implies perfect foresight since the labor income differential depends on the future amount of capital installed and labor employed in both regions.

preference rate ρ. For any set of initial conditions $\{K_E(0), K_W(0), N_E(0)\}$ a competitive equilibrium is given by a set of time paths for quantities $\{K_E(t), K_W(t), N_E(t)\}_{t\in(0,\infty)}$ and prices $\{q_E(t), q_W(t), \lambda_W(t), w_E(t), w_W(t)\}_{t\in(0,\infty)}$ such that the capital stock in both regions evolves according to (5) and it holds in any period that

1. firms maximize the present value of net cash flow;
2. individuals maximize life-time utility;
3. if and only if labor is interregionally mobile, migration benefits equal migration costs, $\lambda_j(t) = mc_i, i, j \in \{E, W\}, i \neq j$;
4. the labor market clears.

Equilibrium conditions 1, 2, and 4 are straightforward. Condition 3 holds, since migration costs are a function in the number of migrants, $M(t)$, in period t according to (14). Using (9), (10), (19) together with $L_i = N_i$, and differentiating (18) with respect to time, the evolution of the economy is then governed by the following differential equation system:

$$\dot{K}_E = \left(\frac{q_E - 1}{\gamma_K} - \delta\right) K_E, \tag{20}$$

$$\dot{K}_W = \left(\frac{q_W - 1}{\gamma_K} - \delta\right) K_W, \tag{21}$$

$$\dot{N}_E = -\gamma_L^{-1}\lambda_W, \tag{22}$$

$$\dot{q}_E = (\bar{r} + \delta)\, q_E - \alpha \left(\frac{K_E}{N_E}\right)^{\alpha-1} - \frac{(q_E - 1)^2}{2\gamma_K}, \tag{23}$$

$$\dot{q}_W = (\bar{r} + \delta)\, q_W - \alpha \left(\frac{K_W}{P - N_E}\right)^{\alpha-1} - \frac{(q_W - 1)^2}{2\gamma_K}, \tag{24}$$

$$\dot{\lambda}_W = \bar{r}\lambda_W - (w_W - w_E)\,, \tag{25}$$

$$K_E(0), K_W(0), N_E(0) \text{ are given.}$$

A steady state equilibrium of the system is defined by the constancy of the three state variables, $\dot{K}_E = \dot{K}_W = \dot{N}_E = 0$, and costate variables, $\dot{q}_E = \dot{q}_W = \dot{\lambda}_W = 0$. Starting with the costate variables, $\bar{q}_E = \bar{q}_W = 1 + \delta\gamma_K$ follows directly from (20) and (21), where a bar denotes steady state values. The migration benefit is constant if and only if $w_W = w_E$,

which requires identical capital-labor ratios in both regions. Setting $\dot{q}_E = \dot{q}_W = 0$ in (23) and (24) respectively and using the expression for $\bar{q}_E$ and $\bar{q}_W$ yields

$$\frac{K_E}{N_E} = \frac{K_W}{P - N_E} = \left(\frac{\alpha}{(\bar{r} + \delta)(1 + \gamma_K \delta) - \frac{\gamma_K \delta^2}{2}} \right)^{\frac{1}{1-\alpha}}, \tag{26}$$

where $N_W = P - N_E$ is used. Thus, (26) implies $\bar{w}_W = \bar{w}_E$ in steady state and therefore $\bar{\lambda}_W = 0$. For a given eastern population size, steady state regional capital endowments are uniquely determined.

For the stability analysis of the dynamic system, equations (20)-(25) are linearized by means of a first-order Taylor approximation and written in matrix form:[14]

$$\begin{pmatrix} \dot{q}_E \\ \dot{q}_W \\ \dot{\lambda}_W \\ \dot{K}_E \\ \dot{K}_W \\ \dot{N}_E \end{pmatrix} = \underbrace{\begin{pmatrix} \bar{r} & 0 & 0 & -F^E_{KK} & 0 & -F^E_{KL} \\ 0 & \bar{r} & 0 & 0 & -F^W_{KK} & -F^W_{KL} \\ 0 & 0 & \bar{r} & F^E_{LK} & -F^W_{LK} & F^E_{LL} - F^W_{LL} \\ \bar{K}_E \gamma_K^{-1} & 0 & 0 & 0 & 0 & 0 \\ 0 & \bar{K}_W \gamma_K^{-1} & 0 & 0 & 0 & 0 \\ 0 & 0 & -\gamma_L^{-1} & 0 & 0 & 0 \end{pmatrix}}_{\mathcal{M}} \begin{pmatrix} q_E - \bar{q}_E \\ q_W - \bar{q}_W \\ \lambda_E - \bar{\lambda}_W \\ K_E - \bar{K}_E \\ K_W - \bar{K}_W \\ N_E - \bar{N}_E \end{pmatrix}. \tag{27}$$

The analytic expression for the six eigenvalues of matrix $\mathcal{M}$ are given by[15]

$$\{\lambda_1, \ldots, \lambda_6\} = \left\{ \begin{array}{cc} 0, & \bar{r}, \\ \frac{1}{2}\left(\bar{r} + \sqrt{\bar{r}^2 + 2(A - \sqrt{A^2 - 4B})}\right), & \frac{1}{2}\left(\bar{r} + \sqrt{\bar{r}^2 + 2(A + \sqrt{A^2 - 4B})}\right) \\ \frac{1}{2}\left(\bar{r} - \sqrt{\bar{r}^2 + 2(A + \sqrt{A^2 - 4B})}\right), & \frac{1}{2}\left(\bar{r} - \sqrt{\bar{r}^2 + 2(A - \sqrt{A^2 - 4B})}\right) \end{array} \right\}, \tag{28}$$

where

$$A = -\left[F^E_{KK}\bar{K}_E\gamma_K^{-1} + F^W_{KK}\bar{K}_W\gamma_K^{-1} + (F^E_{LL} - F^W_{LL})\gamma_L^{-1}\right], \tag{29}$$

$$B = \bar{K}_W\gamma_K^{-1}\gamma_L^{-1}\left[F^W_{KK}\left(F^E_{LL} - F^W_{LL}\right) + \left(F^W_{KL}\right)^2\right] + \bar{K}_E\gamma_K^{-1}\gamma_L^{-1}\left[F^E_{KK}\left(F^E_{LL} - F^W_{LL}\right) - \left(F^E_{KL}\right)^2\right] + \bar{K}_E\bar{K}_W\gamma_K^{-2}F^E_{KK}F^W_{KK}. \tag{30}$$

[14] $F^i_{k,l}$ denotes the second derivative of the production function of region $i \in \{E, W\}$ with respect to $k, l \in \{K_i, L_i\}$ to simplify notations. The derivation is provided in the online-appendix.

[15] The derivation is relegated to the appendix.

For non-negative values of the inner discriminant, $A^2 - 4B$, the model implies meaningful solutions. The dynamic system exhibits three strictly positive $(\lambda_2, \lambda_3, \lambda_4)$ and two strictly negative eigenvalues (λ_5, λ_6) and corresponds to a perfect foresight model with saddle-path stability. Moreover, as $\lambda_1 = 0$ the model exhibits path dependency. There exists a unique steady state equilibrium corresponding to a given set of initial values for the endogenous state variables $\{K_E(0), K_W(0), N_E(0)\}$. Changes in the initial conditions will therefore change the steady state capital and labor endowment in both regions. The model's speed of convergence is determined by the smallest absolute negative eigenvalue, λ_6, which is increasing in the adjustment cost intensities γ_L and γ_K.

3.5 Calibration

α	$\bar{r}$	δ	γ_L	γ_K	P
0.3	0.03	0.05	357.08	10.8	5

Table 2: Baseline calibration

The model parameters are specified to meet the empirical regularities of Germany's post-unification periods. The technology parameter is set to $\alpha = 0.3$ to be in line with empirical estimates on German data (Dreger and Schumacher, 2000; Willman, 2002) and the rate of return to capital is set to $\bar{r} = 0.03$.

The migration cost intensity parameter γ_L is calibrated as follows.[16] Equation (22) states that $\gamma_L = \frac{\lambda_W}{-\dot{N}_E}$. Using $\lambda_E(t) = -\lambda_W(t)$, the initial theoretical migration benefit for an East German resident is expressed by $\lambda_E(0) = \int_0^\infty [w_E(s) - w_W(s)]\, e^{-\bar{r}s} ds$, where $t = 0$ corresponds to 1991 in real-time. Assume that the western wage takes on its steady state value in each period and productivity convergence between East and West Germany takes place at a constant rate κ. Then, with an initial and steady state value convergence may be approximated by $w_E(s) - \bar{w}_W \cong [w_E(0) - \bar{w}_W]\, e^{-\kappa s}$ for $s \geq 0$. Given the time series for real labor productivity convergence in Table 1, this approximation implies $\kappa \cong 0.04$. For an initial productivity gap of roughly 60 percent in 1991, the initial theoretical migration benefit reads $\lambda_E(0) \cong -8.57$. That is, the gain in terms of human wealth for an East German

[16] The calibration strategy basically follows Burda (2006).

worker moving to West Germany in 1991 was roughly 857% of the annual western wage. Note that the calibration strategy is based on productivity rather than observed wage convergence to get a benchmark prediction of economic integration. Using the latter would have implied a lower initial migration benefit as wage convergence exceeded productivity convergence, particularly during the first decade after unification. This empirical fact is taken into account in both extensions of the model in Section 4 and 5. East Germany's labor force accounted for roughly 20 percent of overall Germany in 1991. Thus, the eastern population share, N_E, is normalized to unity and overall population is set to $P = 5$. In the same year, about 2.4 percent of the East German labor force migrated to West Germany (Statistisches Bundesamt, 2005), so that $\gamma_L = \frac{\lambda_W}{-\dot{N}_E} \cong 357.08$.

The steady state condition for the shadow price of capital states that $\gamma_K = \frac{\bar{q}-1}{\delta}$. Dittmann et al. (2010) estimated $q = 1.54$ for the average German shadow value of capital. Given $\delta = 0.05$ for unified Germany (Statistisches Bundesamt, 2010, 2011), the capital adjustment cost intensity reads $\gamma_K = 10.8$.

The baseline set of parameter values are listed in Table 2. The frictions γ_L and γ_K are the key determinants of slowing migration and capital formation, so that capital-labor ratios across both regions are not equalized instantaneously. To analyse the transitional dynamics of the endogenous variables, the dynamical system is solved by applying the relaxation algorithm introduced by Trimborn et al. (2008).[17]

3.6 Numerical Evaluation

In this section I discuss the transitional dynamics of the endogenous variables and compare the theoretical predictions of the benchmark model with the empirical results for relative East-West-German labor productivity convergence, East-German labor force net outmigration, and East-German private gross capital inflows. I also perform a sensitivity analysis with respect to variations in migration and capital adjustment cost intensities. The initial capital endowments in both regions are set in the following way: I assume that West Germany was in steady state equilibrium at the time of unification, so that $K_W(0) = \bar{K}_W$.

[17] The algorithm is implemented in Mathematica. The underlying file is available from the author upon request.

The initial eastern capital endowment, $K_E(0)$, is then chosen such that the model matches initially the 40 percent East German real labor productivity level in 1991.

3.6.1 Transitional Dynamics

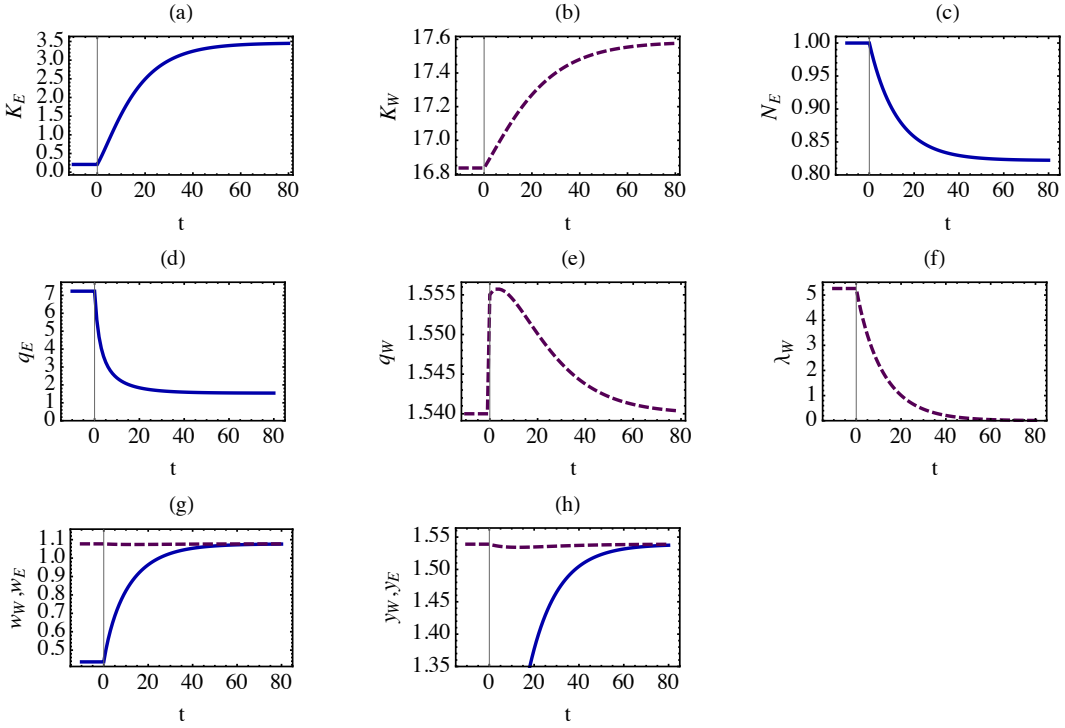


Figure 2: Transitional dynamics implied by the benchmark model. Solid lines: time path eastern variables; dashed lines: time path western variables. Set of parameters as in Table 2.

Figure 2 illustrates the time path of the three state and co-state variables, the eastern and western wage as well as income per capita in both regions. The blue solid lines represent the path of the eastern and the purple dashed lines of the western variables, respectively. Starting from initial values at time zero, the dynamic transition sets in at period one.[18] The initial conditions imply: (i) large capital investment incentives in the east (panel d) associated with capital accumulation over time (panel a), and (ii) positive migration benefits for eastern individuals (panel f) associated with east-west migration flows (panel c).[19] Capital

[18] Previous periods on the negative x-axis are associated with the initial value of the respective variable to highlight the economy's take off. Periods on the x-axis correspond to years.

[19] Note that the initial migration benefit $\lambda_W(0)$ deviates from the calibrated value in section 3.5. The deviation results from the assumption of a constant rate of productivity convergence κ to simplify calibration. However, the model presented is of non-linear form and exhibits a non-constant convergence rate.

inflows and labor outflows lead to a decline of both, the shadow value of the eastern capital stock and the migration benefit, during the transition to the steady state equilibrium. Moreover, east-west migration flows increases capital investment incentives in the west (panel e) associated with capital accumulation over time (panel b). Migration, therefore, affects the long run capital endowment in both regions. The benchmark model implies small repercussion effects of economic integration on western wages (panel g) and western income per capita (panel h).

3.6.2 Evaluation

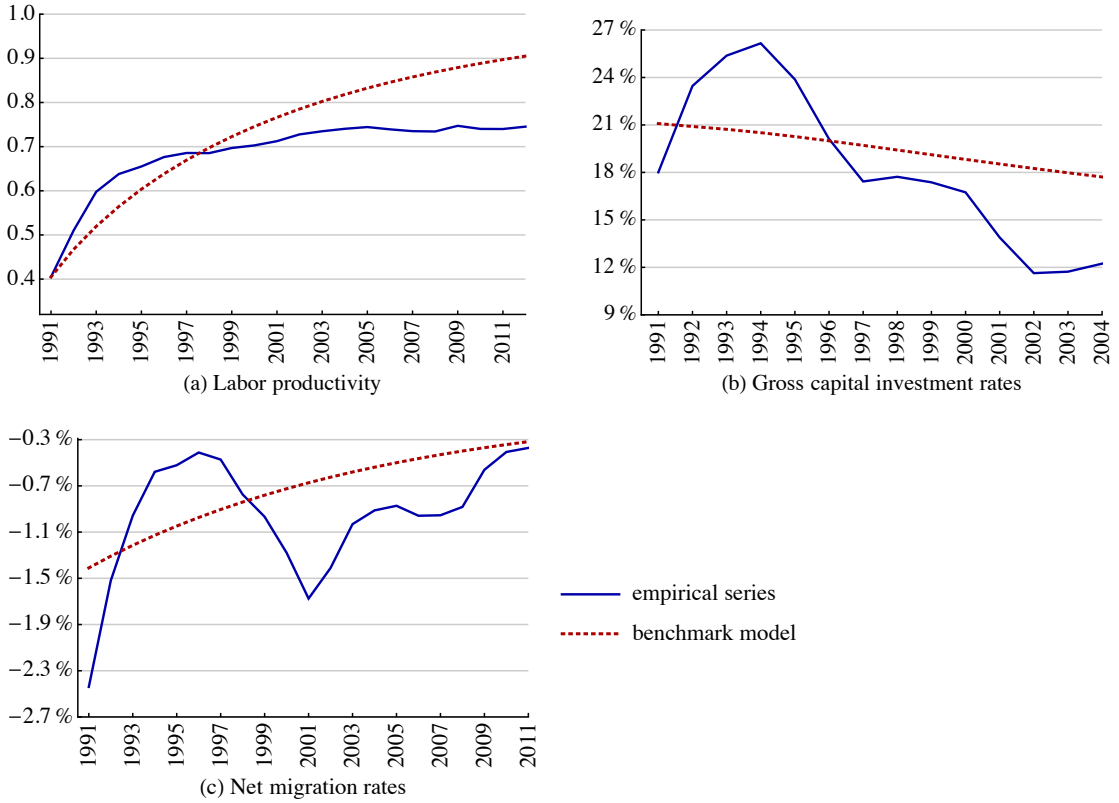


Figure 3: Time paths of labor productivity convergence, gross capital investment rates, and net migration rates implied by the benchmark model. Set of parameters as in Table 2.

Figure 3 displays the time paths of labor productivity convergence, gross capital investment rates, and net migration rates (dotted lines) together with the corresponding empirical time series (solid lines). Starting at a 40 percent labor productivity level, the benchmark model implies faster productivity convergence apart from the first eight years (panel a),

on average more capital inflows (panel b) and less outmigration (panel c) compared to the observed empirical pattern. More specifically, the model predicts total net migration between 1991 and 2011 of about 981,300 persons, compared with 1,186,500 East German migrants over the same period. Thus, the neoclassical model with factor mobility *underestimates* migration and *overestimates* productivity convergence and gross capital investments in unified Germany over the observable time period.

As outlined above, the benchmark model exhibits productivity compensation and full employment in each period. According to Table 1, however, Eastern real wage convergence predominated productivity convergence, particularly during the early years after unification. The high-wage policy increased Eastern firms' production costs and led to massive lay-offs and rising unemployment rates. Thus, this aspect of German Reunification motivates the introduction of a wage-setting behavior into the theoretical framework.

3.6.3 Sensitivity Analysis

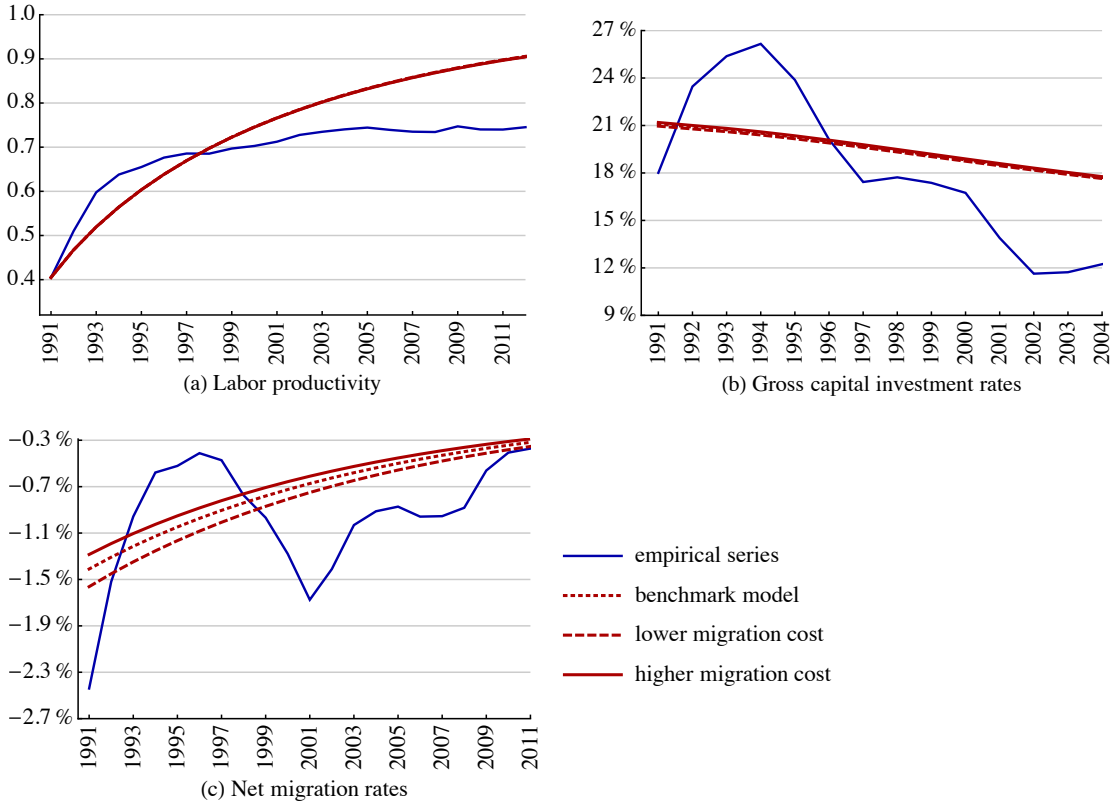


Figure 4: Time paths of labor productivity convergence, gross capital investment rates, and net migration rates, assuming alternative migration cost intensities.

Migration and capital adjustment cost intensities are the key parameter determining the model's dynamics. Therefore, I first consider an increase (decrease) in migration cost intensity γ_L by 10 percent. The time path of labor productivity convergence in panel (a) of Figure 4 parallels the result in Rappaport (2005): a change in labor mobility costs does not affect the speed of productivity convergence. Capital investment rates are only marginally affect, whereas migration rates increase (decrease) significantly due to a decrease (increase) in migration costs.

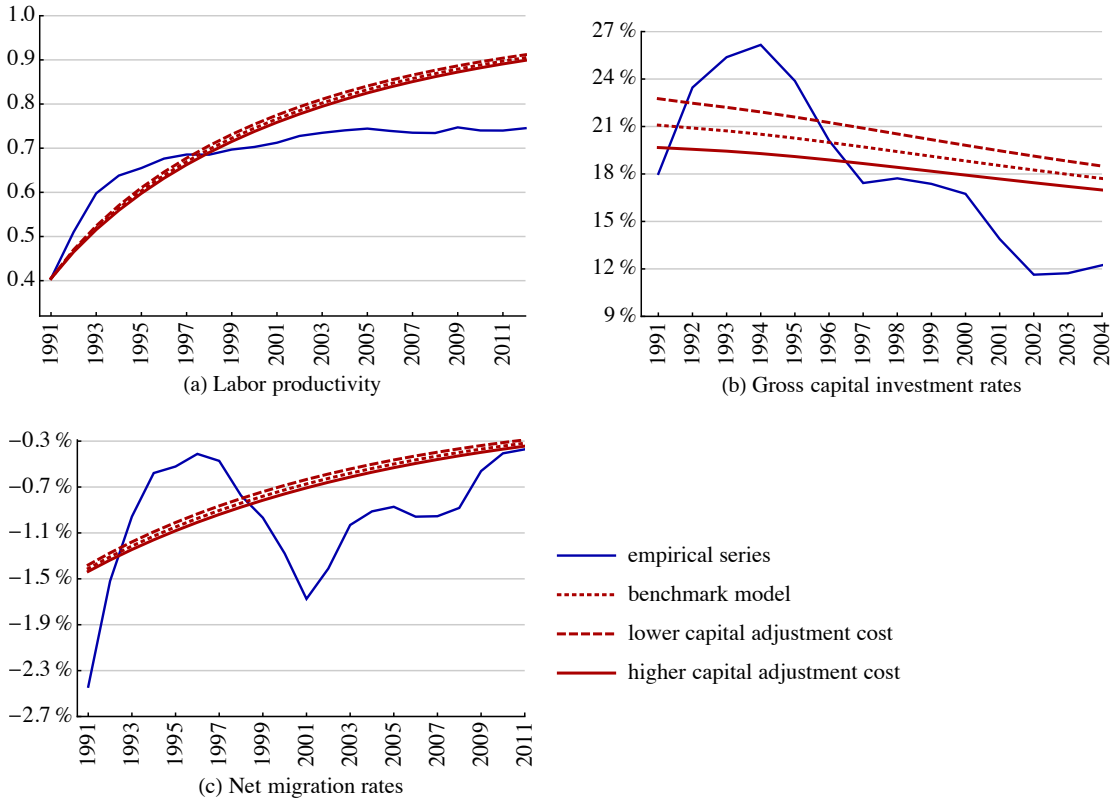


Figure 5: Time paths of labor productivity convergence, gross capital investment rates, and net migration rates, assuming alternative capital adjustment cost intensities.

Figure 5 displays that an increase (decrease) in capital adjustment cost intensity γ_K by 10 percent, lowers (raises) gross capital investment rates (panel b), which in turn decreases (increases) labor productivity convergence marginally (panel a). Variations in γ_K changes the evolution of migration rates only in a minor way (panel c).

4 Introducing Wage-Setting Behavior

Consider the benchmark model described above. In the following extension, I assume that the eastern wage is imposed by the behavioral function $\tilde{w}_E$, which exceeds the market clearing wage outside of a steady state. The eastern employment rate ℓ_E is then endogenously determined in accordance with the behavioral function to satisfy marginal productivity compensation. The western employment rate is normalized to unity and assumed to be constant for native individuals and immigrants, so that $L_W = N_W$ and $L_E = N_E \ell_E$. During the transition, wage-setting behavior is accountable for a lower eastern employment rate (higher unemployment rate) compared to the western region, i.e. $\ell_E < 1$, which corresponds to the empirical observation in unified Germany documented in Table 1. The way wage-setting behavior is introduced here seems plausible in a representative agent framework since the considered agent represents the average individual of his respective region.

4.1 Behavioral Function and Migration Benefit

Assume that East Germany's wage path is a function of its western equivalent, analog to Funke and Strulik (2000), and takes the form of

$$\tilde{w}_E(t) \quad = \quad \theta^{\beta} w_W(t), \qquad 0 < \beta < 1, \tag{31}$$

where $\theta = {}^{y_E}/_{y_W}$ denotes the east's relative labor productivity per worker with $y_E = ({}^{K_E}/_{N_E})^{\alpha} \ell_E^{1-\alpha}$ and $y_W = ({}^{K_W}/_{(P-N_E)})^{\alpha}$.[20] For example, given an initial 40 percent relative labor productivity level and $\beta = 0.6$, equation (31) implies a relative eastern wage of 58 percent at unification time and 81 percent when θ equals 70 percent.

Marginal productivity compensation requires that $\tilde{w}_E = (1-\alpha)\left(\frac{K_E}{N_E \ell_E}\right)^{\alpha}$ holds in each period t. Substituting the expression into (31) and solving for ℓ_E yields the time path of the

[20] For a micro-founded analysis of wage-setting behavior in Eastern Germany see e.g. Burda and Funke (1993).

eastern employment rate:

$$\ell_E(t) \quad = \quad \left(\frac{\frac{K_E(t)}{N_E(t)}}{\frac{K_W(t)}{P-N_E(t)}} \right)^{\frac{\alpha-\alpha\beta}{(1-\alpha)\beta+\alpha}}, \tag{32}$$

which depends on the relative eastern capital-labor ratio. Initial conditions then imply that $\ell_E < 1$ throughout the transition until capital-labor ratios are equalized in steady state, so that $\bar{\ell}_E = \bar{\theta} = 1$ and $\tilde{w}_E = w_W$. Using (32), the eastern wage is given by

$$\tilde{w}_E(t) \quad = \quad (1-\alpha) \left(\frac{K_E(t)}{N_E(t)} \right)^{\frac{\alpha\beta}{(1-\alpha)\beta+\alpha}} \left(\frac{K_W(t)}{P-N_E(t)} \right)^{\frac{\alpha(\alpha-\alpha\beta)}{(1-\alpha)\beta+\alpha}}. \tag{33}$$

Thus, eastern labor income is now represented by the wage sum $\tilde{w}_E \ell_E$ given by (32) and (33) respectively, whereas western labor income, w_W, remains unchanged,

Using the wage sum in the optimization problem of the representative eastern individual in (11) and (12) yields the optimal consumption path:

$$c_E(t) = \begin{cases} \rho \left(a_E(t) + \int_t^\infty e^{-\bar{r}[s-t]} \tilde{w}_E(s) \ell_E(s) ds \right) & \text{if the agent does not migrate} \\ \rho \left(a_E(t) + \int_t^\infty e^{-\bar{r}[s-t]} w_W(s) ds - m c_E \right) & \text{if the agent migrates in period } t \end{cases}. \tag{34}$$

The migration benefit given by the present value labor income differential across both regions, $\lambda_W(t) = \int_t^\infty \left[w_W(s) - \tilde{w}_E(s) \ell_E(s) \right] e^{-\bar{r}[s-t]} ds$, now evolves according to

$$\dot{\lambda}_W(t) \quad = \quad \bar{r} \lambda_W(t) - \left[w_W(t) - \tilde{w}_E(t) \ell_E(t) \right]. \tag{35}$$

4.2 Equilibrium and Stability Analysis

An equilibrium of the dynamic small open economy with wage-setting behavior is defined as follows:

Definition 4.1. Consider the benchmark model described in Section 3 extended by wage-setting behavior according to (31). Suppose the given interest rate $\bar{r}$ is stationary and equals individual's time preference rate ρ. For any set of initial conditions $\{K_E(0), K_W(0), N_E(0)\}$ a competitive equilibrium is given by a set of time paths for quantities $\{K_E(t), K_W(t),$

$N_E(t)\}_{t\in(0,\infty)}$, prices $\{q_E(t), q_W(t), \lambda_W(t), \tilde{w}_E(t), w_W(t)\}_{t\in(0,\infty)}$, and value $\{\ell_E(t)\}_{t\in(0,\infty)}$ such that the capital stock in both regions evolves according to (5) and it holds in any period that

1. firms maximize the present value of net cash flow;
2. individuals maximize life-time utility;
3. if and only if labor is interregionally mobile, migration benefits equal migration costs, $\lambda_j(t) = mc_i, i, j \in \{E, W\}, i \neq j$;
4. the labor market clears.

The dynamic system of the extended model is described by (20)-(22), (24), (35) together with $\dot{q}_E = (\bar{r} + \delta)\, q_E - \alpha \left(\frac{K_E}{N_E \ell_E}\right)^{\alpha-1} - \frac{(q_E-1)^2}{2\gamma_K}$ and $K_E(0), K_W(0), N_E(0)$ given. For $\bar{\ell}_E = \bar{\theta} = 1$ and $\tilde{w}_E = \bar{w}_W$ in steady state equilibrium, the local stability properties of the extended model are identical to the benchmark model as discussed in Section 3.4. However, wage-setting behavior affects the transitional dynamics of the endogenous variables and the steady state distribution of production factors in both regions.

4.3 Calibration

α	$\bar{r}$	δ	γ_L	γ_K	P	β
0.3	0.03	0.05	291.67	10.8	5	0.82

Table 3: Calibration of the model with wage-setting behavior

The baseline calibration in Table 2 remains unchanged except for two parameter values. First, the concavity parameter β of the behavioral function in (31) is specified to be consistent with the German Reunification period. Therefore, dividing both sides of (31) by the western wage at time t and using the average East German relative real wage and real labor productivity level from Table 1 to get $\beta \cong 0.82$. Average values are chosen, so that $\tilde{w}_E$ traces on average the observed time path of East-West real wage convergence between 1991 and 2011.

Second, the migration cost intensity γ_L is adjusted. The initial theoretical migration benefit for an East German household now reads $\lambda_E(0) = \int_0^\infty [\tilde{w}_E(s)\ell_E(s) - \bar{w}_W]\, e^{-\bar{r}s} ds$.

Thus, labor income convergence may be approximated by the time series of real wage convergence in Table 1, which implies a constant convergence rate of $\kappa \cong 0.03$. For a real wage gap of 42 percent in 1991, the initial theoretical migration benefit and the migration cost intensity read $\lambda_E(0) \cong -7$ and $\gamma_L \cong 291.67$, respectively. Note that, using real wage convergence instead of labor productivity convergence implies a lower initial migration benefit and therefore a lower migration cost intensity. All parameter values are summarized in Table 3.

4.4 Numerical Evaluation

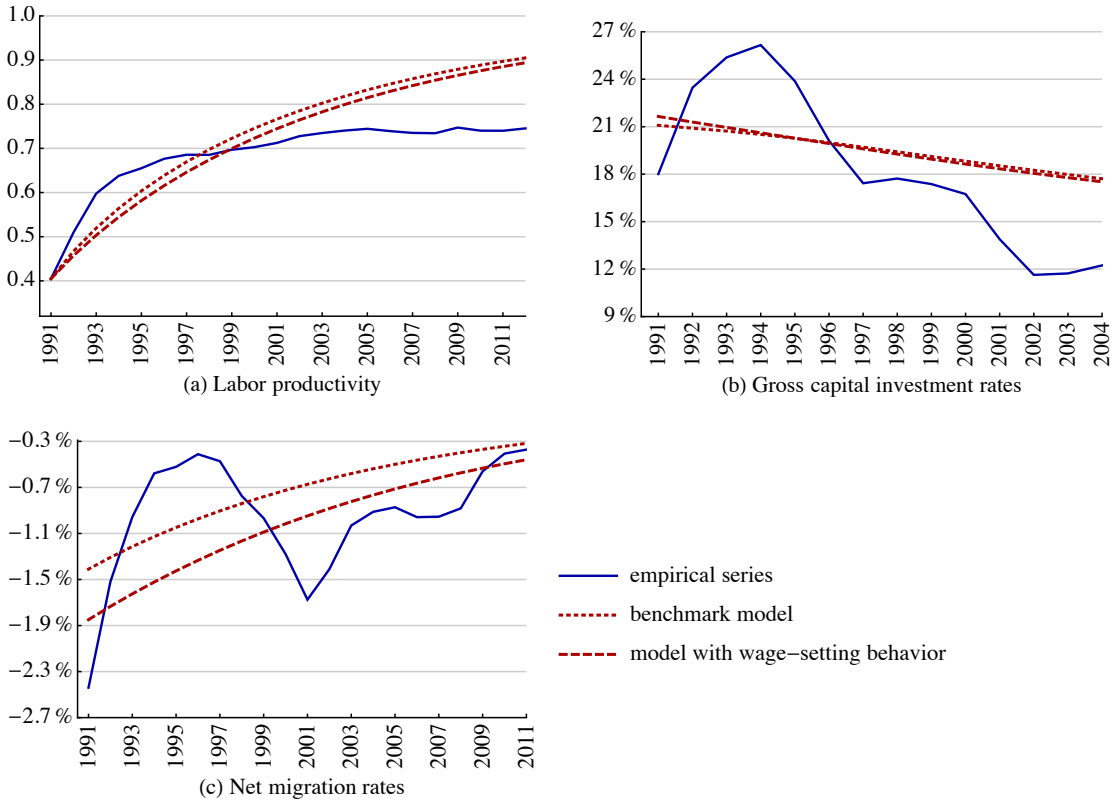


Figure 6: Time paths of labor productivity convergence, gross capital investment rates, and net migration rates implied by the model with wage-setting behavior. Set of parameters as in Table 3.

As displayed by the dashed lines in Figure 6, wage setting slows down labor productivity convergence marginally but increases net migration significantly compared to the benchmark model. Aggregate migration increases from 981,300 (benchmark model) to 1,317,600 persons (compared with 1,186,500 East German migrants) over the observable

time period. That is, wage-setting behavior lowers the eastern employment rate and depresses labor income.[21] Depressed income in turn causes more labor to migrate to the west in each period (panel c). Moreover, the lower employment rate leads to a fall in regional GDP and depresses capital investment incentives which keeps the gross capital investment rate nearly unchanged (panel b). Even though the quantitative effect is rather small, less capital investments, however, delay labor productivity convergence between both integrated regions (panel a).

Massive lay-offs and rising east-west migration rates as a consequence of the high-wage policy required the German government to substantially increase social security transfers to East German households. In a final step, this aspect of Reunification motivates the introduction of a public transfer system into the existing framework and to evaluate the predictions of the further extended model.

5 Introducing Public Social Transfers

Consider the model with wage-setting behavior described in Section 4. In the following extension, I assume that eastern individuals exclusively enjoy public transfers z_E to compensate for their loss of labor income due to lower employment rates during the transition to the steady state. These transfers then reflect parts of the enhanced social security payments to East German households in the aftermath of unification.

5.1 Model Modifications

To finance transfers, assume that the government taxes capital and labor income in both regions with an identical but time-varying tax rate τ. The government runs a balanced budget according to

$$\tau(t)\left[Y_W(t) + Y_E(t)\right] \quad = \quad Z_E(t), \tag{36}$$

where $Z_E = z_E N_E$ denotes aggregate government spendings. Eastern individuals receive transfers of $z_E = (1 - \ell_E)\,\tilde{w}_E$ to compensate for their loss of labor income due to non-

[21] In the appendix (Fig. B.1-B.2) the transitional dynamics implied by both extended models are displayed.

employment $(1-\ell_E)$. Multiplying both sides by the eastern population size at time t and substituting into (36) yields the time path of the tax rate:

$$\tau(t) \quad = \quad \frac{\left[\tilde{w}_E(t) - \ell_E(t)\tilde{w}_E(t)\right] N_E(t)}{Y_W(t) + Y_E(t)}, \tag{37}$$

where ℓ_E and $\tilde{w}_E$ are given by (32) and (33) respectively. During the transition to the steady state, the tax rate is positive as $\ell_E < 1$ and converges to zero as the employment rate converges to unity.

The representative firm in each region now chooses paths for investment I_i and labor L_i, so as to maximize the present value of its *after-tax* net cash flow:

$$\begin{aligned} \max_{L_i, I_i} \quad & \int_0^\infty \left\{ (1-\tau)\left[K_i^\alpha L_i^{1-\alpha} - w_i L_i\right] - I_i \left[1 + \frac{\gamma_K}{2}\left(\frac{I_i}{K_i}\right)\right] \right\} e^{-\bar{r}t} dt \\ s.t. \quad & (7), \quad i = E, W, \end{aligned} \tag{38}$$

where capital adjustment costs are not deductible and initial conditions are given. The evolution of the capital stock in both regions is then governed by

$$\dot{K}_i \quad = \quad \left(\frac{q_i - 1}{\gamma_K} - \delta\right) K_i, \tag{39}$$

$$\dot{q}_i \quad = \quad (\bar{r} + \delta)\, q_i - (1-\tau)\,\alpha \left(\frac{K_i}{N_i \ell_i}\right)^{\alpha-1} - \frac{(q_i - 1)^2}{2\gamma_K}, \tag{40}$$

where $L_i = N_i \ell_i$ and $i = E, W$. A positive tax rate reduces capital investment incentives and the rate of capital accumulation in both regions. The western wage remains given by (8), while the eastern wage is affected by wage-setting behavior and evolves according to (33).

Human wealth of an eastern individual is now comprised by *after-tax* labor income, $(1-\tau)\tilde{w}_E \ell_E$, and public transfers, $(1-\ell_E)\tilde{w}_E$. More specifically, individual wealth reads

$$V_E(t) = \begin{cases} a_E(t) + \displaystyle\int_t^\infty e^{-\bar{r}[s-t]} \left[1 - \tau(s)\ell_E(s)\right] \tilde{w}_E(s) ds & \text{if agent does not migrate} \\ a_E(t) + \displaystyle\int_t^\infty e^{-\bar{r}[s-t]} \left[1 - \tau(s)\right] w_W(s) ds - mc_E & \text{if agent migrates in } t \end{cases}. \tag{41}$$

Using the adjusted individual wealth (41) in the optimization problem of the representative eastern individual in (11) and (12) yields the optimal consumption path of a migrant and non-migrant differing by the present value labor income differential across both regions. Again, this difference determines the migration benefit, which now evolves according to

$$\dot{\lambda}_W(t) \quad = \quad \bar{r}\lambda_W(t) - \{[1-\tau(t)]\, w_W(t) - [1-\tau(t)\ell_E(t)]\, \tilde{w}_E(t)\}\,. \tag{42}$$

5.2 Equilibrium and Stability Analysis

An equilibrium of the dynamic small open economy with wage-setting behavior and tax-financed public transfers is defined as follows:

Definition 5.1. Consider the model described in Section 4 extended by a public transfer system described by (36) and (37). Suppose the given interest rate $\bar{r}$ is stationary and equals individual's time preference rate ρ. For any set of initial conditions $\{K_E(0), K_W(0), N_E(0)\}$ a competitive equilibrium is given by a set of time paths for quantities $\{K_E(t), K_W(t), N_E(t)\}_{t\in(0,\infty)}$, prices $\{q_E(t), q_W(t), \lambda_W(t), \tilde{w}_E(t), w_W(t)\}_{t\in(0,\infty)}$, and values $\{\ell_E(t), \tau(t), z_E(t)\}_{t\in(0,\infty)}$ such that the capital stock in both regions evolves according to (5) and it holds in any period that

1. firms maximize the present value of net cash flow;
2. individuals maximize life-time utility;
3. if and only if labor is interregionally mobile, migration benefits equal migration costs, $\lambda_j(t) = mc_i, i, j \in \{E, W\}, i \neq j$;
4. the labor market clears;
5. the governments budget is balanced according to (36).

The dynamic system of the fully extended model is described by (19), (39), (40), and (42). The local stability properties of the extended model are again identical to the benchmark model as $\bar{\tau} = \bar{z}_E = 0$ and $\tilde{w}_E = \bar{w}_W$ in steady state. However, public transfers affect the transitional dynamics of the endogenous variables and the steady state distribution of production factors in both regions.

5.3 Numerical Evaluation

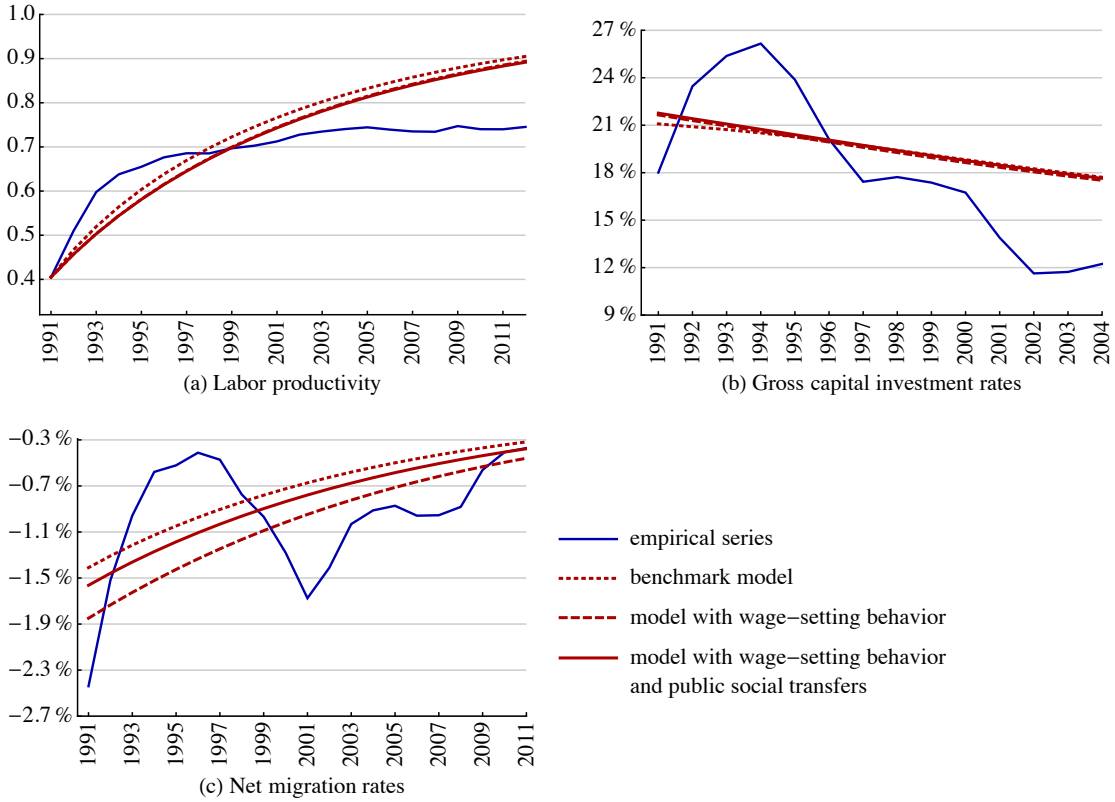


Figure 7: Time paths of labor productivity convergence, gross capital investment rates, and net migration rates implied by the model with wage-setting behavior and public social transfers. Set of parameters as in Table 3.

Compensating eastern individuals for their labor income loss due to wage setting requires a tax rate of about 1.2 percent initially (see panel d of Figure B.2 in the appendix). Tax rate and transfers decrease over time as capital-labor ratios are equalized across both regions. Then, as displayed by the solid (red) lines in Figure 7, public transfers mitigate the effect of wage setting on net migration, the time path moves closer to the observable migration pattern (panel c), whereas the implied labor productivity path (panel a) and the implied gross capital inflow path (panel b) do not change significantly. More generous public transfers would lower net migration rates further, but would also require a higher tax rate that may distorts capital investments and delays labor productivity convergence more strongly. The fully extended model, however, predicts total net migration between 1991 and 2011 of about 1,110,900 persons, compared with 1,186,500 East German migrants over the same period. Thus, the model matches the aggregate migration pattern in unified

Germany successfully.

6 Summary and Conclusion

The massive movement of capital and labor in opposite directions is the most striking characteristic of economic integration of Eastern and Western Germany. Thus, the first aim of this paper was to evaluate the potential of a neoclassical growth model with capital and labor mobility to explain major empirical trends of the German integration episode. I therefore set up a benchmark model and compared it's predictions with the empirical time series for relative East-West-German labor productivity convergence, East-German labor force net outmigration, and East-German private gross capital inflows. As a result, the model implies faster productivity convergence (apart from the first eight years), lower emigration and less capital inflows than observed in unified Germany.

The second aim was then to examine the impact of wage-setting behavior and public social transfers, two policy measures that have been actually implemented, on the macroeconomic development in unified Germany. I first introduced wage-setting behavior into the framework, which, as a result, increased annual net migration rates significantly, slowed down labor productivity convergence marginally, and changed gross capital inflows in a minor way. In a second step, I extended the model by public social transfers in favor of eastern workers. As a result, net migration rates are lowered, whereas implied labor productivity convergence and gross capital inflows did not change significantly.

In summary, I show that the fully extended model is able to replicate aggregate migration pattern in unified Germany. Furthermore, wage-setting behavior has delayed labor productivity convergence between both German regions and would have increased East-West net migration substantially if government interventions had not been carried out. Finally, public social transfers have been successful in reducing the effect of wage setting on East-West net migration.

The paper complements the theoretical literature on the pattern of German East-West convergence by including a key feature, namely migration, into the analysis. The proposed model therefore provides a tractable analytical framework for future research studying

public policy effects on capital and labor mobility in the process of economic integration.

A Appendix: Derivations

The Firm's Maximization Problem

The current-value Hamiltonian corresponding to the optimization problem of a firm of region i (see Definition 3.1) is given by

$$\mathcal{H}_i = K_i^{\alpha} L_i^{1-\alpha} - w_i L_i - I_i \left[1 + \frac{\gamma_K}{2}\left(\frac{I_i}{K_i}\right)\right] + q_i \left(I_i - \delta K_i\right).$$

where q_i is the multiplier (co-state-variable) associated with constraint (7). Necessary optimality conditions are ${}^{\partial \mathcal{H}_i}/_{\partial L_i} = 0$, ${}^{\partial \mathcal{H}_i}/_{\partial I_i} = 0$, $\dot{q}_i = \bar{r} q_i - {}^{\partial \mathcal{H}_i}/_{\partial K_i}$, and the corresponding transversality condition. Thus,

$$w_i = (1-\alpha)\left(\frac{K_i}{L_i}\right)^{\alpha}, \tag{43}$$

$$\frac{I_i}{K_i} = \frac{q_i - 1}{\gamma_K}, \tag{44}$$

$$\dot{q}_i = (\bar{r} + \delta) q_i - \alpha \left(\frac{K_i}{L_i}\right)^{\alpha-1} - \frac{\gamma_K}{2}\left(\frac{I_i}{K_i}\right)^2 \tag{45}$$

$$\lim_{t\to\infty} e^{-\bar{r}t} q_{it} K_{it} = 0. \tag{46}$$

Substituting (44) into (5) and (45) yields (9) and (10).

Eigenvalues of Matrix $\mathcal{M}$

Note that $\mathcal{M}$ is a 2×2 block matrix with four 3×3 matrices. Since both matrices on the diagonal of $\mathcal{M}$ are diagonal in itself the eigenvalues of $\mathcal{M}$ can be calculated analytically in a simple straight forward way. Let $\mathcal{F}$ denote the quadratic upper right matrix and $\mathcal{C}$ the quadratic lower left matrix of $\mathcal{M}$, so that

$$\mathcal{M} = \begin{pmatrix} \bar{r}\mathcal{I} & \mathcal{F} \\ \mathcal{C} & 0\mathcal{I} \end{pmatrix},$$

where $\mathcal{I}$ denotes the identity matrix. The matrix eigenvalues are given by the solution of the characteristic polynomial $\rho_{\mathcal{M}}(\lambda) = \det(\mathcal{M} - \lambda\mathcal{I}) = 0$ with

$$\mathcal{M} - \lambda\mathcal{I} \;=\; \begin{pmatrix} (\bar{r} - \lambda)\mathcal{I} & \mathcal{F} \\ \mathcal{C} & -\lambda\mathcal{I} \end{pmatrix}.$$

Since the diagonal of $\mathcal{M}$ is given by two diagonal matrices the characteristic polynomial $\rho_{\mathcal{M}}(\lambda)$ simplifies to $\rho_{\mathcal{M}}(\lambda) = -\det(\mathcal{FC} + (\bar{r} - \lambda)\lambda\mathcal{I}) = 0$, implying the two roots λ_1 and λ_2. Let $-\varphi$ be the solution to $(\bar{r} - \lambda)\lambda$ such that

$$\lambda_{1,2} \;=\; \frac{1}{2}\left(\bar{r} \pm \sqrt{\bar{r}^2 + 4\varphi}\right). \tag{47}$$

Then, computation further simplifies to the solution of the characteristic polynomial of the 3×3 matrix $\mathcal{FC}$:

$$\rho_{\mathcal{FC}}(\varphi) \;=\; -\det(\mathcal{FC} - \varphi\mathcal{I}) \;=\; 0, \tag{48}$$

where

$$\mathcal{FC} \;=\; \begin{pmatrix} -F^E_{KK}\bar{K}_E\gamma_K^{-1} & 0 & F^E_{KL}\gamma_L^{-1} \\ 0 & -F^W_{KK}\bar{K}_W\gamma_K^{-1} & F^W_{KL}\gamma_L^{-1} \\ F^E_{LK}\bar{K}_E\gamma_K^{-1} & -F^W_{LK}\bar{K}_W\gamma_K^{-1} & -(F^E_{LL} - F^W_{LL})\gamma_L^{-1} \end{pmatrix}.$$

F^i_{jk} denotes the second derivative of the production function of region $i = \{E, W\}$ with respect to $j, k \in \{K_i, L_i\}$. The solution to (48) yields the three roots φ_1, φ_2, and φ_3 and is given by $\rho_{\mathcal{FC}}(\varphi) = \det(\mathcal{F})\det(\mathcal{C}) - B\varphi + A\varphi^2 - \varphi^3 = 0$, where

$$\begin{aligned} A \;&:=\; \mathrm{tr}(\mathcal{FC}) > 0, \\ B \;&:=\; [\det(\mathcal{FC}_{11}) + \det(\mathcal{FC}_{22}) + \det(\mathcal{FC}_{33})] > 0, \end{aligned}$$

and $\det(\mathcal{FC}_{ii})$ is the minor of $\mathcal{FC}$ formed by eliminating row i and column i.[22] Note that

[22] The elements of $\mathcal{F}$ exhibit the following properties: $F^i_{KL} = F^i_{LK} > 0$ and $F^i_{KK} < 0, i = E, W$ as well as $F^W_{LL} > 0$ and $F^E_{LL} < 0$.

the determinant of $\mathcal{F}$ is zero for a Cobb-Douglas production function, so the solution to (48) reduces to $\rho_{\mathcal{FC}}(\varphi) = \varphi\left(\varphi^2 - A\varphi + B\right) = 0$ and is solved by

$$\varphi_1 = 0 \tag{49}$$

$$\varphi_{2,3} = \frac{1}{2}\left(A \pm \sqrt{A^2 - 4B}\right). \tag{50}$$

Substituting (49) and (50) into (47) yields the six eigenvalues of matrix $\mathcal{M}$.

B Appendix: Trajectories

- Figure B.1 displays the trajectories of the endogenous variables implied by the model with wage-setting behavior discussed in Section 4.

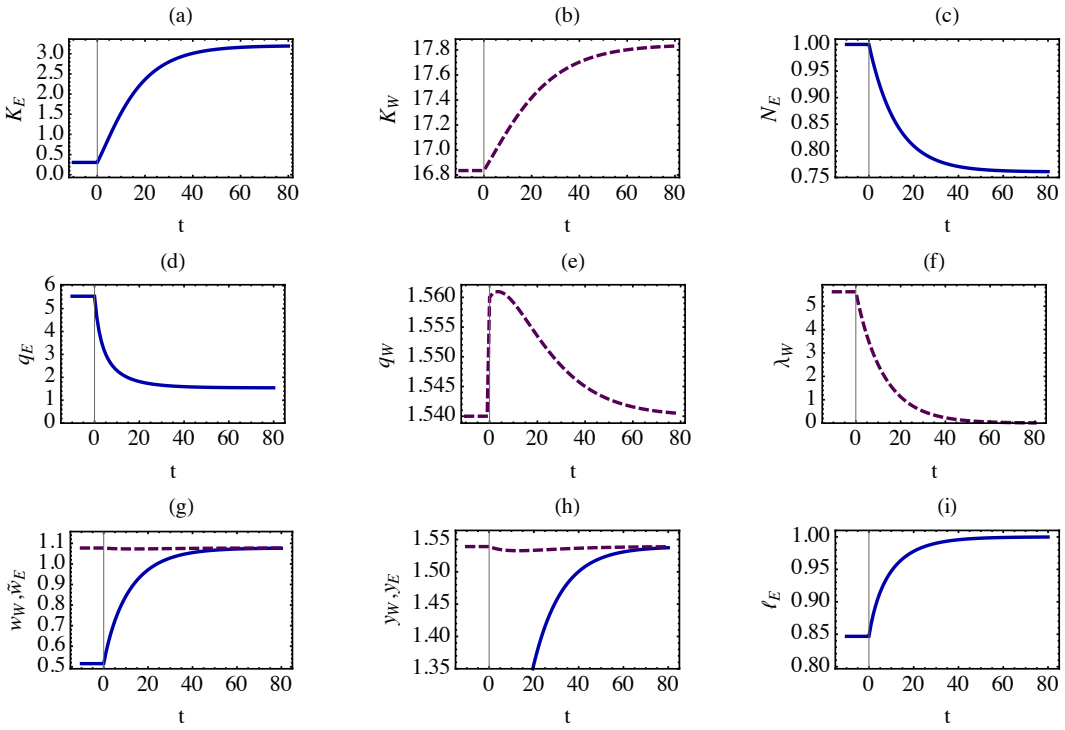


Figure B.1: Transitional dynamics implied by the model with wage-setting behavior. Solid lines: time path eastern variables; dashed lines: time path western variables. Set of parameters as in Table 3.

- Figure B.2 displays the trajectories of the endogenous variables implied by the model with wage-setting behavior and public social transfers discussed in Section 5.

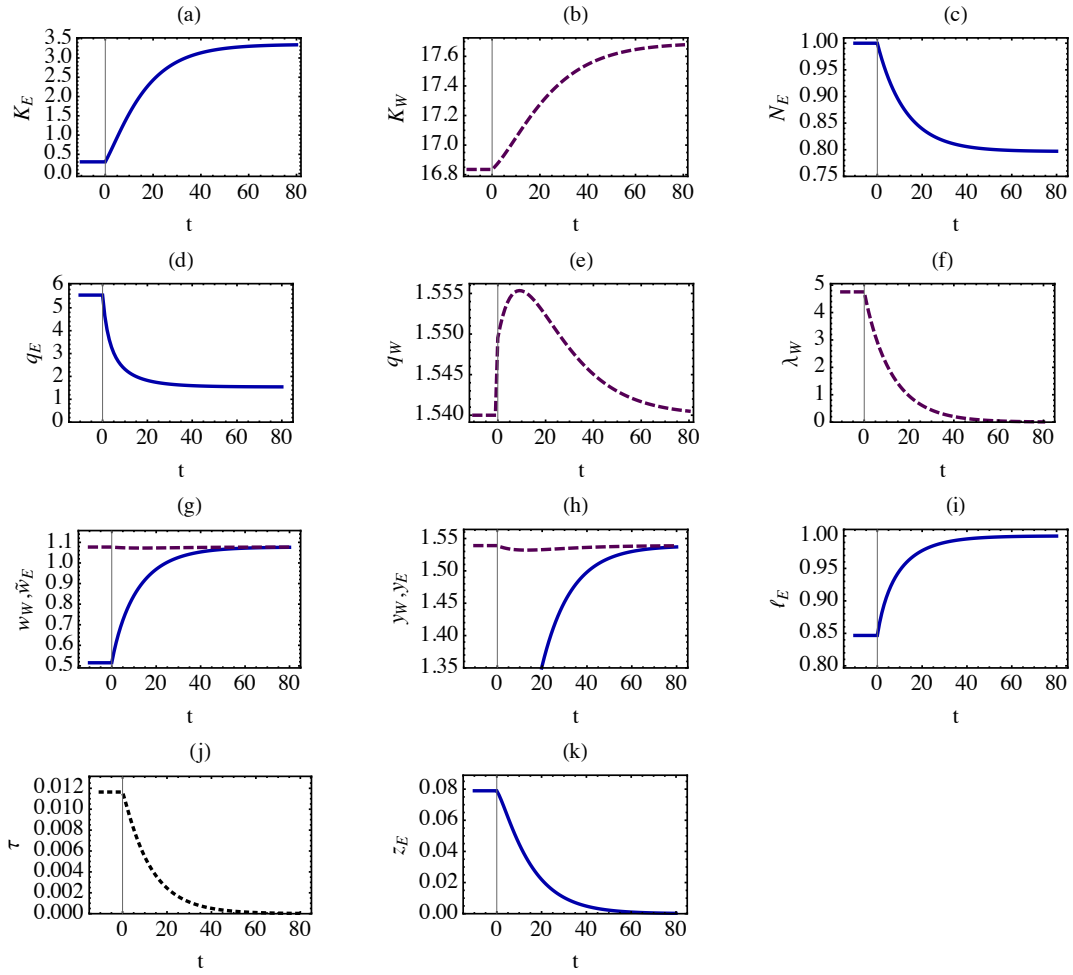


Figure B.2: Transitional dynamics implied by the model with wage-setting behavior and public social transfers. Solid lines: time path eastern variables; dashed lines: time path western variables; dotted line: time path tax rate. Set of parameters as in Table 3.

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Online-Appendix

Here I provide the derivations of the linearized dynamic system applied in the stability analysis in Section 3.4. The first-order Taylor approximation of (20) and (21) yields

$$\dot{K}_i = \bar{K}_i \gamma_K^{-1} (q_i - \bar{q}_i) + \left[\frac{\bar{q}_i - 1}{\gamma_K} - \delta \right] \left(K_i - \bar{K}_i \right), \quad i = E, W.$$

Using $\bar{q}_E = \bar{q}_W = 1 + \delta\gamma_K$ one gets

$$\dot{K}_E = \bar{K}_E \gamma_K^{-1} (q_E - \bar{q}_E), \tag{51}$$

$$\dot{K}_W = \bar{K}_W \gamma_K^{-1} (q_W - \bar{q}_W). \tag{52}$$

Linearizing the evolution of the eastern population size in (22) yields

$$\dot{N}_E = -\gamma_L^{-1} \left(\lambda_W - \bar{\lambda}_W \right). \tag{53}$$

Linearizing (23) and (24) yields

$$\dot{q}_E = \bar{r} (q_E - \bar{q}_E) - F_{KK}^E \left(K_E - \bar{K}_E \right) - F_{KL}^E \left(L_E - \bar{L}_E \right), \tag{54}$$

$$\dot{q}_W = \bar{r} (q_W - \bar{q}_W) - F_{KK}^W \left(K_W - \bar{K}_W \right) - F_{KL}^W \left(L_E - \bar{L}_E \right), \tag{55}$$

where $\bar{q}_E = \bar{q}_W = 1 + \delta\gamma_K$ is used and F_{jk}^i is the second derivative of the production function of region $i = \{E, W\}$ with respect to $j, k = \{K_i, L_i\}$. The second derivatives explicitly read

$$F_{KK}^E = \alpha(\alpha - 1) K_E^{\alpha - 2} L_E^{1-\alpha} < 0, \qquad F_{KL}^E = \alpha(1-\alpha) K_E^{\alpha-1} L_E^{-\alpha} > 0,$$

$$F_{KK}^W = \alpha(\alpha - 1) K_W^{\alpha - 2} (P - L_E)^{1-\alpha} < 0, \qquad F_{KL}^W = -\alpha(1-\alpha) K_W^{\alpha-1} (P - L_E)^{-\alpha} < 0.$$

Linearizing the evolution of the migration benefit in (25) yields

$$\dot{\lambda}_W = \bar{r} \left(\lambda_W - \bar{\lambda}_W \right) + F_{LK}^E \left(K_E - \bar{K}_E \right) - F_{LK}^W (K_W - \bar{K}_W) - (F_{LL}^W - F_{LL}^E) \left(L_E - \bar{L}_E \right), \tag{56}$$

where

$$
\begin{aligned}
F^E_{LK} &= \alpha(1-\alpha)K_E^{\alpha-1}L_E^{-\alpha} > 0, \qquad F^E_{LL} = -\alpha(1-\alpha)K_E^{\alpha}L_E^{-\alpha-1} < 0, \\
F^W_{LK} &= \alpha(1-\alpha)K_W^{\alpha-1}(P-L_E)^{-\alpha} > 0, \qquad F^W_{LL} = \alpha(1-\alpha)K_W^{\alpha}(P-L_E)^{-\alpha-1} > 0.
\end{aligned}
$$

The linearized dynamic system is defined by equations (51)-(56).

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Author
Sebastian BÖHM
University of Fribourg, Faculty of Economics and Social Sciences, Department of Economics, Bd de Pérolles 90, CH-1700 Fribourg, Switzerland. Phone: +41 26 300 8253;
Email: sebastian.boehm@unifr.ch ; Website: http://www.unifr.ch/makro/de/team/post-doc-snf

Abstract
The massive movement of capital and labor in opposite directions is the most striking characteristic of economic integration of Eastern and Western Germany. Beyond that, wage-setting behavior during the early years of unification and massive public social transfers have affected the transition path of the Eastern economy. In this paper, I set up a two-region open economy model with capital and labor mobility, wage-setting behavior, and public social transfers to explain major empirical trends of the German integration episode. I show that the model is able to replicate aggregate migration pattern in unified Germany and that wage-setting behavior has delayed labor productivity convergence between both German regions, whereas public social transfers have reduced the effect of wage setting on East-West net migration.

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University of Fribourg, Switzerland, Faculty of Economics and Social Sciences

Bd de Pérolles 90, CH-1700 Fribourg
Tél.: +41 (0) 26 300 82 00
decanat-ses@unifr.ch www.unifr.ch/ses