07.2014
N° 450

WORKING PAPERS SES

Three-Valued Modal Logic for Qualitative Comparative Policy Analysis with Crisp-Set QCA

© Georg P. Mueller

FACULTÉ DES SCIENCES ECONOMIQUES ET SOCIALES
WIRTSCHAFTS- UND SOZIALWISSENSCHAFTLICHE FAKULTÄT
UNIVERSITÉ DE FRIBOURG | UNIVERSITÄT FREIBURG

1. Crisp-set QCA: Principles and problems

Qualitative Comparative Analysis QCA is a booming methodology, which has become a standard for small-N research. Currently, most scholars use the *fuzzy-set* version of QCA. There is however an interesting alternative, which is developed in this article: *Crisp-set* QCA, from which Charles Ragin (1989) departed about 25 years ago, when he introduced *Qualitative Comparative Analysis* in political science. Crisp set QCA compares binary conditions X_1, X_2, ... with regard to a binary outcome-variable Y and synthesizes the result in a Boolean formula. Tab. 1 presents a typical data matrix suitable as input for this method: deference to persons by gender and status, e.g. at the door to a business meeting. As a comparison of the last two columns of Tab. 1 demonstrates, the Boolean synthesis of this table is obviously

Chief X_2 ==> Deference Y (1)

whereas gender (X_1 = Woman) has no influence on the dependent variable Y = Deference.

Tab. 1: An exemplary dataset:
Deference to persons, by gender and status.

Person	X_1 Woman	X_2 Chief	Y Deference
1	1	1	1
2	0	1	1
3	1	0	0
4	0	0	0

Legend: X_1 = Woman: 1 = yes, 0 = no. X_2 = Chief: 1 = yes, 0 = no. Y = Deference to a person described by X_1 and X_2: 1 = yes, 0 = no, where the table is based on the assumption Y = X_2.

If data-sets have more cases and variables than Tab. 1, for crisp-set QCA it is advisable to use the following formalized three-step procedure (Ragin 1989, Ragin 1998, Rihoux & De Meur 2009):

1. As a first step, the information about all cases with an outcome-value Y=1 (true) has to be *summarized* in a Boolean expression in disjunctive normal form (Muzio and Wesselkamper 1986: chap. 2.3): each case from the data table with the outcome *Y=1* becomes a conjunction of the independent variables X_1, X_2, ..., X_i, ... or their negation $\neg X_i$, if X_i = 0 (false). Thus all these variables are linked by Boolean AND-operators. Case by case, these conjunctions are *subsequently* united by OR-operators and build

one final Boolean formula, specifying the condition from which follows Y=1. Consequently, from the exemplary Tab. 1 we infer

$$(X_1 \text{ AND } X_2) \text{ OR } (\neg X_1 \text{ AND } X_2) \quad (2)$$

where the AND-clauses left and right of the Boolean OR represent the biographical situations of the respective persons 1 and 2 in Tab. 1.

2. Since the complexity of the disjunctive normal form resulting from step 1 rapidly increases with the number of the independent variables X_1, X_2, ... , there is a need to *simplify* this formula in a second step. For this purpose crisp-set QCA uses the Quine-McCluskey algorithm (Mendelson 1970: chap. 4). The simplification can be done by hand or with computer software like fs/QCA[1)] (Drass & Ragin 2013). Thus from the exemplary formula (2) follows [2)]

$$(X_1 \text{ AND } X_2) \text{ OR } (\neg X_1 \text{ AND } X_2) = (X_1 \text{ OR } \neg X_1) \text{ AND } X_2 = X_2 \quad (3)$$

3. In a third and final step, the simplified formula has to be *explored* with regard to its logical implications. For example from formula (3) follows

$$X_2 \Longrightarrow Y \quad (4)$$

This means that for deference Y in the workplace only status X_2 is relevant, but not gender X_1.

Tab. 2: A modified exemplary dataset, based on Tab. 1.

Case	X_1 Woman	X_2 Chief	Y Deference	Y*= Rec. Y
1	1	1	?	--
2	0	1	1	1
3a	1	0	0	0
3b	1	0	0	0
3c	1	0	1	0
4	0	0	0	0

Legend: Y*: Recoding of Y by canonical crisp-set QCA -methodology. Other definitions: see Tab. 1.

In principle, the aforementioned procedure of crisp-set QCA may be applied to any qualitative data-set with dichotomized dependent and independent variables. The attainability of planned goals or the presence or absence of unintended consequences would be typical applications in *public policy analysis.* Nonetheless, crisp-set QCA also has limits and problems:

a) For some configurations (X_1, X_2, ... , X_n) the dependent variable Y may be missing (Ragin 1989: 104 ff.). Case 1 in Tab. 2 exemplifies this situation, which in the terminology of QCA is also called a *logical remainder* (Schneider & Wagemann 2012: chap. 6). In *public policy analyses,* there are at least three sources for this kind of missing instantiation. One of them is *missing implementation*: no governmental agency has ever tried to realize the policy under discussion. The second is *technical censoring:* the policy has only very recently been implemented and effects take time to develop and are thus not yet visible. Finally, missing instantiation may be the result of *missing evaluation* of policies, which really have been used in the past.

b) For certain other configurations, the dependent variable Y may have *contradictory* or *inconsistent* values 0 and 1, depending on the cases compared (Schneider & Wagemann 2012: chap. 5; Ragin 1989: 113 ff.). In Tab. 2, for example, the cases 3a, 3b, and 3c are contradictory. This situation is not so unlikely in policy evaluations, if explanatory binary variables X_i have been neglected, which would otherwise have split the contradictory configuration (X_1, X_2, ... , X_n). The problem is further accentuated by the fact that from the point of view of pure Boolean logic, the existence of just *one* reliable case with a deviant value Y is sufficient to constitute a contradiction.

There are two obvious strategies in order to solve the problems (a) and (b) (Schneider & Wagemann 2012: chap. 5.1, 5.2, 6). One of them is the *elimination* of contradictory or missing values (see Y* of case 1 in Tab. 2). The other strategy is *imputation.* When following this second strategy, the dependent variable of a contradictory configuration is, for example, set equal to the most frequent value of this configuration (cf. Y* and Y of case 3c in Tab. 2) or logical remainders are assigned theoretically plausible counterfactual values. Both strategies are problematic because of their tendency to oversimplify the analysis. Eliminating or changing contradictory cases according to majority principles deprives qualitative analyses of new insights from deviant cases, which deserve special attention, as demonstrated by grounded theory (Strauss & Corbin 1998, Charmaz 2006). Similarly, the imputation of logical remainders is often based on debatable ad-hoc theories and tends to stabilize existing knowledge by avoiding its falsification.

Hence this article proposes a different approach: the coding of contradictions and logical remainders as *indeterminate* cases, which are subsequently treated by three-valued Boolean and modal logic (Beall & Fraassen 2003, Bergmann 2008, Mueller 2008). Apart from integrating the indeterminate truth into scientific analysis, this approach also opens new opportunities to operationalize concepts like the necessary, possible or impossible consequences of a given set of conditions. These innovations are also of value for comparative *public policy analysis,* where there is an increasing usage of QCA (see e.g. Kangas (1996) or Ragin (1996)).

2. An overview of three-valued modal logic

Three-valued logic was originally introduced by Jan Lukasiewicz (1970) in order to describe with a code other than *t* or *f* the truth of future events with *uncertain* outcomes. Obviously, this third code *i* also satisfies important needs of *public policy analyses,* where problems of technical censoring or non-implementation may hide the future truth, as we have seen above.

The idea that the third truth-value i stands for the hidden truth, which can only be disclosed in the future, is also present in the Lukasiewicz extension of classical Boolean logic to three truth-values, as presented in Tab. 3 (Bergmann 2008: 76). As this table indicates, the result Z of the Boolean operations OR, NOT, or AND has a *certain* truth-value *true* or *false,* if the experimental replacement of a table entry X = i or Y = i by the two possible alternative values *true* or *false* has in conventional *two*-valued logic no influence on the truth of Z (see in Tab. 3 e.g. Z = (X OR Y) = 1 for entries X=1 and Y=i). However, if the uncertainty about X or Y is propagated to the outcome Z, because it *varies* with the mentioned experimental changes in the values of X and Y, the resulting Z has the value *i = indeterminate* (see in Tab. 3 e.g. Z = (X AND Y) = i for entries X=1 and Y=i).[3)]

Tab. 3: The definitions of Boolean operators in three-valued logic.[4)]

		Z =		
X	Y	¬ X	X OR Y	X AND Y
1	1	0	1	1
1	i		1	i
1	0		1	0
i	1	i	1	i
i	i		i	i
i	0		i	0
0	1	1	1	0
0	i		i	0
0	0		0	0

Legend: ¬ : Negation; 0 = false; 1 = true; i = indeterminate. Source: Bergmann 2008: 76.

The logic system of Lukasiewicz has been challenged by other multi-valued logical calculi, e.g. of Bochvar and Kleene (Rescher 1969: chap. 2). Nonetheless we will hold to the intuitively very convincing approach of Lukasiewicz, mainly because of the reduced importance of truth-table Tab. 3 for the concerns of this article: by the use of *modal logic,* as proposed here, it is possible to avoid to work with the Lukasiewicz truth-table Tab. 3.

Modal logic formalizes the *necessity* and *possibility* of propositions. By the respective logical operators NEC (necessity) and POS (possibility) it maps the indeterminate truth i of these propositions to the conventional values true = 1 or false = 0. The precise definitions of these modal operators are based on Rescher (1969: 25) and given in Tab. 4. If there is the *possibility* of Y and consequently POS(Y) = 1, Y is according to Tab. 4 either true or indeterminate. The latter truth-value i obviously implies the possibility that Y may prove to be true, once we know more about reality. If we assume the *necessity* of Y and consequently NEC Y = 1, Y must be true and not indeterminate or even false (see Tab. 4).

Tab. 4: The definitions of the modal operators NEC and POS.

Y	¬ Y	POS Y	NEC Y	POS ¬ Y	NEC ¬ Y
0	1	0	0	1	1
i	i	1	0	1	0
1	0	1	1	0	0

Legend: ¬ : Negation; POS: Possibility; NEC: Necessity; 0 = false; 1 = true; i = indeterminate.
Source: Rescher 1969: 25.

In combination with the negation ¬Y, there are in total four modal transformations of a three-valued logical variable Y: POS Y, NEC Y, POS ¬Y, and NEC ¬Y. If these expressions are the right-hand of a conventional Boolean implication X ==> ... , they allow us to classify the conditional variable X. In the case of X ==> NEC Y, X is a *strict trigger* of Y and X ==> NEC Y will in the following sections of the paper be denoted as X —> Y (see Glossary). If X ==> NEC ¬Y, X is said to be a *strict inhibitor* of Y, for which we will use the abbreviation X –//–> Y. Similarly, in what follows, the expression X ==> POS Y will be denoted as X ----> Y. Thus, X is a *potential trigger* of Y. Finally, if X ==> POS ¬Y, X is a *potential inhibitor* of Y and X ==> POS ¬Y will be denoted as X --//--> Y. It is obvious, that this typology of strict or potential triggers and inhibitors (see Glossary) may be of considerable use for *policy analyses* of the attainability of planned goals as well as the conditions by which undesired consequences can be avoided.

3. QCA with three-valued modal logic

Although the indeterminate truth-value i was originally introduced in order to code uncertain future outcomes, it is of course also applicable to contradictory configurations, where there is a similar uncertainty about the correct value of the outcomes Y. Hence this paper proposes to replace in the original data table all missing and all contradictory *outcomes* by the *indeterminate value i.* Thus, instead of omitting case 1 of Tab. 2, it is assigned in Tab. 5 a recoded outcome Y'= i. Similarly, instead of using the majority principle in order to assign in Tab. 2 the contradictory cases 3a,b,c a common value Y*= 0, we assume in Tab. 5 that these cases have an indeterminate value Y'= i.

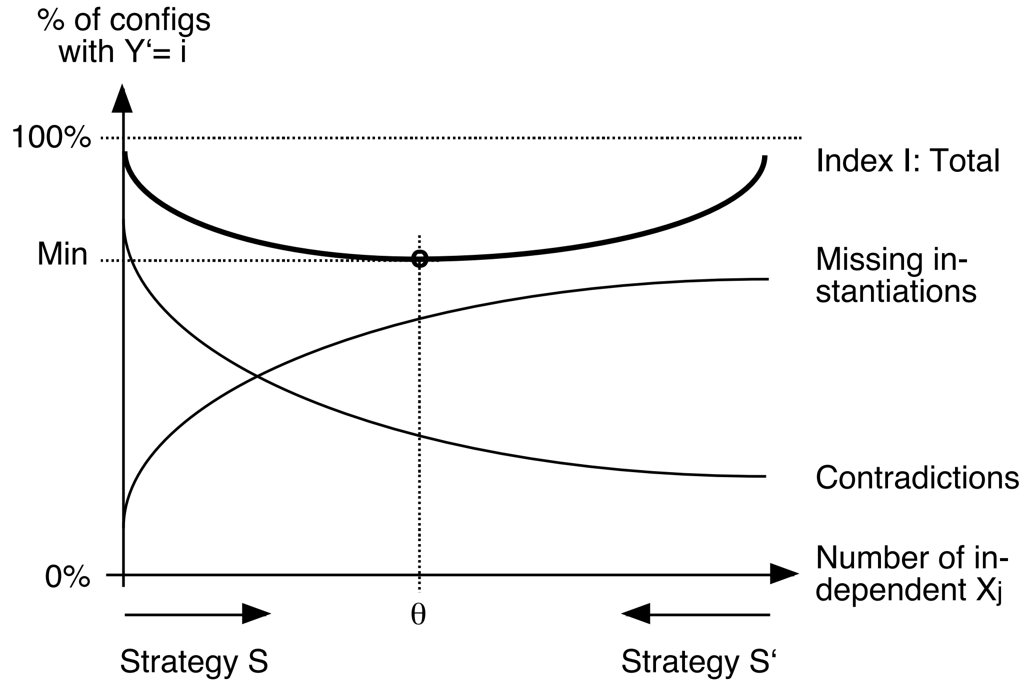
Tab. 5: An exemplary application of the tools of three-valued modal logic to Tab. 2.

Case	X_1 = Woman	X_2 = Chief	Y = Deference	Y' = Rec. Y	NEC Y'	NEC ¬ Y'	POS Y'	POS ¬ Y'
1	1	1	?	i	0	0	1	1
2	0	1	1	1	1	0	1	0
3a	1	0	0					
3b	1	0	0					
3c	1	0	1	i	0	0	1	1
4	0	0	0	0	0	1	0	1

Legend: Y ': Recoding of Y by three-valued modal logic. Other columns: see previous tables.

Tab. 5 also illustrates two problems of the use of the truth-code i. The *first problem* is an inflation of i-coded outcomes, caused by contradictions (type-1 i-inflation) or missing instantiations (type-2 i-inflation). The greater the number of explanatory independent variables, the higher for a given number of observations the percentage of configurations (X_1, X_2, ... , X_n) with missing instantiations and codes Y'= i (see Fig. 1). One of the strategies against this type-2 i-inflation is a reduction in the number of explanatory variables by constructing *generic meta-variables,* generally based on OR-aggregations of the original variables X_j. However, as Fig. 1 illustrates, this strategy S' may produce a type-1 i-inflation: the smaller the number of independent variables, the higher the risk of contradictions, which are assigned the code i. One of the "natural" solutions to this problem is the introduction of additional explanatory variables, which split existing configurations with contradictory outcomes into two internally consistent parts (see Ragin 1989: 113). Of special interest for this purpose are *control-variables*. As in traditional statistical regression analysis, the introduction of such variables into the data analysis allows us to separate the normal cases from the *exceptional* ones (Lewis-Beck 1995: 55). Unfortunately, this strategy S may again result in an increase in the total percentage of codes Y'= i, mainly due to a growing number of missing instantiations (see Fig. 1). Hence, if both types of i-inflation are taken together, there is a u-shaped relationship between the number of explanatory variables X_j and the *index I,* defined in Fig. 1 as the *total* percentage of configurations with Y'= i. In sum, the parameter I is an expression of our *ignorance* about the social phenomenon under investigation. With scientific progress its value should be brought down from an initial level close to 100% to the lowest possible level *Min* (see Fig. 1). Hence with the right mix of the two mentioned strategies S and S‘, the researcher should try to get as close as possible to the corresponding optimal point θ on the horizontal axis of Fig. 1.

Fig. 1: Sources and counter-strategies for i-inflations.



Legend: See text.

The *second problem* from the recoding of contradictory or missing outcomes with the indeterminate value i is the *inability* of the Quine-McCluskey algorithm in general and the related fs/QCA software (Drass & Ragin 2013) in particular to process the resulting three-valued data-table. Fortunately, the problem refers *only* to the *dependent* outcome variable Y‘ (see Tab. 5): in QCA cases with missing values for the *explanatory* variables X_j are generally not considered, when completing the data-table for empirical analysis. This allows us to solve the problem by transforming the *dependent* variable Y‘ as well as its negation ¬Y‘ by the modal operators NEC and POS (see Tab. 4). As a consequence, there are four new dependent variables NEC Y‘, NEC ¬Y‘, POS Y‘, and POS ¬Y‘ (see Tab. 5) with only binary values 0 and 1, which can subsequently be treated with Ragin‘s standard procedure for crisp-set QCA. Hence, instead of one equation, the Quine-McCluskey procedure based on three-valued modal logic yields as many as *four.* This diversity obviously calls for *integration* of the knowledge about the original variable Y.

The idea of using necessary and possible triggers and inhibitors for the common outcome Y (see Section 2) offers a solution to this integration problem, especially if we accept as triggers and inhibitors not only particular variables X_j but entire Boolean functions $f(X_1,X_2, ...)$. Thus after applying the Quine-McCluskey procedure to the recoded data-table (e.g. Tab. 5), we replace in the resulting Boolean formulas:

a) *$f(X_1,X_2, ...)$ ==> NEC Y‘* by *$f(X_1,X_2, ...)$ —> Y* and consider $f(X_1,X_2, ...)$ as a strict trigger of Y;

b) *$f(X_1,X_2, ...)$ ==> POS Y‘* by *$f(X_1,X_2, ...)$ ----> Y* and consider $f(X_1,X_2, ...)$ as a potential trigger of Y;

c) *$f(X_1,X_2, ...)$ ==> NEC ¬Y‘* by *$f(X_1,X_2, ...)$ –//–> Y* and consider $f(X_1,X_2, ...)$ as a strict inhibitor of Y;

d) *$f(X_1,X_2, ...)$ ==> POS ¬Y‘* by *$f(X_1,X_2, ...)$ --//--> Y* and consider $f(X_1,X_2, ...)$ as a potential inhibitor of Y (see Glossary at the end of the article).

In case (a), the *strict triggering* of Y by $f(X_1,X_2, ...)$ means that $f(X_1,X_2, ...)$ is a *sufficient condition* of Y. Thus $f(X_1,X_2, ...)$ implies the necessity NEC Y’, which means that Y’ is true and not only indeterminate or even false. Similarly, $f(X_1,X_2, ...)$ is a *necessary condition* of Y, if $\neg f(X_1,X_2, ...)$ is a *strict inhibitor* of Y and thus $\neg f(X_1,X_2, ...)$ –//–> Y (see case (c)). Finally, $f(X_1,X_2, ...)$ is a *necessary and sufficient condition* of Y if $\neg f(X_1,X_2, ...)$ –//–> Y and $f(X_1,X_2, ...)$ —> Y are both true.

If $f(X_1,X_2, ...)$ is a Boolean *disjunction,* which combines the original variables X_1, X_2, ... by Boolean OR operators, each of the variables X_1, X_2, ... can be interpreted as a *separate* strict or potential trigger or inhibitor of the same type as $f(X_1,X_2, ...)$: if in this situation one of the X_j is true, the whole *disjunction* $f(X_1,X_2, ...)$ is also true and thus triggers or inhibits the outcome Y. If for example the Quine-McCluskey algorithm yields for Tab. 5

$(X_1$ OR $X_2)$ ==> POS Y‘ (5)

$(X_1$ OR $X_2)$ is a potential trigger of Y, denoted by

$(X_1$ OR $X_2)$ ----> Y (6)

and consequently X_1 and X_2 are both separate potential triggers of Y:

X_1 ----> Y (7a)

X_2 ----> Y (7b)

Certain triggers and inhibitors are of special interest for *public policy analysis.* (Rossi & Freeman 1993: chap. 3). *Triggers* are crucial when interpreting the results of a QCA with regard to unintended *negative side effects* (Dunn 2004: 76 ff.). Of similar interest are the *inhibitors of planned outcomes,* often represented by the power of stakeholders, since they threaten the attainability of policy goals (Dunn 2004: 193 ff.). In both cases, QCA based on three-valued *modal* logic may help to alleviate the resulting policy problems: for example, it may give answers to questions as to what turns a *strict* trigger of a negative side effect into a less serious *potential* trigger or what has to be added to a potential *inhibitor* of a planned goal in order to make it a strict *trigger* of this goal.

4. An exemplary analysis of welfare policy

In this section we present a reanalysis of a work by Kangas (1996), in order to show the use and the advantages of three-valued modal logic in *comparative policy analysis*. The original paper of Kangas investigated the effects of the following independent variables

C = Christian democratic party power,

W = Working class mobilization, and

F = Fragmentation of the right / bourgeois party block

on the binary dependent variable

H = Level of health insurance as an outcome of the welfare state.

The data of the 18 countries analyzed by Kangas (1996: 354) are presented in Tab. 6.

As this table demonstrates, the data used by Kangas (1996) are far from being ideal for crisp-set QCA. For configuration 8 there is no instantiation of H (see Tab. 6). Similarly, the configurations 4a and 4b on the one hand and 6a and 6b on the other are pairwise identical but have contradictory values for the outcome H. Although Kangas did not disclose too many of the methodological details of his research, he seems to have discarded contradictory and missing cases. This is reasonable in view of the high level of inconsistency of the cases 4a / 4b and 6a / 6b as well as the theoretical deficits for making a sound imputation about the value of H of case 8. In this way Kangas (1996: 354) finally arrived at the following conclusion:

$$C \text{ AND } \neg F \Longrightarrow H \qquad (8)$$

Tab. 6: Health insurance provisions in 1950, by political configuration.

Configuration Nr.	Work-class W	Christ. party C	Fragm. right F	Health security H	H re-coded H‘	Countries with corresponding configuration (Year 1950)
1	0	0	0	0	0	Canada, USA
2	0	0	1	0	0	Ireland, Japan
3	0	1	0	1	1	France, Germany, Italy
4a	0	1	1	1	i	Netherlands
4b	0	1	1	0	i	Switzerland
5	1	0	0	0	0	Australia, New Zealand, UK
6a	1	0	1	0	i	Denmark, Finland, Sweden
6b	1	0	1	1	i	Norway
7	1	1	0	1	1	Austria, Belgium
8	1	1	1	?	i	None

Legend: W = Working class mobilization: 1=high, 0=low. C = Christian democratic party strength: 1=high, 0=low. F = Fragmentation of the right/bourgeois bloc: 1=high, 0=low. H = Level of health insurance by public welfare: 1=high, 0=low. H‘ = Three-valued recoding of H, for details see text. Source: Kangas (1996), p. 354, tab. 14.2.

Thus, according to the Boolean formula (8) a well-developed health insurance system requires a strong Christian democratic party as part of a non-fractionalized bourgeois right, where the latter is assumed to support the progressive health policy of its Christian democratic ally.

Three-valued QCA, as proposed in this article, arrives at *additional* conclusions by making full use of the original data-set. Instead of dropping the aforementioned configurations 4a, 4b, 6a, 6b, and 8, three-valued QCA keeps them all for qualitative analysis, but only after replacing missing instantiations and contradictions with the indeterminate value i. The result is a recoded variable H, which is denoted in Tab. 6 and 7 as H‘ and has three truth-values 0 = *false,* 1 = *true,* and i = *indeterminate.* The latter value is subsequently mapped to 0 or 1 by modal transformations of H‘, which yield four new, dependent variables NEC H‘, POS H‘, NEC ¬H‘, and POS ¬H‘ (see Tab. 7).

Tab. 7: The result of the modal transformations of H‘ in Tab. 6.

Configuration Nr.	Work-class W	Christ. party C	Fragm. right F	Health sec. rec. H‘	Strict triggering NEC H‘	Potential triggering POS H‘	Strict inhibition NEC ¬H‘	Potential inhibition POS ¬H‘
1	0	0	0	0	0	0	1	1
2	0	0	1	0	0	0	1	1
3	0	1	0	1	1	1	0	0
4	0	1	1	i	0	1	0	1
5	1	0	0	0	0	0	1	1
6	1	0	1	i	0	1	0	1
7	1	1	0	1	1	1	0	0
8	1	1	1	i	0	1	0	1

<u>Legend</u>: NEC: Necessity. POS: Possibility. ¬: NOT. Other definitions: See Tab. 6. Configuration 4 represents the pair of configurations 4a,b of Tab. 6. Similarly, configuration 6 corresponds to the configurations 6a and 6b of Tab. 6.

Due to their binary nature, it is possible to explain NEC H‘, POS H‘, NEC ¬H‘, and POS ¬H‘ with the classical Quine-McCluskey algorithm (Mendelson 1970: chap. 4), as described in Section 1. The respective results for Tab. 7 are as follows:

C OR (W AND F) ==> POS H‘ (9a)

C AND ¬ F ==> NEC H‘ (9b)

¬ C OR F ==> POS ¬H‘ (9c)

(¬ C AND ¬ F) OR (¬ C AND ¬ W) ==> NEC ¬H‘ (9d)

In order to have only *one* instead of four dependent variables, we considered the expressions to the left of the implication ==> as strict or potential triggers and inhibitors and adapted the respective formulas accordingly (see Section 3):

C OR (W AND F) ----> H (10a)

C AND ¬ F —> H (10b)

¬ C OR F --//--> H (10c)

(¬ C AND ¬ F) OR (¬ C AND ¬ W) –//–> H (10d)

Thus, there is a correspondence between (9a) to (9d) on the one hand and (10a) to (10d) on the other, which facilitates the interpretation of the original output of the Quine-McCluskey procedure.

From (10a) it follows that not only (C OR (W AND F)) but also the Christian democratic party strength C alone is a *potential* trigger of progressive health services H. If at the same time ¬F is also true, C as a part of (C AND ¬F) is, according to formula (10b), even a *strict* trigger of H: in this situation a strong Christian democratic party dominates a non-fractionalized strong bourgeois bloc, which is thus assumed to support the progressive health policy of its Christian democratic senior partner. This obviously corresponds to the main finding by Kangas (1996: 354) using a two-valued QCA (see equation (8)).

A similar consideration also holds for the *absence* ¬C of a strong Christian democratic party, which is in formula (10c) a *potential* inhibitor of H. If this absence is in (10d) combined with the absence ¬F of the fractionalization of the bourgeois block, (¬C AND ¬F) is a *strict* inhibitor of H: in this case a strong non-fractionalized right, which is in addition uncontrolled by a weak Christian democratic party, follows its own interest and thus becomes a strict inhibitor of a progressive health policy H.

On the grounds of what has been said so far, the effects of a strong *bourgeois right,* indicated by the absence ¬F of fractionalization, depends very much on the strength of the Christian democratic party: if the Christian democrats are *strong,* (C AND ¬F) is a *strict trigger* of a progressive health policy H (see equation 10b). If the Christian democrats are *weak*, (¬C AND ¬F) and the whole left-hand side of equation (10d) are true and consequently the bourgeois right becomes a *strict inhibitor* of H.

According to the equations (10a) to (10d), the role of the *working class* is even more modest. Due to equation (10a), a *strong* working class W is only a *potential* trigger of H, where this effect can only be observed, if the right bourgeois block is fractionalized and thus F = true. If the working class is *weak,* then ¬W has an inhibitory effect on H, though only if the Christian democratic party is also weak and thus (¬C AND ¬W) becomes a strict inhibitor of a progressive health policy H (see equation (10d)).

5. Summary and conclusions

This article proposes new solutions to old problems of crisp-set QCA: in order to avoid the imputation or elimination of cases when treating logical remainders and contradictions, the article proposes accepting uncertainty by the use of Lukasiewicz's indeterminate *truth-value i* together with the modal operators *NEC* and *POS.*

This approach has three major advantages. First, it is more honest with regard to our *ignorance* about social reality by accepting the mentioned indeterminacy. Second, three-valued QCA is more in alignment with *qualitative* research, where exceptional or contradictory observations are not statistical outliers or accidents but clues guiding researchers to new theoretical strands (Strauss & Corbin 1998, Charmaz 2006). Finally it makes less *debatable assumptions,* when preparing the data-input for the Quine-McCluskey minimization, e.g. by not using weak ad-hoc theories for the imputation of missing or contradictory values. All three advantages are illustrated in Section 4 by a reanalysis of Kangas' (1996) data about the political conditions for advancing the welfare state and its health policy.

Our exemplary reanalysis not only confirms the main finding of Kangas' work (compare equations (8) and (10b)), but supplements this information with three other explanatory equations (10a), (10c), and (10d). As shown above, this means additional knowledge from three-valued QCA, which is however not always crisp but also partially fuzzy and thus points to our lack of genuine knowledge (see equations (10a) and (10c)). Nevertheless, by comparing potential and strict triggers and inhibitors, it is possible to identify variables and conditions, which turn potentially fuzzy policy outcomes into crisp, strict results. This is obviously a major advantage for *social policy analysis.*

Needless to say that three-valued QCA also carries a risk of *agnosticism* as a result of i-inflations, when i-coded outcomes prevail and the index I approaches the ceiling of 100% (see Fig. 1). This risk increases with the number of explanatory variables as well as the number of empirical observations. Consequently, this paper also discusses counter-measures such as the splitting of configurations and the unification of related explanatory variables into new generic terms. In principle, both strategies against i-inflations seem to be viable, the details of their implementation, however, will require further research in the future.

Glossary of logical symbols and expressions

f	False, also denoted by 0.
t	True, also denoted by 1.
i	Indeterminate truth in three-valued logic.
X AND Y	Boolean conjunction of X and Y. For definition in 3-valued logic see Tab. 3.

X OR Y Boolean disjunction of X and Y. For definition in 3-valued logic see Tab. 3.

¬ X Boolean negation of X. For definition in 3-valued logic see Tab. 3.

X ==> Y Implication in 2-valued logic: Y follows from X.

NEC X Necessity of X in modal logic. For definition see Tab. 4.

POS X Possibility of X in modal logic. For definition see Tab. 4.

X ----> Y Potential triggering of Y by X.

X --//--> Y Potential inhibition of Y by X.

X —> Y Strict triggering of Y by X.

X –//–> Y Strict inhibition of Y by X.

Notes

1: fs/QCA is not only for fuzzy-set QCA but still contains modules for doing crisp-set QCA, on which e.g. Section 4 of this article relies on.

2: (X_1 OR ¬ X_1) AND X_2 = X_2, since (X_1 OR ¬ X_1) is tautologically true such that the truth of (X_1 OR ¬ X_1) AND X_2 depends only on X_2.

3: This principle does not hold for the three-valued implication ==>, which Lukasiewicz (1970) intentionally defined in such a way that *i ==> i* is true. This irregularity is not relevant for this article, since it does not make use of Lukasiewicz's three-valued implication.

4: Based on the three-valued logic of *Lukasiewicz* (1970). Other logic systems have partly differing definitions.

References

Beall, J. C. & van Fraassen, Bas C. (2003): *Possibilities and Paradox: An Introduction to Modal and Many-Valued Logic, chap. 5.* Oxford: Oxford University Press.

Bergmann, Merrie (2008): *An Introduction to Many-Valued and Fuzzy Logic.* Cambridge: Cambridge University Press.

Charmaz, Kathy (2006): *Constructing Grounded Theory: A Practical Guide Through Qualitative Analysis.* London: Sage.

Drass, Kriss A. & Ragin, Charles (2013): *fs/QCA Software, Release 2.0.* In: http://www.u.arizona.edu/~cragin/fsQCA/software.shtml (accessed at June, 11, 2013).

Dunn, William N. (2004): *Public Policy Analysis.* Upper Saddle River: Pearson–Prentice Hall.

Kangas, Olli (1996): *The Politics of Social Security: On Regressions, Qualitative Comparisons, and Cluster Analysis.* In: Thomas Janoski and Alexander Hicks (eds.), The Comparative Political Economy of the Welfare State, chap. 14. Cambridge: Cambridge University Press.

Lewis-Beck, Michael S. (1995): *Data Analysis: An Introduction.* Thousand Oaks: Sage.

Lukasiewicz, Jan (1970 [1920]): *Selected Works.* Ed. by L. Borkowski. Amsterdam: North-Holland.

Mendelson, Elliot (1970): *Boolean Algebra and Switching Circuits.* New York: McGraw-Hill.

Mueller, Georg P. (2008): *Three-Valued Modal Logic for Reconstructing the Semantic Network Structure of a Corpus of Coded Texts.* In: Thomas Friemel (ed.), Why Context Matters, pp. 37-54. Wiesbaden: VS Verlag.

Muzio, J. C. & Wesselkamper, T. C. (1986): *Multiple-Valued Switching Theory.* Bristol: Adam Hilger Ltd.

Ragin, Charles (1989): *The Comparative Method: Moving Beyond Qualitative and Quantitative Strategies.* Berkeley: University of California Press.

Ragin, Charles (1996): *A Qualitative Comparative Analysis of Pension Systems.* In: Thomas Janoski and Alexander Hicks (eds.), The Comparative Political Economy of the Welfare State, chap. 13. Cambridge: Cambridge University Press.

Ragin, Charles (1998): *Using Qualitative Comparative Analysis to Study Configurations.* In: Udo Kelle (ed.), Computer-Aided Qualitative Data Analysis: Theory, Methods and Practice, chap. 13. London: Sage.

Rescher, Nicolas (1969): *Many-valued Logic.* New York: McGraw-Hill.

Rihoux, Benoît & De Meur, Gisèle (2009): *Crisp-Set Qualitative Comparative Analysis.* In: Rihoux, Benoît & Ragin, Charles (eds.), Configurational Comparative Methods, chap. 3. Los Angeles: Sage.

Rossi, Peter & Freeman, Howard (1993): *Evaluation: A Systematic Approach.* Newbury Park: Sage.

Schneider, Carsten & Wagemann, Claudius (2012): *Set-Theoretic Methods for the Social Sciences.* Cambridge: Cambridge University Press.

Strauss, Anselm & Corbin, Juliet (1998): *Basics of Qualitative Research: Techniques and Procedures for Developing Grounded Theory.* Thousand Oaks: Sage.

Author

Georg P. MUELLER

The author has a Ph.D. from the University of Zurich (Switzerland), where he studied sociology, mathematics, and philosophy. He currently works as senior lecturer (Maître d'enseignement et de recherche) at the Department of Communication and Media Research (DCM) of the University of Fribourg, were he teaches research methodology and statistics. His research interests include public policy analysis, the construction of social indicators for social monitoring and early warning, and the development of mathematical models of social processes. E-mail: Georg.Mueller@unifr.ch

Abstract

Contradictory and missing outcomes are problems common to many qualitative comparative studies, based on the methodology of crisp-set QCA. They also occur in public policy analyses, e.g. if important background variables are omitted or outcomes of new policies are technically censored. As a new solution to these problems, this article proposes the use of three-valued modal logic, originally introduced by the Polish philosopher Jan Lukasiewicz (1970). In addition to *true* and *false, indeterminate* is the third truth-value in this alternative approach, which serves to code missing or contradictory data. Moreover, modal operators allow a differentiation between strict and possible triggers and inhibitors of policy outcomes. The advantages of three-valued modal logic in crisp-set QCA are illustrated by an empirical example from comparative welfare policy analysis. Its conclusions allow comparisons with the corresponding results from a conventional crisp-set QCA of the same data-set.

Keywords

Three-valued modal logic, qualitative comparative analysis (QCA), public policy, social security, international comparisons

JEL Classification

C65, H43, H55, Z18

Citation proposal

Mueller, Georg P. 2014. «Three-Valued Modal Logic for Qualitative Comparative Policy Analysis with Crisp-Set QCA». Working Papers SES 450, Faculty of Economics and Social Sciences, University of Fribourg (Switzerland)

Working Papers SES

Last published :

443 Isakov D., Weisskopf J.-P.: Do not wake sleeping dogs: Pay-out policies in founding family firms; 2013

444 Suter P., Gmür M.: Member Value in Co-operatives; 2013

445 Ravasi C.: Les top managers internationaux des grandes entreprises suisses: profils et parcours de carrière; 2013

446 Sudharshan D., Furrer O., Arakoni R. A.: Robust Imitation Strategies; 2013

447 Ravasi C., Salamin X., Davoine E.: The challenge of dual career expatriate management in a specific host national environment: An exploratory study of expatriate and spouse adjustment in Switzerland based MNCs; 2013

448 Schöni O., Seger L.: Comparing Mobile Communication Service Prices Among Providers: A Hedonic Approach: 2014

449 Isakov D., Parietti S.: Analyse des rémunérations des dirigeants de sociétés suisse cotées en bourse entre 2007 et 2012; 2014

Catalogue and download links:

http://www.unifr.ch/ses/wp

http://doc.rero.ch/collection/WORKING_PAPERS_SES