Supplementary material to

'Reanalysing glacier mass balance measurement series'

Zemp et al.

Supplement A. Co-registration of digital elevation models resulting from geodetic surveys based on Nuth and Kääb (2011).

A simple test to determine whether two DEMs are misaligned is to display the elevation differences as a map in grey-scale colour. The two DEMs are misaligned if the resulting image looks like a shaded-relief image of the terrain (see, e.g., Fig. 1 in Nuth and Kääb, 2011). A co-registration of two DEMs allow correcting for this misalignment. For this description, a master (higher-quality DEM) and slave (lower-quality DEM) is defined however the choice is arbitrary as long as the final difference is between DEMs that are aligned. The elevation differences (*dh*) of the slave DEM with respect to the master DEM are related to terrain slope (α) and aspect (ω) as follows:

$$\frac{dh}{\tan(\alpha)} = a \cdot \cos(b - \omega) + c \,, \qquad \text{(A1)}$$

where *a* is the magnitude of the misalignment, *b* is the direction of the misalignment, and *c* is the mean elevation bias ($z_{adjustment}$) divided by the tangent of the mean slope of the sample. The input data (*dh*, α and ω) should be a sample from non-glacierized stable terrain that contains a distribution of at least half of all possible aspects, uniformly distributed, if possible. Terrain slope and aspect are ideally computed from the master DEM, however, since the process is iterative, the choice of DEM for slope and aspect is arbitrary. The unknown co-registration parameters (*a*, *b*, and *c*) can be solved using a least-squares minimization, common to programmes such as Excel, MATLAB and R. Since this is an analytical solution based on a non-analytical terrain surface, the process should be repeated until the solution converges. Thus, once the initial solution is determined, the slave DEM should be translated by the magnitude (*a*) and direction (*b*) of the co-registration vector and adjusted for the mean vertical bias, and the DEMs re-differenced. The translation of the parameters a, b, and c into x, y, and z adjustments is as follows:

$$x_{adjustment} = a \cdot \sin(b) \qquad \text{(A2)}$$

$$y_{adjustment} = a \cdot \cos(b) \qquad \text{(A3)}$$

$$z_{adjustment} = \frac{c}{\tan(\bar{a})} \qquad \text{(A4)}$$

Equation A1 can then be solved again. Typically no more than two to three iterations are required depending upon the quality of the terrain sample.

If 3D co-registration is successful, then the bias of the co-registration ($z_{adjustment}$ in Eq. A4) is removed and there remains the uncertainty of the vertical co-registration adjustment(s). One approach to estimate this potential un-removed vertical error is to introduce additional elevation datasets, either as a control or as a part of a time series. When three or more datasets are available, co-registration can be performed between each of these, and the summation of the 3D co-registration vectors returns the residual error remaining within the series. Studies have shown that remaining vertical errors can reach magnitudes of at least 1–3 m (Nuth et al., 2012; Berthier et al., 2012) and should be included in the uncertainty assessment.

There are a few exceptions in which the solution of the 3D co-registration problem fails. The first is on flat or low-sloped terrain; i.e., slopes less than three to five degrees that are present in the stable terrain sample. Second, many older maps and DEMs created for glaciological applications do not contain a sufficient sample of the surrounding topography. In these cases, co-registration may be performed using alternate (image matching) methods, described in Berthier et al. (2007), for example.

Supplement B. A method to determine the spatial auto-correlation based on Rolstad et al. (2009).

The uncertainty in the spatially averaged elevation difference is estimated by following these steps:

1. Create an elevation difference grid of the bedrock region surrounding the glacier.
2. Detrend the grid, if necessary, using a polynomial model to remove bias (as described in Supplement A).
3. Estimate the spatial auto-correlation, such as by statistically assessing the grid to determine the semi-variogram parameters of nugget c_0, partial sill c_1 and range a_1 by fitting a spherical semi-variogram model to the empirically derived semi-variogram. Standard geostatistical software packages are available to do this. For reference, the standard deviation of the elevation error derived over bedrock $\sigma_{\Delta z}$ is related to the semi-variogram parameters by:

   $$\sigma_{\Delta z}^2 = c_0 + c_1 , \quad \text{(B1)}$$

4. If the correlation range a_1 is greater than the representative radius L of the averaging area $S=\pi L^2$, then the uncertainty of the spatially averaged elevation difference σ_S is to be calculated, cf. Equation 11 in Rolstad et al. (2009), using

   $$\sigma_S^2 = c_0 \frac{\Delta h^2}{L^2} + c_1 \left(1 - \frac{L}{a_1} + \frac{1}{5}\left(\frac{L}{a_1}\right)^3\right) \qquad a_1 > L , \quad \text{(B2)}$$

   where *Δh* is the pixel size.

If the correlation range is less than the representative radius of the averaging area (a_1<L), as may be the case when determining the geodetic mass balance over large areas, then σ_S is determined using

$$\sigma_S^2 = c_0 \frac{\Delta h^2}{L^2} + \frac{1}{5} c_1 \frac{a_1^2}{L^2} \qquad a_1 < L . \qquad \text{(B3)}$$

As discussed in Rolstad et al. (2009) there may be more than one scale of spatial correlation related to the derivation of the DEMs. It must be emphasized that it is generally the largest correlation scale that has the greatest impact on the spatially averaged uncertainty.

References

Berthier, E., Arnaud, Y., Kumar, R., Ahmad, S., Wagnon, P., and Chevallier, P.: Remote sensing estimates of glacier mass balances in the Himachal Pradesh (Western Himalaya, India). Remote Sens. Environ., 108, 327–338, 2007.

Nuth, C. and Kääb, A.: Co-registration and bias corrections of satellite elevation data sets for quantifying glacier thickness change. The Cryosphere, 5, 271–290, 2011.

Rolstad, C., Haug, T., and Denby, B.: Spatially integrated geodetic glacier mass balance and its uncertainty based on geostatistical analysis: application to the western Svartisen ice cap, Norway. J. Glaciol., 55, 666–680, 2009.

Note that in the table 0-values were reported as 'assumed to be zero' or 'not specified'.

Note that for the statistics below, only independent PoR are analysed, omitting the overall PoR for glaciers with more than one PoR.

Average	11	-466	0	140	0	278	12	62	-577	3	50	34	-3	9	1	1
RMS	12	677	0	146	0	320	19	105	729	46	58	45	28	17	17	3
Stdv	6	498	0	42	0	161	15	86	451	46	30	30	28	14	17	3
Min	4	-1757	0	79	0	100	-13	0	-1626	-196	10	1	-112	0	-17	0
Max	32	708	0	224	0	539	36	319	201	141	150	140	74	58	100	14

Average	-4	1	-454	340	-574	66	120	1416	1219	1		72	497		64	403
RMS	6	2	667	367	724	75	226	4160	1273	3		77	556		69	451
Stdv	4	2	496	141	447	37	194	3965	372	2		29	253		28	205
Min	-10	0	-1727	143	-1701	21	-196	-2200	489	-2.59		0	180		0	146
Max	0	10	708	566	210	209	731	23392	2045	12.22		95	1108		90	900

Abbreviations:

Glacier co(	see Table 1 in paper
PoR	period of record
B	balance
glac	glaciological
geod	geodetic
a	annual
ε	epsilon, systematic error
σ	sigma, random error
point	point location
spatial	spatial integration
ref	glacier reference area changing over time
DEM	digital elevation model
dc	density conversion
sd	survey differences
int	internal balance
bas	basal balance
corr	corrected for systematic errors
Δ	discrepancy, cf. Eq. 19
σ.common	common variance, cf. Eq. 20
δ	reduced discrepancy, cf. Eq.21
H0	hypothesis that B.glac = B.geod
α	probability of rejecting H0 although the results of both methods are equal, i.e. unnecessary appointment of seris for calibration (risk typ I)
β	probability of maintaining H0 although the results of both methods are different, i.e. non-recalibration of erroneous series (risk type II)
ε.limit	lowest detectable bias, cf. Eq. 25