Published in "EPL - Europhysics Letters 98(4): 40002 , 2012"
which should be cited to refer to this work.

Testing spatial noncommutativity via magnetic hyperfine structure induced by fractional angular momentum of Rydberg system

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Abstract – An approach to solve the critical problem of testing quantum effects of spatial noncommutativity is proposed. Magnetic hyperfine structures in a Rydberg system induced by fractional angular momentum originated from spatial noncommutativity are discussed. The orders of the corresponding magnetic hyperfine splitting of the spectrum $\sim 10^{-7}$–10^{-8} eV lie within the limits of accuracy of current experimental measurements. Experimental tests of physics beyond the standard model are the focus of broad interest. We note that the present approach is reasonably achievable with current technology. The proof is based on very general arguments involving only the deformed Heisenberg-Weyl algebra and the fundamental property of angular momentum. Its experimental verification would constitute an advance in understanding of fundamental significance, and would be a key step towards a decisive test of spatial noncommutativity.

Introduction. – As one of the current candidates in tracking down new physics beyond the standard model, quantum mechanics in noncommutative space (NCQM) [1–21] should be verifiable[1]. Modifications of spatial noncommutativity (NC) to normal quantum theory depending on vanishingly small NC parameters, which lead to NC quantum effects, are usually far beyond experimental accuracy. Therefore, a widely held view is that NCQM can only make predictions outside the range of experimental observation. However, the conclusion is premature [20]. Indeed, attempts in recent experiments performed by Connerade *et al.* [20] suggest that there may be a way to test for NCQM.

Recently, it has been found [14,19,21] that the vanishingly small NC constants [5,12], which usually appear in NC corrections of any physical observable, cancel out in the fractional angular momentum (FAM) originated from spatial noncommutativity under well-defined conditions. It turns out that FAM results in the unusual zero-point value $\hbar/4$. This provides a distinct signature of spatial noncommutativity, which survives into the normal quantum scale. The difficulty involved in testing spatial noncommutativity via FAM is that direct measurements of FAM are a challenge enterprise.

With particular emphasis on feasible experimental tests, this paper proposes an approach of testing spatial noncommutativity via measuring magnetic hyperfine structures (MHFS [22,23]) induced by FAM in a Rydberg system. The orders of the corresponding splitting of MHFS $\sim 10^{-7}$ -10^{-8} eV lie within the limits of accuracy of current experimental measurements, and can be detected by using existing technology. The significant advance of the proposed method is that it solves a critical outstanding problem of NC quantum effects being unmeasurable, paves the way for notable progress and will lead to the first real test of spatial noncommutativity. Our proof is based on a very general argument involving only the deformed Heisenberg-Weyl algebra and the fundamental property of angular

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[1] This paper focuses on the low energy relics of noncommutative quantum theory and construct formalism, which closely relates to a way testable by current experiments. It is enough to work in deformed formalism at the NCQM level.

momentum. Therefore, if it is achieved experimentally, this will constitute an advance in understanding of fundamental significance.

Review of FAM originated from spatial noncommutativity [14,15,19,21]**.** – We investigate ion motion in the laboratory system, trapped in a uniform magnetic field $\mathbf{B}$ aligned along the z-axis and an electrostatic potential [21]

$$V_{eff} = V_{eff,2} + V_{eff,z} = \frac{m_I}{2}(\omega_\rho^2 x_i x_i + \omega_z^2 z^2), \qquad (1)$$

(the summation convention is used, $i, j = 1, 2$), where m_I is ion mass, ω_ρ and ω_z are characteristic frequencies, respectively, in the (x_1, x_2)-plane and z-direction. The vector potential A_i of $\mathbf{B}$ is chosen as $A_i = -B\epsilon_{ij}x_j/2$, $A_z = 0$. The Hamiltonian $H(x,p)$ of the trapped ion can be decomposed into $H = H_2 + H_z$, where $H_z(z, p_z) = p_z^2/2m_I + m_I\omega_z^2 z^2/2$, and

$$H_2(x,p) = H_{k,2} + V_{eff,2} = \frac{1}{2m_I}p_i^2 + \frac{1}{2}\omega_c\epsilon_{ij}p_i x_j + \frac{1}{2}m_I\omega_P^2 x_i^2, \qquad (2)$$

where $H_{k,2} = \sum_i (p_i - q^* A_i)^2/2m_I$ is the mechanical kinetic energy operator which is different from the canonical kinetic energy operator $p_i p_i/2m_I$. H_2 is a two-dimensional Chern-Simons Hamiltonian with the cyclotron frequency $\omega_c = q^*B/m_I$, effective charge $q^* = Z^*e(>0)$ and the characteristic frequency $\omega_P = (\omega_\rho^2 + \omega_c^2/4)^{1/2}$. In the following, we focus on H_2.

The deformed Hamiltonian $H_2(\hat{x},\hat{p})$ in noncommutative space can be obtained by reformulating the corresponding undeformed $H_2(x,p)$ in terms of deformed canonical variables $\hat{x}_i$ and $\hat{p}_i$ which satisfy two-dimensional deformed Heisenberg-Weyl algebra

$$[\hat{x}_i, \hat{x}_j] = i\xi^2\epsilon_{ij}\theta, \quad [\hat{x}_i, \hat{p}_j] = i\hbar\delta_{ij}, \quad [\hat{p}_i, \hat{p}_j] = i\xi^2\epsilon_{ij}\eta,$$

where θ and η are the constant parameters of spatial noncommutativity, independent of position and momentum; ϵ_{ij} is a two-dimensional antisymmetric unit tensor with $\epsilon_{12} = -\epsilon_{21} = 1$, $\epsilon_{11} = \epsilon_{22} = 0$. The scaling factor ξ is defined as $\xi = (1 + \theta\eta/4\hbar^2)^{-1/2}$.

The deformed Heisenberg-Weyl algebra can be realized by x_i and p_i as follows:

$$\hat{x}_i = \xi\left(x_i - \frac{1}{2\hbar}\theta\epsilon_{ij}p_j\right), \quad \hat{p}_i = \xi\left(p_i + \frac{1}{2\hbar}\eta\epsilon_{ij}x_j\right),$$

where x_i and p_i satisfy the undeformed Heisenberg-Weyl algebra $[x_i, x_j] = [p_i, p_j] = 0$, $[x_i, p_j] = i\hbar\delta_{ij}$. The deformed $H_2(\hat{x},\hat{p})$ can be further expressed by x_i and p_i as $\hat{H}_2(x,p)$:

$$\begin{aligned}\hat{H}_2(x,p) &= \hat{H}_{k,2}(x,p) + \hat{V}_{eff,2}(x) \\ &\equiv \frac{1}{2M}\left(p_i + \frac{1}{2}G\epsilon_{ij}x_j\right)^2 + \frac{1}{2}Kx_i^2 \\ &= \frac{1}{2M}p_i^2 + \frac{1}{2M}G\epsilon_{ij}p_i x_j + \frac{1}{2}M\Omega_P^2 x_i^2, \qquad (3)\end{aligned}$$

where the effective parameters M, G, Ω_P and K are defined as

$$\begin{aligned}\frac{1}{2M} &\equiv \xi^2\left(\frac{1}{2m_I}c_1^2 + \frac{1}{16\hbar^2}m_I\omega_\rho^2\theta^2\right), \\ \frac{G}{2M} &\equiv \xi^2\left(\frac{1}{m_I}c_1c_2 + \frac{1}{4\hbar}m_I\omega_\rho^2\theta\right), \\ M\Omega_P^2 &\equiv \xi^2\left(\frac{1}{m_I}c_2^2 + \frac{1}{2}m_I\omega_\rho^2\right), \\ K &\equiv M\Omega_P^2 - \frac{1}{4M}G^2,\end{aligned}$$

and $c_1 = 1 + m_I\omega_c\theta/4\hbar$, $c_2 = m_I\omega_c/2 + \eta/2\hbar$. $\hat{H}_2$ can be changed into two uncoupled harmonic modes [14,21].

Similarly, the deformed angular momentum $J_z(\hat{x},\hat{p}) = \epsilon_{ij}\hat{x}_i\hat{p}_j$ can be expressed by undeformed variables x_i and p_i as

$$\hat{J}_z(x,p) = \epsilon_{ij}x_i p_j - \frac{1}{2\hbar}\xi^2(\theta p_i p_i + \eta x_i x_i).$$

The corrections due to spatial noncommutativity are terms $O(\theta)$ and/or $O(\eta)$, which lead to $\hat{J}_z$ taking a fractional value. The existing upper bounds of θ and η are $\theta/(\hbar c)^2 \le (10\,\mathrm{TeV})^{-2}$ [5] and $|\sqrt{\eta}| \le 1\,\mu\mathrm{eV}/c$ [12]. $O(\theta)$ and $O(\eta)$ are vanishingly small, so that the corrections of spatial noncommutativity are beyond the limits of measurable accuracy of experiments.

Reduction for massive system. – We found a testable effect of spatial noncommutativity in the reduced system of $\hat{H}_2$. Because $\hat{H}_2$ and $\hat{H}_{k,2}$ do not commute, different from the massless model considered in [24], the difficulty of reduction for the massive model is how to treat the mechanical kinetic energy $\hat{H}_{k,2}$. To get rid of this difficulty, the reducing procedure is adopted in the following steps.

The ion oscillates harmonically with an axial frequency along the z-axis (its energy alternates between kinetic and potential energy). In the (1, 2)-plane, it executes a superposition of a fast circular cyclotron motion of an effective cyclotron frequency with a small radius (its energy is almost exclusively kinetic energy), and a slow circular magnetron drift motion of an effective magnetron frequency in a large orbit (its energy is almost exclusively potential energy). $\hat{V}_{eff,2}$ is reduced by reducing the amplitudes of the radio-voltage and the dc voltage applied between the electrodes of the ring and two end caps of the combined trap. We use, *e.g.*, Doppler cooling to slow the energy of ion down to the mK and then cool the ion to the ground state of $\hat{H}_2$ with the sideband cooling [25,26]. By synchronizing the laser field with $\hat{V}_{eff,2}$ reduction, the ion is kept in the ground state of the reducing $\hat{H}_2$. In $\hat{V}_{eff,2} \to 0$ the axial and the magnetron-like motions disappear, only the cyclotron motion survives. Thus $\hat{H}_2 \to \hat{H}_2^{(\hat{V}\to 0)} = \hat{H}_{k,2}$, and the energy of the survived motion is the ground value $\hat{\mathcal{E}}_{k,0}$.

Taking $\hat{H}_2^{(\hat{V}\to 0)}=\hat{\mathcal{E}}_{k,0}$ as the initial condition, the reduced system is obtained by resetting an electric field $\tilde{\mathbf{E}}$ of harmonic potential $m_I(\tilde{\omega}_\rho^2 x_i x_i+\tilde{\omega}_z^2 z^2)/2$, which leads to a full Hamiltonian $\tilde{H}_2=\hat{H}_{k,2}+\tilde{K}x_i x_i/2$ (in eq. (3) we replace ω_ρ with $\tilde{\omega}_\rho$, then K and G are replaced with $\tilde{K}$ and $\tilde{G}$). $\tilde{\mathbf{E}}$ satisfies the condition that the ion is trapped in the first stability range of the Paul trap. Thus $\tilde{\mathbf{E}}$ is weak. The original $\mathbf{B}$ is fixed such that the corresponding energy interval $\Delta\hat{\mathcal{E}}_k=\hat{\mathcal{E}}_{k,1}-\hat{\mathcal{E}}_{k,0}$ is large enough so that $\tilde{\mathbf{E}}$ cannot disturb the ion from the ground state $|\hat{\mathcal{E}}_{k,0}\rangle$ to the first excited state $|\hat{\mathcal{E}}_{k,1}\rangle$ of $\hat{H}_2^{(\hat{V}\to 0)}$. Thus the system remains in the ground state. In the subspace $\{|\hat{\mathcal{E}}_{k,0}\rangle_i\}$ of the ground state, for any state $|\psi\rangle=\sum_i c_i|\hat{\mathcal{E}}_{k,0}\rangle_i$ we obtain $\tilde{H}_2|\psi\rangle=(\hat{H}_{k,2}+\tilde{K}x_i x_i/2)|\psi\rangle=(\hat{\mathcal{E}}_{k,0}+\tilde{K}x_i x_i/2)|\psi\rangle$. Therefore, in the subspace $\{|\hat{\mathcal{E}}_{k,0}\rangle_i\}$ of the ground state, $\tilde{H}_2$ is reduced to

$$\tilde{H}_2\to\hat{\mathcal{E}}_{k,0}+\frac{1}{2}\tilde{K}x_i x_i\equiv\tilde{H}_2^{(0)}. \tag{4}$$

The reduced system $\tilde{H}_2^{(0)}$ is a constrained one [21]. The Lagrangian corresponding to $\tilde{H}_2$ is $\tilde{L}_2=M\dot{x}_i\dot{x}_i/2+\tilde{G}\epsilon_{ij}\dot{x}_i x_j/2-\tilde{K}x_i x_i/2$. The reduced Lagrangian corresponding to $\tilde{H}_2^{(0)}$ is $\tilde{L}_2^{(0)}=\tilde{G}\epsilon_{ij}\dot{x}_i x_j/2-\tilde{K}x_i x_i/2-\hat{\mathcal{E}}_{k,0}$. The definition of canonical momenta $p_i\equiv\partial\tilde{L}_2^{(0)}/\partial\dot{x}_i$ does not determine the velocities $\dot{x}_i$ as functions of p_i and x_j, but gives the relations between p_i and x_j:

$$\tilde{\varphi}_i\equiv p_i+\frac{1}{2}\tilde{G}\epsilon_{ij}x_j=0. \tag{5}$$

According to Dirac's formalism of quantizing a constrained system, such relations are primary constraints [27,28]. Because the Poisson brackets $\{\tilde{\varphi}_i,\tilde{\varphi}_j\}_P=\tilde{G}\epsilon_{ij}\neq 0$, the Dirac brackets are determined, $\{x_i,p_j\}_D=\delta_{ij}/2$, etc. The constraints $\tilde{\varphi}_i$ are strong conditions. They are used to eliminate dependent variables: four variables $(x_i,p_i),(i=1,2)$ are reduced to two independent ones (*e.g.*, x_1,p_1). Using these constraints to eliminate dependent variables, the corresponding quantum commutators of independent variables $\tilde{x}\equiv\sqrt{2}x_1$ and $\tilde{p}\equiv\sqrt{2}p_1$ are $[\tilde{x},\tilde{p}]=i\hbar$, etc. Then $\tilde{H}_2^{(0)}$ is rewritten as 1-dimensional harmonic Hamiltonian plus $\hat{\mathcal{E}}_{k,0}$. The full Hamiltonian $\tilde{H}_2$ has two harmonic modes [14,21]. The reduction to the reduced phase space alters the symplectic structure. It leads to one mode of $\tilde{H}_2$ going to infinity, decoupling from the system, and only one mode $\tilde{H}_2^{(0)}$ surviving[2]. $\tilde{H}_2^{(0)}$ has a reduced set of eigenstates, and the eigenvalues of $\hat{J}_z$ then become

$$\tilde{\mathcal{J}}_n=\hbar\tilde{\mathcal{J}}\left(n+\frac{1}{2}\right),\qquad (n=0,1,2,\cdots), \tag{6a}$$

$$\tilde{\mathcal{J}}=1-\frac{m_I\omega_c\theta}{4\hbar}-\frac{\eta}{m_I\omega_c\hbar+m_I^2\tilde{\omega}_\rho^2\theta+\eta}, \tag{6b}$$

where the two terms $O(\theta)$ and $O(\eta)$ are corrections due to the spatial noncommutativity, which are inaccessible to experiment because they are vanishingly small.

In the case of both position-position and momentum-momentum noncommuting, there is an effective intrinsic magnetic field $B_{eff}\sim\eta$ [21]. Thus a further limiting process of diminishing the external magnetic field B (ω_c) to zero is meaningful, and the surviving system has nontrivial dynamics. In this limit we have $\eta/(m_I\omega_c\hbar+m_I^2\tilde{\omega}_\rho^2\theta+\eta)\to\eta/(m_I^2\tilde{\omega}_\rho^2\theta+\eta)$. Using the consistency condition[3] $\eta=m_I^2\tilde{\omega}_\rho^2\theta$, this leads to a cancellation between the NC parameters θ and η, so that this term equals $1/2$, and $\tilde{\mathcal{J}}=1/2-m_I\omega_c\theta/4\hbar$, where $1/2$ dominates $\tilde{\mathcal{J}}$. Therefore, the dominant value of the zero-point angular momentum $\tilde{\mathcal{J}}_0$ assumes a fractional value[4]: $\hbar/4$. This is a distinct NC signal, which is within the limits of measurable accuracy of current experiments.

MHFS induced by FAM $\tilde{\mathcal{J}}_0$. – We consider a doubly charged alkaline-earth ion I^{++} caught in a combined-field trap. The trapping mechanism is provided by a uniform magnetic field $\mathbf{B}$ aligned along the z-axis and an electrostatic potential (1). For an alkaline-earth atom, the outer subshell has two s electrons, and the inner shells are completely filled. When the two s electrons of the outer shell are ionized, the resulting double-ion I^{++} also has rotational symmetry and resembles an effective spherical nucleus. We consider an electron injected into the trap and the captured electron together with this ion forms a singly charged ion I^+ which is still stably trapped. It is required that the principal quantum number n of the captured electron is large enough so that the system is a

[2] We compare dynamics in the present reduction and the reduction in the massless limit of [24]. The Lagrangian $\tilde{L}_2$, the reduced $\tilde{L}_2^{(0)}$ and the constraints $\tilde{\varphi}_i$ are similar to the Lagrangian L, eq. (1), the reduced L_0, eq. (5) and the constraints C^i, eq. (17) of [24]. The reduction $\tilde{L}_2\to\tilde{L}_2^{(0)}$ is similar to the reduction $L\to L_0$ of [24]. In both reductions, therefore, the similar Chern-Simons–type behavior and the truncated states decoupling are obtained.

[3] The proportionality of the NC parameters θ and η is determined by fundamental principles. At the quantum mechanics level, the general structures of the deformed annihilation and creation operators which satisfy a complete and closed deformed bosonic algebra at the non-perturbation level were obtained in ref. [16]. The proportionality $\eta=K\theta$ between the NC parameters θ and η is clarified from the consistency of the deformed Heisenberg-Weyl algebra with the deformed bosonic algebra. θ is a fundamental constant. K depends on some dynamical parameters of the Lagrangian. From the definition of momenta being the partial derivatives of the Lagrangian with respect to the NC coordinates, the dependence of η on the dynamical parameters of the considered system is understood.

[4] There is a subtle point related to taking the meaningful limits $\theta,\eta\to 0$ and $B\to 0$. In the limits $\theta,\eta\to 0$, the deformed dynamics in NC space is reduced to an undeformed one in commutative space. The reduced system $\tilde{H}_2^{(0)}$ is a constrained one. The deformed Poisson brackets of the constraints are $\{\tilde{\varphi}_i,\tilde{\varphi}_j\}_P=\tilde{G}\epsilon_{ij}$. In the limits $\theta,\eta\to 0$, they are reduced to undeformed ones in commutative space, $\{\varphi_i,\varphi_j\}_P=m_I\omega_c\epsilon_{ij}$. If we followed with $B\to 0(\omega_c\to 0)$, we would obtain $\{\varphi_i,\varphi_j\}_P=0$, thus Dirac brackets of canonical variables would not be determined, and the system would not survive at the quantum level. This indicates that in eq. (6b) when we take $\theta,\eta\to 0$ first to yield the conventional result, it makes no sense to follow with $B\to 0$. On the other hand, if we take $B\to 0$ first, the deformed Poisson brackets are reduced to $\{\varphi_i,\varphi_j\}_P=(m_I^2\omega_\rho^2\theta+\eta)\epsilon_{ij}/\hbar$. This shows that the subsequent limit $\theta,\eta\to 0$ also is meaningless. Therefore, *only in NC space* nontrivial dynamics of the reduced system $\tilde{H}_2^{(0)}$ survives at the quantum level in the limit $B\to 0$.

Rydberg one. In a reasonable approximation, the energy spectrum of the Rydberg electron is calculated on a similar basis as for a hydrogen-like system.

According to the above analysis, in the case where both position-position and momentum-momentum operators are noncommuting, and under the aforementioned conditions, the trapped ion I^{++} possesses FAM $\tilde{\mathcal{J}}_0$. Correspondingly, there is a zero-point magnetic momentum $\tilde{\mu}_0$,

$$\tilde{\mu}_0 = \frac{Z^* e}{2m_I}\tilde{\mathcal{J}}_0 = \frac{Z^*\mu_N}{A\hbar}\tilde{\mathcal{J}}_0, \tag{7}$$

where $m_I = Am_P$ (m_P is the proton mass and A is the nuclear mass number), $\mu_N = e\hbar/2m_p$ is the nuclear magneton.

The magnetic interaction between the magnetic momentum $\tilde{\mu}_0$ and the magnetic fields of the Rydberg electron induces the magnetic hyperfine structures of the energy spectrum of the Rydberg electron. Thus the measurement of FAM $\tilde{\mathcal{J}}_0$, through the corresponding $\tilde{\mu}_0$, is turned into measuring MHFS of the Rydberg electron. Similar to MHFS generated by nuclear spin [22,23], splitting of MHFS induced by $\tilde{\mathcal{J}}_0$ of the ion I^{++} can be calculated in two equivalent approaches [22]: investigating the interaction of the ion I^{++} on the Rydberg electron, or discussing the equivalent interaction of the Rydberg electron on the ion I^{++}. In the following, we apply the second approach.

To get a clean signal of such induced MHFS, we choose some even-even nucleus, because the nuclear spin of an even-even nucleus is zero.

The magnetic hyperfine interaction [22,23]. In the center-of-mass system the magnetic hyperfine splitting of the energy spectrum of the Rydberg electron induced by $\tilde{\mathcal{J}}_0$ of the ion I^{++} is described by the effective hyperfine interaction Hamiltonian $H_{in}^{(hfs)}$ between $\vec{\tilde{\mu}}_0 = -(Z^*\mu_N/A\hbar)(0,0,\tilde{\mathcal{J}}_0)$ of the ion I^{++} and the magnetic fields generated at the position of the ion I^{++} by the Rydberg electron. The corresponding splitting and intervals of the electronic energy spectrum are $\Delta E_{nljm_j}^{(hfs)} = \langle nljm_j|H_{in}^{(hfs)}|nljm_j\rangle = A_{nlj}\tilde{\mathcal{J}}_0 m_j\hbar$, $\Delta E_{nlj}^{(hfs)}(\Delta m_j) \equiv \Delta E_{nljm_j}^{(hfs)} - \Delta E_{nljm_{j'}}^{(hfs)} = A_{nlj}\tilde{\mathcal{J}}_0\Delta m_j\hbar$, where $\Delta m_j = m_j - m_{j'}$.

We consider the even-even nucleus of magnesium ($Z = 12, A = 24$). When two s electrons at the M shell are ionized, the ion Mg^{++} has a spherical configuration. The Rydberg electron should fill shells of $n > 3$. We estimate the magnetic hyperfine splitting and intervals of the spectrum of the Rydberg electron of $n = 6, l = 0$.

For an s electron, $l = 0, j = 1/2, m_j = \pm 1/2, \Delta m_j = 1$. Owing to the non-vanishing electronic charge density at the ion Mg^{++}, the only contribution to the Hamiltonian $H_{int}^{(hfs)}$ comes from the Fermi contact interaction. From $A_{n0\frac{1}{2}} = (8/3)(m_e/Am_p)\alpha^4(m_e c^2)(Z^*/n)^3/\hbar^2$, it follows that the magnetic hyperfine splitting and intervals have orders

$$\Delta E_{60\frac{1}{2}\frac{\pm 1}{2}}^{(hfs)} = \pm\frac{1}{2}A_{60\frac{1}{2}}\tilde{\mathcal{J}}_0\hbar \sim \pm 5.5\times 10^{-8}\,\mathrm{eV}, \tag{8a}$$

$$\Delta E_{60\frac{1}{2}}^{(hfs)}(1) = \Delta E_{60\frac{1}{2}\frac{1}{2}}^{(hfs)} - \Delta E_{60\frac{1}{2}\frac{-1}{2}}^{(hfs)} \sim 1.1\times 10^{-7}\mathrm{eV}. \tag{8b}$$

Measurements of $\Delta E_{nljm_j}^{(hfs)}$ and/or $\Delta E_{nlj}^{(hfs)}(\Delta m_j)$ directly determine FAM $\tilde{\mathcal{J}}_0$, thus providing signals of spatial noncommutativity.

Testing spatial noncommutativity via MHFS by FAM $\tilde{\mathcal{J}}_0$. – An ionic core with a closed shell configuration such as Mg^{++} is a conceptually ideal system to testing spatial noncommutativity. Mg^{++} is trapped by a combination of an electrostatic potential (1) and a uniform magnetic field $\mathbf{B}$ aligned along the z-axis [21]. According to the mentioned approach, the reduced system $\hat{H}_2^{(0)}$ is realized. In the well-defined limits, the surviving system has nontrivial dynamics, and FAM of Mg^{++} is $\tilde{\mathcal{J}}_0$. To make $\tilde{\mathcal{J}}_0$ observable, we inject an electron into the trap, capturing it in a high Rydberg state of an appropriate principal quantum number n by Mg^{++}. The coupling between $\tilde{\mu}_0$ of Mg^{++} and the magnetic fields generated at the position of Mg^{++} by the Rydberg electron will induce the magnetic hyperfine splitting of the electronic energy spectrum, which is a signal of spatial noncommutativity. Their orders are $\sim 10^{-7}$–10^{-8} eV, which lie within the limits of measuring accuracy of current experiments. This experiment can be achieved by existing technology, for example, high-resolution laser spectroscopy. Considering the pollution from other interactions during the measurement, we should pick up the true signal contributing the magnetic hyperfine splitting induced by FAM. This is achieved by the experiments which are performed twice: one with the magnetic field detuned to zero and one without the detuning process.

Summary. – NCQM is a candidate of possible new physics. At first sight, it seems that NCQM is unverifiable. However, we found that MHFS induced by FAM is one of the most important effects of spatial noncommutativity which, under well-defined conditions, lies within the range of normal laboratory measurements. Physics beyond the standard model is speculative. Its experimental tests are the focus of broad interest, especially the MHFS approach is reasonably achievable with current technology. Comparing with the experiments performed on quasi-bound Rydberg states in crossed fields [20], via a Chern-Simons process [19], using modified electron momentum spectroscopy [21] and others, MHFS is the most effective approach. Based on the unique feature of the MHFS approach, its experimental observation will be a key step towards a decisive test of confirming or ruling out spatial noncommutativity.

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JZZ's work is supported by NSFC (No. 10575037) and SEDF; LKG's work is supported by NSFC (No. 60490280) and NFRPC (No. 2005CB724500); HPL's work is supported by NSFC (No. 10774162); WC's work is supported by the Swiss National Foundation (Grant No. 200020-125124).

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