

## Second Generation of Composite Fermions and the Self-Similarity of the Fractional Quantum Hall Effect

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In the framework of a recently developed model of interacting composite fermions, we investigate different composite-fermion phases. Their interaction potential allows for the formation of both solid and new quantum-liquid phases, which are interpreted in terms of second-generation composite fermions and which may be responsible for the fractional quantum Hall states observed at unusual filling factors, such as  $\nu = 4/11$ . Projection of the composite-fermion dynamics to a single level, involved in the derivation of the Hamiltonian of interacting composite fermions, reveals the underlying self-similarity of the model.

*Keywords:* quantum Hall effect; fermions in reduced dimensions

### 1. Introduction

Most fractional quantum Hall effects (FQHE) may be understood as an integer quantum Hall effect (IQHE) in terms of a quasi-particle, a so-called composite fermion (CF), which consists of an electron and a vortex-like object with vorticity  $2s$ .<sup>1</sup> The formation of CFs is due to the presence of strong correlations between the interacting electrons in a partially filled Landau level (LL), the filling of which is characterized by the ratio  $\nu = n_{el}/n_B$  between the electronic and the flux densities,  $n_{el}$  and  $n_B = B/(h/e)$ , respectively. Because of its fractional charge, the CF experiences a reduced coupling  $(eB)^* = eB/(2sp + 1)$  to the external magnetic field  $B$

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and, in an approximate sense, forms LLs (CF-LLs) itself.<sup>2</sup> The FQHE arises when the CF filling factor  $\nu^* = \hbar n_{el}/(eB)^*$  is an integer  $p$ . Because the electronic and CF filling factors are related by  $\nu = \nu^*/(2s\nu^* + 1)$ , this type of FQHE is expected for the series  $\nu = p/(2sp + 1)$ .

Recently, Pan *et al.* have observed a FQHE at  $\nu = 4/11$ , which is not part of the series and which may not be described as an IQHE of CFs, but should rather be viewed as a FQHE of CFs at  $\nu^* = 1 + 1/3$ .<sup>3</sup> This discovery has renewed the interest in a possible self-similarity of the FQHE, which was investigated by Mani and v. Klitzing on the basis of scaling transformations.<sup>4</sup> It is natural to interpret such new FQHE states in terms of higher-generation CFs: the FQHE of CFs at  $\nu = 4/11$  may thus be viewed as an IQHE of second-generation CFs (C<sup>2</sup>Fs).<sup>5,6,7</sup>

Here, we investigate the self-similarity of the FQHE within a model of interacting spin-polarized composite fermions at  $\nu^* \neq p$ , which we have recently derived in the framework of Murthy and Shankar's Hamiltonian theory.<sup>2</sup> The self-similarity is found in the mathematical structure of the model: its Hamiltonian has the same form as that of electrons restricted to a single LL if one replaces the electronic by the CF interaction potential. Furthermore, the CF density operators, restricted to a single CF-LL, satisfy the same commutation relations as the projected operators of the electron density, in terms of a renormalized magnetic length  $l_B^* = \sqrt{\hbar/(eB)^*}$ .<sup>6</sup> However, the CF interaction potential, which has been derived within the model, and that for electrons in a single LL have a *different* form and cannot be related to each other by simple rescaling of the magnetic length. The existence of higher-generation CFs is therefore not guaranteed by the self-similarity of the model. Nevertheless, detailed energy calculations for competing CF quantum-liquid and solid phases, such as bubbles and stripes of CFs, have been performed within the model and indicate the stability of some C<sup>2</sup>F states.<sup>8</sup>

## 2. Model of Interacting Composite Fermions

At  $p < \nu^* < p + 1$ , one is confronted with a ground-state degeneracy in a model of non-interacting CFs. This degeneracy is lifted by the residual CF interactions, which may be taken into account in the Hamiltonian<sup>6</sup> ( $l_B = \sqrt{\hbar/eB} \equiv 1$ )

$$\hat{H}(s, p) = \frac{1}{2A} \sum_{\mathbf{q}} v_{s,p}^{CF}(\mathbf{q}) \bar{\rho}^{CF}(-\mathbf{q}) \bar{\rho}^{CF}(\mathbf{q}), \quad (1)$$

with the CF-interaction potential, in terms of Laguerre polynomials  $L_p(x)$ ,

$$v_{s,p}^{CF}(\mathbf{q}) = \frac{2\pi e^2}{\epsilon(q)q} e^{-q^2 l_B^{*2}/2} \left[ L_p \left( \frac{q^2 l_B^{*2} c^2}{2} \right) - c^2 e^{-q^2/2c^2} L_p \left( \frac{q^2 l_B^{*2}}{2c^2} \right) \right]^2 \quad (2)$$

and the vortex charge  $c^2 = 2ps/(2ps + 1)$ .<sup>6</sup> This model describes low-energy excitations within the same CF-LL, and the restricted CF density operators  $\bar{\rho}^{CF}(-\mathbf{q})$  satisfy the commutation relations  $[\bar{\rho}^{CF}(\mathbf{q}), \bar{\rho}^{CF}(\mathbf{k})] = 2i \sin[(\mathbf{q} \times \mathbf{k})_z l_B^{*2}/2] \bar{\rho}^{CF}(\mathbf{q} + \mathbf{k})$ . In contrast to prior investigations, where the interaction potential has been constructed from a few numerically determined pseudopotentials,<sup>9</sup> here, it has been

derived in a set of transformations,<sup>6</sup> in the framework of the Hamiltonian theory of the FQHE.<sup>2</sup> Inter-CF-LL excitations are taken into account with the help of a  $q$ -dependent dielectric function  $\epsilon(q)$  in the CF interaction potential (2).<sup>6</sup>

Because of its similarity with the electronic model, the Hamiltonian (1) may be analysed with the help of the same techniques as the one of electrons restricted to a single level.<sup>8</sup> The energy  $E_{coh}^L(s, p; \tilde{s})$  of the quantum-liquid phases at  $\bar{\nu}^* = 1/(2\tilde{s}+1)$ , where  $\tilde{s}$  is an integer, and  $\bar{\nu}^* = \nu^* - p$  is the partial filling of the  $p$ -th CF-LL, may be calculated in Laughlin's wave function approach.<sup>10</sup> At  $\bar{\nu}^* \neq 1/(2\tilde{s}+1)$ , the energy  $\Delta_{s,p}^{qp/qh}(\tilde{s})$  of the excited quasiparticles [for  $\bar{\nu}^* > 1/(2\tilde{s}+1)$ ] or quasiholes [for  $\bar{\nu}^* < 1/(2\tilde{s}+1)$ ] has to be taken into account. The latter may be interpreted as C<sup>2</sup>Fs or C<sup>2</sup>F-holes excited to a higher level. These excitations raise the energy of the quantum-liquid phases away from  $\bar{\nu}^* = 1/(2\tilde{s}+1)$ , where local minima in form of cusps are obtained. They are at the origin of the incompressibility of the quantum liquids. The energy of CF-solid phases may be calculated in the Hartree-Fock approximation, which has been used in the study of electron-solid phases in higher LLs.<sup>11,12,13</sup> The expressions for the energies of the quantum-liquid and CF solid phases may be found in Ref. 8.

### 3. Results

The results for the energies of the different CF phases are shown in the Fig.1. In contrast to Ref. 8, the energies are calculated with a screened interaction potential. The quantum-liquid (C<sup>2</sup>F) phases are stable around  $\nu^* = 1 + 1/3$  and  $1 + 1/5$ , which correspond to the electronic fillings  $\nu = 4/11$  and  $6/17$ , respectively. Whereas a spin-polarized FQHE at  $\nu = 4/11$  has been observed by Pan *et al.*, only a tiny local minimum in the longitudinal resistance hints to the existence of a possible  $6/17$  state.<sup>3</sup> Indeed, this C<sup>2</sup>F state is close in energy to a CF Wigner crystal ( $M = 1$ ), the energy of which is lowered if one takes into account an underlying impurity potential: a solid phase may take advantage of the minima of such a potential by deformation of its crystalline structure, in analogy with the electronic case.<sup>13</sup> It is therefore not clear whether the quantum liquid at  $\nu = 6/17$  survives in the presence of impurities. At half-filling, one finds a CF stripe phase, which may give rise to an anisotropic longitudinal resistance, which has recently been observed at  $\nu = 13/8$ .<sup>14</sup> While these results are similar to those obtained for the competing electronic phases in the first excited LL,<sup>13</sup> as one may expect from self-similarity arguments, there are differences due to the different form of the CF interaction potential. In  $p = 1$ , the  $1/3$  state, *e.g.*, is more stable than the  $1/5$  state in the CF model, and a two-CF bubble phase ( $M = 1$ ) is unstable (see figure), in contrast to the first excited electronic LL.<sup>13</sup>

Note that the CF Wigner crystal has a lower energy than the quantum liquid above  $\nu \simeq 0.44$ . This transition point is shifted to lower densities if one takes into account impurity effects and the repulsive interaction between the excited C<sup>2</sup>Fs, which leads to non-linear slopes in the quantum-liquid energies at  $\bar{\nu}^* \neq 1/(2\tilde{s}+1)$ .

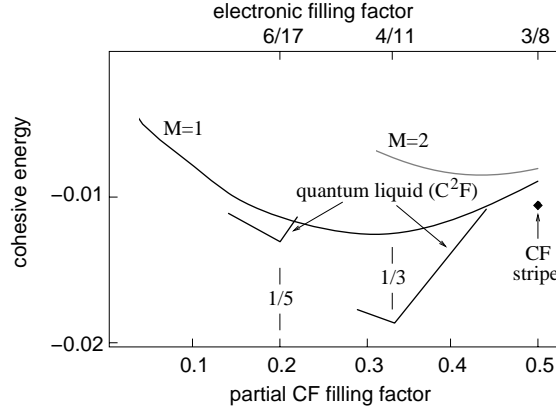


Fig. 1. Cohesive energies of the different CF phases, in units of  $e^2/\epsilon l_B$ .

Impurities favor the CF Wigner crystal also at small values of  $\bar{\nu}^*$ . Therefore, the  $C^2F$  phases are surrounded by insulating CF Wigner crystals, and this may lead to a *reentrance phenomenon of the FQHE*, in analogy with the reentrance of the IQHE in the first excited LL, observed by Eisenstein *et al.*<sup>15</sup>

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### References

1. J. K. Jain, *Phys. Rev. Lett.* **63**, 199 (1989).
2. G. Murthy and R. Shankar, *Rev. Mod. Phys.* **75**, 1101 (2003).
3. W. Pan *et al.*, *Phys. Rev. Lett.* **90**, 016801 (2003).
4. R. G. Mani and K. v. Klitzing, *Z. Phys. B* **100**, 635 (1996).
5. J. H. Smet, *Nature* **422**, 391 (2003).
6. M. O. Goerbig, P. Lederer, and C. Morais Smith, *Phys. Rev.* **B69**, 155324 (2004); preprint: cond-mat/0401340.
7. A. Lopez and E. Fradkin, *Phys. Rev.* **B69**, 155322 (2004).
8. M. O. Goerbig, P. Lederer, and C. Morais Smith, preprint: cond-mat/0403016.
9. S.-Y. Lee *et al.*, *Phys. Rev. Lett.* **87**, 256803 (2001); *Phys. Rev.* **B66**, 085336 (2002).
10. R. B. Laughlin, *Phys. Rev. Lett.* **50**, 1395 (1983).
11. A. A. Koulakov, M. M. Fogler, and B. I. Shklovskii, *Phys. Rev. Lett.* **76**, 499 (1996); M. M. Fogler, A. A. Koulakov, and B. I. Shklovskii, *Phys. Rev.* **B54**, 1853 (1996).
12. R. Moessner and J. T. Chalker, *Phys. Rev.* **B54**, 5006 (1996).
13. M. O. Goerbig, P. Lederer, and C. Morais Smith, *Phys. Rev.* **B69**, 115327 (2004).
14. F. Fischer *et al.*, *Physica* **E22**, 108 (2004);  $\nu = 13/8$  in the upper spin-branch of the lowest LL is the particle-hole symmetric filling of  $\nu = 3/8$ .
15. J. P. Eisenstein *et al.*, *Phys. Rev. Lett.* **88**, 076801 (2002).