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Quantum Phases in Partially Filled Landau Levels

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We compare the energies of different electron solids, such as bubble crystals with triangular and square symmetry and stripe phases, to those of correlated quantum liquids in partially filled intermediate Landau levels. Multiple transitions between these phases when varying the filling of the top-most partially filled Landau level explain the observed reentrance of the integer quantum Hall effect. The transitions between electron-solid and quantum-liquid phases, as well as those between bubble crystals with different number of electrons per lattice site, are first-order. This leads to a variety of measurable phenomena such as the phase coexistence between a Wigner crystal and a two-electron bubble phase in a Landau-level filling-factor range $4.16 \lesssim \nu \lesssim 4.28$, which has recently been investigated in transport measurements under micro-wave irradiation.

Keywords: quantum Hall effect; electronic crystals; phase transitions

1. Introduction

Two-dimensional electrons in a perpendicular magnetic field B have attracted much interest during the last two decades since the discovery of the integer and fractional quantum Hall effects (IQHE and FQHE, respectively). In spite of the similarity between the two effects, their origin is quite different: on the one hand, the IQHE is a manifestation of the energy quantization of electrons (mass m and charge $-e$) in highly degenerate Landau levels (LLs), with a level separation of $\hbar eB/m$. The ratio $\nu = n_{el}/n_B$ between the electronic density n_{el} and the density of states per level, $n_B = B/(h/e)$, determines the filling of the LLs, and the IQHE occurs if

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$\nu = N$, with integral N. Electrons promoted to a higher LL when varying the magnetic field, become localized by impurities and therefore do not contribute to the electronic transport. This leads to a plateau in the Hall resistance, accompanied by a vanishing longitudinal resistance. On the other hand, the FQHE is due to a strongly correlated quantum liquid[1] formed by the electrons in a partially filled LL and occurs at some of the "magical" filling factors $\nu = p/(2ps+1)$ [and at their particle-hole symmetric fillings $\nu = 1 - p/(2ps+1)$], with integral s and p. Also in the first excited LL, fractional quantum Hall states have been observed at $\bar{\nu} = 1/3, 2/3, 1/5$, and $4/5$, where $\bar{\nu} = \nu - N$ denotes the filling of the topmost level.

In higher LLs, the strong Coulomb repulsion between electrons in the partially filled level may lead to phases different from quantum liquids: calculations in the Hartree-Fock approximation have revealed the existence of electron-solid phases, such as stripes around $\bar{\nu} = 1/2$ and bubble crystals with varying electron number per lattice site.[2,3] A stripe phase has indeed been observed in transport measurements, which show a large anisotropy in the longitudinal magneto-resistance around $\nu = 9/2, 11/2, 13/2, \ldots$[4] Eisenstein *et al.* have furthermore measured a non-monotonic behavior of the Hall resistance in the first excited LL $n = 1$:[5] the FQHE at $\bar{\nu} = 1/3$ and $1/5$ is surrounded by insulating electronic phases, which cause an integer quantization of the Hall resistance, as for the neighboring IQHE. This reentrant IQHE is reminiscent of an effect observed before in the second excited LL.[6] We have shown that the effect may be understood in terms of an alternation between quantum-liquid and electron-solid phases when varying the filling of the topmost LL.[7] Here, we furthermore investigate bubble crystals with different symmetry. Whereas the quantum-liquid phases are favored at $\bar{\nu} = 1/(2s+1)$, at $\bar{\nu} \neq 1/(2s+1)$ quasi-particles are excited and raise the energy of the quantum liquids above that of the competing electron solids. The latter are insulating because they are pinned by residual impurities in the host material.

2. Energy Calculation for the Different Phases

In order to describe the low-energy degrees of freedom, which, at non-zero values of the partial filling factor $\bar{\nu}$, consist of intra-LL excitations, we adopt a model of spin-polarized electrons,

$$\hat{H} = \frac{1}{2} \sum_{\mathbf{q}} v_n(q) \bar{\rho}(-\mathbf{q}) \bar{\rho}(\mathbf{q}), \qquad \text{with} \quad v_n(q) = \frac{2\pi e^2}{\epsilon q} \left[F_n(q) \right]^2 , \tag{1}$$

where only the components of the density operator in the n-th LL are taken into account, $\rho_n(\mathbf{q}) = F_n(q) \bar{\rho}(\mathbf{q})$.[a] The LL form factor $F_n(q) = L_n(q^2/2) \exp(-q^2/4)$ is given in terms of Laguerre polynomials $L_n(x)$. The quantum-mechanical properties of the model are revealed by the unusual commutation relations for the projected

[a] We use a system of units, in which the magnetic length $l_B = \sqrt{\hbar/eB} \equiv 1$

density operators, $[\bar{\rho}(\mathbf{q}), \bar{\rho}(\mathbf{k})] = 2i \sin[(\mathbf{q} \times \mathbf{k})_z/2]\, \bar{\rho}(\mathbf{q}+\mathbf{k})$. This model allows for a common description of all LLs.

The electron-solid phases are characterized by an order parameter $\Delta(\mathbf{q})$, which determines the density profile of the phase, given by the local filling factor $\bar{\nu}(\mathbf{r})$. The latter is related to the order parameter by Fourier transformation, $\Delta(\mathbf{q}) \equiv \langle \bar{\rho}(\mathbf{q}) \rangle / n_B A = \int d^2 r \bar{\nu}(\mathbf{r}) \exp(i\mathbf{q}\cdot\mathbf{r})/A$, and the cohesive energy of the electron-solid phases becomes in the Hartree-Fock approximation[2,3,7]

$$E_{coh}^{sol}(n;\bar{\nu}) = \frac{n_B}{2\bar{\nu}} \sum_{\mathbf{q}} u_n^{HF}(q) |\Delta(\mathbf{q})|^2, \tag{2}$$

where the Hartree-Fock potential $u_n^{HF}(\mathbf{q})$ takes into account quantum-mechanical exchange effects.

The bubble crystal with an arbitrary lattice symmetry is characterized by the local filling factor $\bar{\nu}(\mathbf{r}) = \Theta(r_B - |\mathbf{r} - \mathbf{R}_j|)$, where $\Theta(x)$ is the step function, and $\mathbf{R}_j$ are the lattice vectors. The volume of the elementary lattice cell $A_{pc} = 2\pi M/\bar{\nu}$ is determined by the partial filling factor and the bubble radius $r_B = \sqrt{2M}$ containing M electrons. The order parameter of the bubble crystal

$$\Delta_M^B(\mathbf{q}) = \frac{2\pi\sqrt{2M}}{Aq} J_1(q\sqrt{2M}) \sum_j e^{i\mathbf{q}\cdot\mathbf{R}_j}$$

yields the cohesive energy

$$E_{coh}^B(n;M,\bar{\nu}) = \frac{n_B \bar{\nu}}{M} \sum_l u_n^{HF}(\mathbf{G}_l) \frac{J_1^2(\sqrt{2M}|\mathbf{G}_l|)}{|\mathbf{G}_l|^2}, \tag{3}$$

where the lattice symmetry is specified only by the reciprocal lattice vectors $\mathbf{G}_l$. The term $\mathbf{G}_l = 0$ is omitted from the sum because it is canceled by the homogeneous background if one supposes overall charge neutrality of the system.

In the case of stripes with width a oriented parallel to the y-direction, the ansatz $\bar{\nu}(\mathbf{r}) = \Theta(a/2 - |x - x_j|)$ leads to the order parameter,

$$\Delta^S(\mathbf{q}) = \frac{2}{L_x} \delta_{q_y,0} \frac{\sin(q_x \Lambda_S \bar{\nu}/2)}{q_x} \sum_j e^{iq_x j \Lambda_S},$$

where $\Lambda_S = a/\bar{\nu}$ is the stripe periodicity. This yields the cohesive energy

$$E_{coh}^S(n;\Lambda_S,\bar{\nu}) = \frac{n_B}{2\pi^2\bar{\nu}} \sum_{l\neq 0} u_n^{HF}\left(q = \frac{2\pi}{\Lambda_S} l\right) \frac{\sin^2(\pi\bar{\nu} l)}{l^2}, \tag{4}$$

which is to be minimized with respect to the variational parameter Λ_S.

The quantum-liquid phases, which may not be characterized by an order parameter, are well described by Laughlin's wave functions.[1] Their cohesive energy is given in terms of Haldane's pseudopotentials,[8] $V_{2m+1}^n = (2\pi/A) \sum_{\mathbf{q}} v_n(q) L_{2m+1}(q^2) \exp(-q^2/2)$,

$$E_{coh}^{q-l}(n;s,\bar{\nu}) = \frac{\bar{\nu}}{\pi} \sum_{m=0}^{\infty} c_{2m+1}^s V_{2m+1}^n + [\bar{\nu}(2s+1) - 1] \Delta^n(s), \tag{5}$$

where the expansion coefficients c^s_{2m+1} specify the Laughlin wave function. The second term in Eq. (5) takes into account the energies $\Delta^n(s)$ of the excited quasi-particles of charge $1/(2s+1)$ [at $\bar{\nu} > 1/(2s+1)$] and quasi-holes of charge $-1/(2s+1)$ [at $\bar{\nu} < 1/(2s+1)$], in units of the electronic charge. They may be calculated analytically in the Hamiltonian theory of the FQHE, established by Murthy and Shankar.[9]

3. Results

Here, we concentrate on some aspects of the phases in the first and second excited LLs, $n = 1$ and $n = 2$, respectively. A more detailed discussion, including a quantitative study of the role of impurities, may be found in Ref. 7.

Fig. 1(a) shows the energies for different electronic phases in $n = 1$. The quantum-liquid phases are favored around $\bar{\nu} = 1/3$ and $1/5$, whereas in an intermediate range, $0.23 < \bar{\nu} < 0.3$, a Wigner crystal (WC, $M = 1$) has a lower energy. In this range, one therefore observes an integer quantization of the Hall resistance, whereas one finds the FQHE around $\bar{\nu} = 1/3$ and $1/5$.[5] Above $\bar{\nu} \sim 0.38$, the FQHE disappears because the quantum liquid has a higher energy than a two-electron bubble crystal, which competes with a stripe phase. The latter has a lower energy as one approaches half-filling. Experimentally, however, an anisotropic longitudinal resistance, which is the signature of stripe phases,[4] has only been observed in a tilted magnetic field.[10] Our energy calculations suggest that quantum-liquid phases may also be found below $\bar{\nu} = 1/5$ in the absence of impurities. However, the energy of the WC is lowered by impurities, due to the deformation of its lattice structure. This effect is most relevant in the dilute limit of small $\bar{\nu}$, and the FQHE is therefore not stable in this limit,[7] where one observes the IQHE.[5] The energies for the bubble crystals are shown both in the case of a triangular (continuous lines) and a square lattice symmetry (broken lines). The energy difference between these two cases is extremely tiny (on the order of 1%). From classical considerations, one would expect that a triangular lattice has a lower energy than a square lattice.[11] Our energy calculations indicate that this is correct in the low-$\bar{\nu}$ limit, whereas at larger densities a WC with square-lattice symmetry has a lower energy than the triangular one. A similar behavior is found for the two-electron bubble crystal. However, this change of symmetry occurs at filling-factor values, where other phases have a lower energy; the square-lattice symmetry of the WC, *e.g*, is favored only above $\bar{\nu} \sim 0.3$, where quantum-liquid, two-electron bubble, and stripe phases are the ground state. Even if our results thus suggest that square electron crystals are not stable, they may be favored if the host material has exactly this symmetry, such as in $GaAs$ heterostructures.

The energy results for $n = 2$ are shown in Fig. 1(b). In contrast to $n = 1$, a quantum liquid is unstable around $\bar{\nu} = 1/3$, where a two-electron bubble crystal has the lowest energy. Our energy calculations suggest that a FQHE might be found around $\bar{\nu} = 1/5$ or $1/7$. Note, however, that the energies of the quantum-liquid phases are

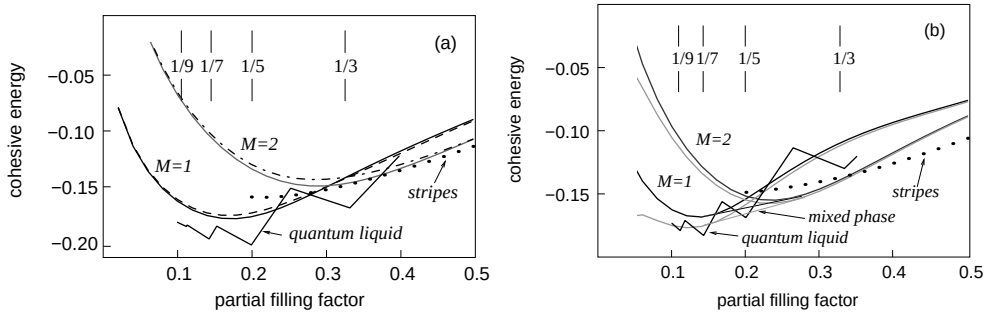

Fig. 1. Cohesive energies of the different phases, in units of $e^2/\epsilon l_B$. (a): phases in $n = 1$. For the bubble phases, both the triangular (continuous lines) and the square crystal (broken lines) are shown. (b): phases in $n = 2$. The gray lines indicate the bubble-crystal energies in the presence of an impurity potential, and the tangents represent a mixed phase.

very close to that of the WC and, in the case of $\bar{\nu} = 1/5$, to a mixed phase of a WC and a two-electron bubble crystal, which is represented by the tangent. It is therefore not clear whether the quantum liquid remains stable in the presence of impurities, which lower the energy of the crystal phases, as shown by the dashed curves. They have been calculated for an impurity strength $V_0/\xi = 0.005e^2/\epsilon l_B^2$, where V_0 is the characteristic energy of a short-range Gaussian potential with correlation length ξ.[7] Experimentally, a small maximum in the longitudinal resistance around $\bar{\nu} = 1/5$ indicates an incipient melting of a crystal phase.[6] This feature has recently been studied in more detail by Gervais *et al.*,[12] who found that this maximum, which decreases when lowering the temperature T, splits into two peaks separated by a small local minimum precisely at $\bar{\nu} = 1/5$ with increasing T. A reminiscent T-dependent effect has been observed in the WC regime in the lowest LL.[13] Even if this effect may indicate a quantum-liquid ground state in extremely pure samples, it may also be understood in different terms: whereas the crystal, which in this scenario remains the $T = 0$ ground state, melts at rather low T (on the order of the energy difference between the WC and the quantum-liquid phase), the quantum coherence of the liquid displaying FQHE features is only destroyed at higher T.[14]

4. Phase Transitions

Our energy calculations suggest that the transitions between the different phases are *first order*. The first-order phase transitions between the quantum-liquid and the insulating bubble crystals may cause a hysteretical behavior in the Hall resistance around the transition points, which, to the knowledge of the authors, has not been reported yet. Also the phase transitions between bubble crystals with different M per site are first order, in agreement with time-dependent Hartree-Fock calculations by Côté *et al.*[15] This leads to a phase coexistence, or a mixed phase, around the

transition points in a filling-factor range, which is described by a tangent on the energy curves, *e.g.* at $0.15 \lesssim \bar{\nu} \lesssim 0.26$ in $n = 2$ [c.f. Fig 1(b)]. Experimentally, there is evidence for such a mixed phase, which is revealed by a double-peak structure in transport measurements under micro-wave irradiation, recently performed by Lewis *et al.*[16]

5. Conclusions

In conclusion, we have performed energy calculations for competing quantum phases in intermediate LLs. An alternation between insulating electron-solid and quantum-liquid phases, which display the FQHE, is at the origin of the observed reentrance of the IQHE in $n = 1$ and $n = 2$.[6,5] The transitions between the different phases are found to be first-order and may lead to a variety of observable phenomena. In the case of transitions between bubble crystals with different electron number per site, a phase coexistence is expected.[7] This scenario is supported by recent micro-wave experiments, in which a double-peak structure has been observed in the longitudinal conductivity in a filling-factor range $0.16 \lesssim \bar{\nu} \lesssim 0.28$,[16] in good agreement our theoretical investigations.[7]

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